the green rays may be necessary to supply the plant with the energy required. A too intense illumination of the leaf, by concentration of the sun's rays upon it, destroys the chlorophyll. According to Pringsheim, this is caused by the chemical rays, but Prof. Stahl considers that the effects of the heat rays have been overlooked, and he insists on this as an important factor in the problem. The variation in the colour of foliage leaves, according to whether they are in the sun or in the shade, is partly due, he thinks, to this danger of overheating. In the special case of the red and brown seaweeds, he considers that the colours are not entirely due to an adaptation to the quality of the light, but also to its intensity.

How far the author's conclusions are justified remains to be seen, but he adduces a considerable amount of evidence in favour of them, which he discusses in a most interesting and suggestive way.

Prof. Stahl suggests that the etiolation, and the yellow coloration of leaves in autumn, may be due to the need of economy in food materials. Willstätter has shown that, in its purest form, green chlorophyll contains C, H, O, N, and Mg. The yellow colouring matters contain only C, H, and O, so that, by keeping back the green chlorophyll in the spring and re-absorbing it in the autumn, a saving would be effected in nitrogen and magnesium, which are of great value to the plant.

Some interesting experiments are described to show that this actually does take place. If leaves which are just on the point of turning yellow, but are still green, are removed from the plant and kept in a damp chamber, they retain their green colour, whilst neighbouring leaves, still attached to the plant, become yellow. So, also, if slits are cut in the leaf, so that the principal veins are severed, the portions of leaf thus cut off from the main conducting vessels remain green, whilst the other parts turn yellow. The results of experiments made by various observers, and others recently made at the author's suggestion, in the agricultural laboratory at Jena, are brought forward to show that potassium and nitrogen, phosphoric acid, iron, chlorine, and silica, are more or less reduced in amount in the yellow as compared with the green leaf. The significance of these facts, which no doubt lend considerable support to Prof. Stahl's interesting hypothesis, is fully discussed, but that the etiolation of young leaves and the yellow coloration of old leaves are so definitely associated with the plant's need for economy cannot, from the evidence before us, be said to be so clearly established as Prof. Stahl seems to think. H. W.

THE FOUNDATIONS OF GEOMETRY.

Grundlagen der Geometrie. By D. Hilbert. Third edition. Pp. vi + 280. (Leipzig and Berlin : B. G. Teubner, 1909.) Price 6 marks.

THIS fascinating work has long since attained the rank of a classic, but attention may be directed to this new edition, which has various additions, mainly bibliographical, and seven supplements, which are reprints of papers by the author on topics related to that of his famous essay. Two of these can be

enjoyed by readers with no exceptional mathematical knowledge. In the one on the equality of the base angles of an isosceles triangle, Dr. Hilbert proves, *inter alia,* the remarkable fact that, even if we assume Euclid's theory of proportion, we cannot prove his propositions on equalities of area, unless we assume the truth of prop. 4, bk. i., of the " Elements " in the wider sense—that is, when one triangle has to be turned over to make it fit the other. It is also pointed out (p. 68) that two tetrahedra can be constructed with equal heights, and bases of equal area, which cannot be cut up into congruent polyhedra, and to which congruent polyhedra cannot be added in such a way that the solids thus produced can be sliced up into congruent parts. Consequently it is impossible to build up a theory of equality of volumes strictly analogous to Euclid's theory of equality of areas.

Another supplement of general interest, and easily understood, is that on the notion of number. The most noticeable thing here is the remark that the commutative law of addition $(a+b=b+a)$ can be deduced from the distributive laws of multiplication, together with the axiom $a.1 = 1.a = a$; thus

$$(a+b)(1+1) = (a+b).1 + (a+b)1 = a+b+a+b$$
$$(a+b)(1+1) = a(1+1) + b(1+1) = a+a+b+b ;$$

therefore $a+b+a+b = a+a+b+b$, and hence $b+a = a+b$.

The seventh supplement, on the foundations of logic and arithmetic, deserves very careful study, both by mathematicians and by philosophers. The main feature of this is that an aggregate is defined as *any* object of thought, and the notion of " element of an aggregate " is a derived one. Dr. Hilbert objects to Dedekind's method in his well-known tract on number, because it postulates the aggregate of " all objects of thought " as a definite conception. A sort of promise is given that the author will expand the ideas of this essay in greater detail, and it is earnestly to be hoped that this intention will be carried out. In connection with these discussions there is one point that deserves attention ; a finite intelligence thinks *in time,* and cannot rid itself of that idea. Now, if we take the statements (1) I am conscious ; (2) I am conscious that I am conscious ; (3) I am conscious that I am conscious that I am conscious ; (1) is the most elementary possible thought from a metaphysical point of view, (2) is the most elementary form of reflection, and if we admit that any thought can be reflected upon, we at once get the natural scale in the form t, tr, tr^2, tr^3, &c. It is not impossible that some such reasoning was in the mind of Rowan Hamilton when he made the statement, which puzzled De Morgan, that " Algebra is the science of Pure Time.'" Until time is defined in terms of simpler entities, it is open to question whether any generation of the natural scale is really more fundamental than the above. Of course, there may be methods which are preferable in the eyes of a mathematician who wishes to avoid metaphysical discussion ; but the fact remains that there is a metaphysical aspect of the question which must be faced before a final answer is reached.

G. B. M.