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Gravitational mass of positron from LEP synchrotron losses

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General relativity (GR) is the current description of gravity in modern physics. One of the cornerstones of GR, as well as Newton's theory of gravity, is the weak equivalence principle (WEP), stating that the trajectory of a freely falling test body is independent of its internal structure and composition. WEP is known to be valid for the normal matter with a high precision. However, due to the rarity of antimatter and weakness of the gravitational forces, the WEP has never been confirmed for antimatter. The current direct bounds on the ratio between the gravitational and inertial masses of the antihydrogen do not rule out a repulsive nature for the antimatter gravity. Here we establish an indirect bound of 0.13% on the difference between the gravitational and inertial masses of the positron (antielectron) from the analysis of synchrotron losses at the Large Electron-Positron collider (LEP). This serves as a confirmation of the conventional gravitational properties of antimatter without common assumptions such as, e.g., coupling of gravity to virtual particles, dynamics of distant astrophysical sources and the nature of absolute gravitational potentials.

Since the formulation of the general theory of relativity (GR) by Einstein^{1,2} in 1915, many efforts have been made to test the theory and no experimental contradiction has been found³. One of the main statements in the foundation of GR (and even Newton's law of universal gravitation) is the equivalence of the gravitational and inertial masses, the so-called weak equivalence principle (WEP). WEP was successfully tested and proven at the 2×10^{-13} level for normal matter³. However, since the theoretical prediction of antimatter by Dirac⁴ in 1928 and its first experimental observation by Anderson⁵ in 1933, there is no conclusive evidence of the gravitational properties of antimatter^{6,7}. This is despite the work with antiparticles becoming a commonplace practice in physics experiments and even medicine⁸.

Direct observation of cold-trapped antihydrogen⁷ by the ALPHA collaboration at CERN sets the limits on the ratio between the gravitational, m_g , and inertial, m , masses of the antihydrogen, $-65 < m_g/m < 110$, including systematic errors, at the 5% significance level. This ratio does not exclude the possibility of, e.g., repulsion of the antihydrogen by Earth (antigravity)! There are many indirect arguments against antigravity⁹. However, for the antimatter, all of them exploit some additional assumptions, such as gravitational properties of virtual particles, physical significance of the absolute values of the gravitational potentials, CPT-invariance, etc. One of the cleanest indirect tests is the comparison of decay parameters of the kaon-antikaon system in the presence of periodic (annual, monthly and diurnal) gravitational potential variations¹⁰. The equality between the kaon and antikaon gravitational masses was established at the 1.8×10^{-9} level¹⁰. However, the gravitational properties of the kaon itself and any other strange matter are not known.

In this article, we constrain possible violations of WEP for antimatter (more precisely, positrons) by analyzing synchrotron losses for 80 GeV positrons at the Large Electron-Positron collider (LEP) at CERN. The advantage of the accelerator experiments is the large relativistic γ -factor. The large γ -factor is known to reveal possible Lorentz-violating effects^{11–13} and to suppress electromagnetic interactions¹⁴, which otherwise overwhelm gravitational forces¹⁵. In addition, the accelerator experiments (in comparison to astrophysical observations) do not require additional and somewhat controversial assumptions on the dynamics of high-energy sources¹⁶. Analysis presented in this article, obviously, does not diminish the importance of the direct methods, since it is by itself indirect and, hence, model-dependent.

Theory in brief

In the absence of gravity, a positron with charge e , velocity \mathbf{v} , energy \mathcal{E} , inertial rest mass m_e and acceleration $\dot{\mathbf{v}}$ perpendicular to \mathbf{v} loses its energy through the synchrotron radiation with the power¹⁷

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$$P = \frac{2}{3} \frac{e^2 \dot{\mathbf{v}}^2}{c^3} \left(\frac{\mathcal{E}}{m_e c^2} \right)^4, \quad (1)$$

where c is the speed of light (in what follows we will work in the natural units, $c = \hbar = 1$). The case $\dot{\mathbf{v}} \perp \mathbf{v}$ corresponds to, e.g., synchrotron radiation in a constant homogeneous magnetic field. The gravitational field of the Earth (Sun or other distant massive celestial objects) around the accelerator can be considered homogeneous and described by an isotropic metric for a static weak field,

$$ds^2 = \mathcal{H}^2 dt^2 - \mathcal{H}^{-2} (dx^2 + dy^2 + dz^2), \quad (2)$$

where $\mathcal{H}^2 = 1 + 2\Phi$, and Φ is the gravitational potential, defining the acceleration of free-falling bodies. For a massive particle (in our case positron) with gravitational mass $m_{e,g}$, one can write the gravitational potential as¹²

$$\Phi_m = \Phi \frac{m_{e,g}}{m_e}, \quad \mathcal{H}_m^2 \equiv 1 + 2\Phi_m, \quad (3)$$

which will modify the dispersion relation of the positron with momentum \mathbf{p} and energy $\mathcal{E} \gg m_e$ and the relation between energy and mass¹²,

$$\mathbf{p}^2 = (1 - 2\kappa)(\mathcal{E}^2 - m_e^2), \quad \mathcal{E} = \frac{m_e \mathcal{H}^{-1} \mathcal{H}_m}{\sqrt{1 - \mathcal{H}^4 \mathcal{H}_m^{-4} v^2}}, \quad (4)$$

where $\kappa = 2\Phi \Delta m_e / m_e$, $\Delta m_e = m_{e,g} - m_e$. We consider no change in the photon dispersion relation due to strong constraints on the variation of the speed of light^{18–20}. Parameter κ here plays a role of an anomalous redshift (or blueshift) and vanishes in the limit $m_{e,g} \rightarrow m_e$. We also assume $|\kappa| \ll 1/\gamma^2$, which is the case at LEP, as will be seen later. This modification can be described by the isotropic version of Standard Model Extension^{21–23} (SME) with $c_{00} = 3c_{ii} = 3\kappa/4$ and other Lorentz-violating parameters set to zero. Eq. (3) is a way to generalize the gravitational coupling of a massive test particle to the background which reproduces the Newton's gravitational law and its relativistic extension. Other modifications of the metric and new accelerator phenomenology emerging from them are to be studied elsewhere. [For a more general form of the background metric, $ds^2 = \Sigma^2 dt^2 - \Xi^{-2} (dx^2 + dy^2 + dz^2)$ and corresponding metric coefficients Σ_m and Ξ_m for a massive particle, the Eqs. (4) become $\mathbf{p}^2 = \left(\frac{\Xi}{\Xi_m} \right)^2 \left[\mathcal{E}^2 \left(\frac{\Sigma}{\Sigma_m} \right)^2 - m_e^2 \right]$ and $\mathcal{E} = m_e \left(\frac{\Sigma_m}{\Sigma} \right) \left[1 - \left(\frac{\Xi \Sigma}{\Xi_m \Sigma_m} \right)^2 v^2 \right]^{-1/2}$. Our analysis should be considered preliminary in the sense that we do not study the cases of WEP violation that cannot be incorporated in the change of the Newton's constant or difference between the gravitational and inertial masses.

In an ultrarelativistic case, $|v| \approx 1$, the ratio \mathcal{E}/m_e from Eq. (4) modifies the synchrotron radiation power (1) by an amount ΔP , such that

$$\Delta P/P = 4\kappa\gamma^2, \quad (5)$$

where $\gamma \equiv 1/\sqrt{1 - v^2}$. This (naive) derivation leads to the same result as a much more rigorous analysis of the synchrotron radiation within the SME^{24,11}. Now, let us imagine that two sets of experimental data “1” and “2” will restrict the values of κ by $|\kappa| < \kappa_{1,2} \equiv |\Delta P/P|_{1,2}/(4\gamma^2)$ at the moments when the gravitational potential acquires values Φ_1 and $\Phi_2 = \Phi_1 + \Delta\Phi$, respectively. The difference $\Delta\Phi$ can be related to the periodic variations in the distances between Earth and other celestial bodies. The experiments are assumed to reproduce the conventional synchrotron radiation power within the uncertainties $(\Delta P/P)_{1,2}$ determining $\kappa_{1,2}$. The restriction on the gravitational mass is then given by

$$\left| \frac{\Delta m_e}{m_e} \right| < \frac{\kappa_1 + \kappa_2}{2|\Delta\Phi|}. \quad (6)$$

On the scale of several months, one can take the change in the solar potential on the Earth's surface as the leading contribution to $\Delta\Phi$. Due to the eccentricity of the Earth's orbit, the distance between Earth and Sun, $d_{SE} \approx 1$ AU (astronomical unit), varies by the amount $\Delta d_{SE} \ll d_{SE}$. This changes the solar potential, $\Phi_\odot = -G_N M_\odot / d_{SE} \approx -9.9 \times 10^{-10}$, by $\Delta\Phi = -\Phi_\odot \Delta d_{SE} / d_{SE}$, where G_N is the Newton's constant and M_\odot is the solar mass. Substituting this into (6), we obtain the relation between the fractional deviation in the masses and the fractional uncertainty in the measured synchrotron radiation power in two experiments,

$$\left| \frac{\Delta m_e}{m_e} \right| < \frac{|\Delta P/P|_1 + |\Delta P/P|_2}{8\gamma^2 |\Phi_\odot \Delta d_{SE} [\text{AU}]|}, \quad (7)$$

where we also assumed the γ -factor to be the same in both experiments.

Analysis of the experiment

In order to apply the formula (7), we consider an analysis of the LEP Energy Working Group²⁵ for the LEP 2 programme in the last few years of LEP operation. The primary physics motivation for the LEP 2 programme

Fill number	Date of the fill	d_{SE} [AU]	$E^{Q_s} - E^{NMR}$ [MeV]	ΔE [MeV]
6114	13 Aug 1999	1.013	-4	30
6338	15 Sep 1999	1.006	10	30
8315	29 Aug 2000	1.010	-10	62
8445	10 Sep 2000	1.007	-52	38
8809	18 Oct 2000	0.996	-43	52

Table 1. Variations in the distance between Earth and Sun during some of the $E = 80$ GeV LEP runs together with the total error in the energy determination assigned by the LEP Energy Working Group²⁵. The distances are obtained from the NASA's Jet Propulsion Laboratory (JPL) solar system data³⁰.

was precise determination of the W boson mass, $M_W \approx 80.4$ GeV. For this purpose, the relative error of the centre-of-mass energy for the accelerated electrons and positrons was reduced to $\Delta E_{CM}/E_{CM} = 1.2 \times 10^{-4}$ for the majority of the LEP 2 runs²⁵. There were three complementary approaches used by the group: precise measurement of the bending magnetic field by nuclear magnetic resonance (NMR) probes and the flux-loop²⁵; spectrometry, i.e. beam deflection by a precisely known magnetic field in a bending magnet; and analysis of the synchrotron tune as a function of the beam energy, energy loss per turn and the accelerating radiofrequency (RF) voltage. A full description of the methods and results is given in the Report²⁵; the LEP machine description can be found in the technical design reports^{26–28}.

The first two methods would not be affected by the Lorentz-violation, since the magnetic field modification is ruled out by the atomic clock experiments²⁹. The third method is based on the measurement of the synchrotron tune, Q_s , which is defined as a ratio of the frequency of longitudinal fluctuations in the beam (due to the synchrotron radiation loss and boost from the accelerating RF system) to the revolution frequency. The fitting formula for Q_s used for the determination of the beam energy E was²⁵

$$Q_s^4 = \left(\frac{\alpha_c h}{2\pi} \right)^2 \left\{ \frac{g^2 e^2 V_{RF}^2}{E^2} + M g^4 V_{RF}^4 - \frac{\tilde{U}^2}{E^2} \right\}, \quad (8)$$

where α_c is the momentum compaction factor (measure for the change in the orbit length with momentum), h is the harmonic number of the accelerator (ratio between the RF frequency and the revolution frequency), V_{RF} is the amplitude of RF voltage, $g \approx 1$ is a fitting parameter whose role is to account for uncertainties in the accelerating RF system, $M \sim 10^{-7}$ is a parameter accounting for the fact that the RF voltage is not distributed homogeneously around the accelerator but in four straight sections near to even numbered access points²⁸ (parameter is obtained from simulations) and \tilde{U} is the total energy loss per turn (synchrotron radiation in dipole magnets, quadrupole magnets, parasitic mode losses, etc., either modeled or measured directly). The total 1σ error of this method, ΔE , as well as the difference between the fitted energy ($E = 80$ GeV) and the NMR model, $E^{Q_s} - E^{NMR}$, for several LEP runs are shown in Table 1.

Following Altschul¹¹, we notice that since E was more accurately known from the other complementary methods, one can reinterpret Eq. (8) as a fit to \tilde{U} (which we, for conservative estimates, consider to be dominated by the radiation in dipoles and, therefore, proportional to P). Treating E as an exact quantity, the previously estimated relative error for E becomes a relative error for \tilde{U} and, hence, for P .

Results

Choosing two sets of measurements (fills 6114 and 6338) and taking into account the difference between NMR and Q_s values for energies, we obtain $\left| \Delta P/P \right|_{1,2} < 9 \times 10^{-4}$ as a 2σ bound for both measurements, which with use of Eq. (7) finally translates to

$$\left| \frac{\Delta m_e}{m_e} \right| < 1.3 \times 10^{-3}, \quad (9)$$

i.e. a 0.13% limit on a possible deviation from WEP. This significantly improves our previous 4% limit¹² coming from the absence of the vacuum Cherenkov radiation and photon stability in accelerator experiments. Since the energy calibration at LEP was performed for both electrons and positrons, the same limit is applied for the electrons. Also, note that $|\kappa_{1,2}| < 9.2 \times 10^{-15} \ll 1/\gamma^2 = 4.1 \times 10^{-11}$, so the previously made assumption is justified.

The weak equivalence principle was introduced in some form already in the 17th century by Newton in his *Principia*. It was further employed by Einstein in the 1910's. Furthermore, the antimatter was discovered in the 1930's. However, only now, with the advances in accelerator physics, are we finally capable to draw certain conclusions on how to bring together the concepts of gravity and antimatter.

References

1. Einstein, A. The Field Equations of Gravitation. *Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.)* **1915**, 844–847 (1915).
2. Einstein, A. The Foundation of the General Theory of Relativity. *Annalen Phys.* **49**, 769–822 (1916).
3. Will, C. M. The Confrontation between General Relativity and Experiment. *Living Rev. Rel.* **17**, 4 (2014).
4. Dirac, P. A. The Quantum theory of electron. *Proc. Roy. Soc. Lond.* **A117**, 610–624 (1928).

5. Anderson, C. The Positive Electron. *Phys. Rev.* **43**, 491–494 (1933).
6. Fischler, M., Lykken, J. & Roberts, T. Direct observation limits on antimatter gravitation (2008). arXiv:0808.3929. Available online, <http://arxiv.org/pdf/0808.3929.pdf>. Date of access: 20/07/2015.
7. Amole, C. *et al.* Description and first application of a new technique to measure the gravitational mass of antihydrogen. *Nature Commun.* **4**, 1785 (2013).
8. Positron Emission Tomography. *A Guide for Clinicians* (ed. Das, B. K. E.). (Springer, India, Private Ltd, 2014).
9. Nieto, M. & Goldman, J. T. I The Arguments against 'antigravity' and the gravitational acceleration of antimatter. *Phys. Rept.* **205**, 221–281 (1991).
10. Apostolakis, A. *et al.* Tests of the equivalence principle with neutral kaons. *Phys. Lett.* **B452**, 425–433 (1999).
11. Altschul, B. Bounding Isotropic Lorentz Violation Using Synchrotron Losses at LEP. *Phys. Rev.* **D80**, 091901 (2009).
12. Kalaydzhyan, T. Gravitational mass of relativistic matter and antimatter, *Phys. Lett. B* **751**, 29 (2015).
13. Kalaydzhyan, T. Testing general relativity on accelerators, *Phys. Lett. B* **750**, 112 (2015).
14. Chao, A. *Physics of collective beam instabilities in high-energy accelerators*. (Wiley-VCH, 1993).
15. Witteborn, F. & Fairbank, W. Experiments to determine the force of gravity on single electrons and positrons. *Nature* **220**, 436–440 (1968).
16. Pakvasa, S. Simmons, W. A. & Weiler, T. J. Test of Equivalence Principle for Neutrinos and Anti-neutrinos. *Phys. Rev.* **D39**, 1761 (1989).
17. Jackson, J. *Classical Electrodynamics*. (Wiley, 1998).
18. Ackermann, M. *et al.* A limit on the variation of the speed of light arising from quantum gravity effects. *Nature* **462**, 331–334 (2009).
19. Biller, S. *et al.* Limits to quantum gravity effects from observations of TeV flares in active galaxies. *Phys. Rev. Lett.* **83**, 2108–2111 (1999).
20. Schaefer, B. E. Severe limits on variations of the speed of light with frequency. *Phys. Rev. Lett.* **82**, 4964–4966 (1999).
21. Colladay, D. & Kostelecky, V. A. CPT violation and the standard model. *Phys. Rev.* **D55**, 6760–6774 (1997).
22. Colladay, D. & Kostelecky, V. A. Lorentz violating extension of the standard model. *Phys. Rev.* **D58**, 116002 (1998).
23. Kostelecky, V. A. Gravity, Lorentz violation, and the standard model. *Phys. Rev.* **D69**, 105009 (2004).
24. Altschul, B. Lorentz violation and synchrotron radiation. *Phys. Rev.* **D72**, 085003 (2005).
25. Assmann, R. *et al.* Calibration of centre-of-mass energies at LEP 2 for a precise measurement of the W boson mass. *Eur. Phys. J.* **C39**, 253–292 (2005).
26. LEP Design Report, Volume 1: The LEP Injector Chain. *Preprints: CERN-LEP-TH-83-29, CERN-PS-DL-83-31, CERN-SPS-83-26, LAL-RT-83-09*. Available online, <https://cds.cern.ch/record/98881>. Date of access: 20/07/2015 (1983).
27. LEP Design Report: Vol. 2. The LEP Main Ring. *Preprint: CERN-LEP-84-01*. Available online, <http://cds.cern.ch/record/102083>. Date of access: 20/07/2015 (1984).
28. Altarelli, Guido, Sjöstrand, T. & Zwirner, F. (eds) Physics at LEP2, v.1. *Preprint: CERN-AC/96-01*. Available online, <https://cdsweb.cern.ch/record/300671>. Date of access: 20/07/2015 (1996).
29. Mattingly, D. Modern tests of Lorentz invariance. *Living Rev. Rel.* **8**, 5 (2005).
30. Jet propulsion laboratory horizons on-line solar system data. <http://ssd.jpl.nasa.gov/?horizons>. Date of access: 20/07/2015.

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