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Graphene plasmonic lens for manipulating energy flow

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SUBJECT AREAS:

NANOPHOTONICS AND
PLASMONICSOPTICAL MATERIALS AND
STRUCTURESReceived
6 November 2013Accepted
27 January 2014Published
12 February 2014Correspondence and
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Manipulating the energy flow of light is at the heart of modern information and communication technologies. Because photons are uncharged, it is still difficult to effectively control them by electrical means. Here, we propose a graphene plasmonic (GP) lens to efficiently manipulate energy flow by elaborately designing the thickness of the dielectric spacer beneath the graphene sheet. Different from traditional metal-based lenses, the proposed graphene plasmonic lens possesses the advantages of tunability and excellent confinement of surface plasmons. It is found that the proposed lens can be utilized to focus and collimate the GP waves propagating along the graphene sheet. Particularly, the lens is dispersionless over a wide frequency range and the performance of lens can be flexibly tuned by adjusting the bias voltage. As an application of such a lens, the image transfer of two point sources with a separation of $\lambda_0/30$ is demonstrated.

Surface plasmon polaritons (SPPs) are electromagnetic surface waves resulting from the coupling of the electromagnetic field to the collective plasma excitations of metal¹. Due to the ability to overcome the diffraction limit of light in microchip-sized devices, SPPs are considered as one of the most promising candidates for nanophotonic components^{2,3}. In recent years, varieties of plasmonic devices with special functionalities are proposed and investigated^{4–8}. In these researches, the noble metals, such as silver and gold, have been popular materials to support SPP waves in the infrared and visible regimes. However, the noble metals possess enormous loss in the frequency regimes of interest. Especially, it is difficult to vary the permittivity functions of the noble metals^{9,10}. These insurmountable obstacles severely limit the propagation length of SPP waves and the tunability of the metallic plasmonic devices.

Graphene, which is a single sheet of carbon atoms in a hexagonal lattice, has drawn much attention for its exceptional electrical and optical properties¹¹. The complex conductivity ($\sigma_g = \sigma_{g,r} + i\sigma_{g,i}$) of graphene is determined by Kubo's formula, which expresses the dependence on the radian frequency ω , charged particle scattering rate Γ , temperature T , and chemical potential μ_c ^{12,13}. Comparing with noble metals, the most advantage of graphene for plasmonic applications is the capability to tune surface conductivity by gate voltage, electric field, magnetic field, and/or chemical doping¹⁴. In addition, the effective mode index for the TM-polarized GP waves along the graphene sheet can reach to ~ 100 in the mid-infrared regime¹⁵. As a result, the GP waves can be tightly confined to the graphene sheet, with effective wavelength (λ_{GP}) much smaller than free space wavelength (λ_0). Meanwhile, the imaginary part of the wavenumber is relatively small, which means that the damping loss of GP waves is relatively low and the propagation length could be dozens of λ_{GP} ¹⁶. Due to the above advantages, graphene has been proved to be a suitable alternative material to noble metals. Recently, a lot of GP devices have been proposed and investigated^{17–25}. For example, Engheta *et al.* reported that by designing and manipulating spatially inhomogeneous conductivity patterns across a flake of graphene, one can treat graphene as a one-atom-thick platform for infrared metamaterials¹⁵. The inhomogeneous conductivity can be obtained by applying different gate voltages or by designing the thickness of dielectric spacer underneath the graphene sheet. Wang *et al.* studied the optical coupling of GPs between graphene sheets. They employed the coupling of GPs to design the optical splitters, spatial switches, and Mach-Zehnder interferometers¹⁶. Perruisseau-Carrier *et al.* proposed a terahertz plasmonic antenna by using a graphene stack. The GP antenna possesses the advantages of high input impedance and easy fabrication²⁶. All these works have proved that graphene possesses excellent confinement of SPPs and high tunability, which makes it an ideal material to realize various plasmonic devices working from the terahertz to the visible regimes¹⁷.

The ability to manipulate optical fields and the energy flow of light is at the heart of modern information and communication technologies, as well as quantum information processing schemes²⁷. However, because photons are uncharged, it is still elusive to effectively control them by electrical means²⁷. In recent years, many metal-based plasmonic devices for subwavelength light management have been proposed theoretically and demonstrated

Figure 1 | (a) Schematic diagram of the Selfoc lens. h represents the thickness of dielectric spacer. (b) Lateral profile of effective mode index and a representative case of transforming plane source to be point source in graphene (output intensity is shown in top figure).

experimentally^{28–30}. For example, Liu *et al.* numerically demonstrated the efficiently manipulating SPPs based on transformation optics in visible regime²⁸. They showed that the propagation of SPPs could be flexibly controlled by carefully tuning the dielectric material properties adjacent to a metal. Zentgraf *et al.* experimentally demonstrated plasmonic Luneburg lens and Eaton lens by tailoring the topology of a dielectric layer adjacent to the metal surface²⁹. They proved that the optical properties in these lenses are changed gradually and thus the scattering could be significantly reduced. Different from the above metal-based plasmonic lenses, we propose a graphene-based self-focus (Selfoc) lens for manipulating the energy flow of GP waves in infrared (IR) regime. By applying external gate voltage, the proposed Selfoc lens can be used to focus and collimate GP waves that propagate along the graphene. In addition, we investigate the dispersion of the proposed Selfoc lens. It is found that such a lens is dispersionless over a wide frequency range. Finally, to illustrate the application of the proposed lens, we demonstrate that the image transfer of two point sources separated by a distance of $\lambda_0/30$ (λ_0 is the incident wavelength in vacuum) can be realized on the graphene. The proposed graphene-based Selfoc lens can be flexibly tuned by controlling the external bias voltage and possesses high confinement

of SPPs, which paves the way for effectively manipulating energy flow of light at nanoscale.

Results

Structure model and analytical theory. As shown in Fig. 1, the proposed Selfoc lens consists of a monolayer graphene with chemical potential μ_c on top of a heavily doped silicon (Si) substrate separated by a dielectric spacer. The dielectric can be set as silicon dioxide with the relative permittivity of $\epsilon_{r2} = 3.9$ ^{31,32}, and the thickness of the spacer h changes gradually from the position of $z = 0$ to periphery along the $\pm z$ -axis directions. The surrounding medium is assumed to be air and GP waves are excited and propagate along the $+x$ -axis direction. A bias voltage is applied between the graphene sheet and doped Si substrate to change the doping of graphene by the electric-field effect^{10,18}.

We start our study by analyzing the dispersion relation for GPs propagating along a monolayer graphene. In our analysis, graphene is treated as an ultra-thin metallic slab with a thickness of Δ . It should be noted that Δ is not the real thickness of graphene (~ 0.33 nm) and it is introduced only for numerical computation purpose^{16,25}. This slab can be characterized by an equivalent permittivity of $\epsilon_{g,eq}$ which

Figure 2 | Evolution of effective mode indices of GP waves as a function of the dielectric spacer thickness with $V_g = 20$ V (a), and the bias voltage with $h = 200$ nm (b) for different frequencies. In calculations, the temperature T is 300 K and relaxation time of graphene sheet is 0.5 ps.

Figure 3 | Lateral profile of the effective mode index n_{GP} (green line) and the thickness of the dielectric spacer h (blue line). In calculations, the bias voltage is 20 V and the incident frequency is 40 THz.

is given by¹⁶

$$\varepsilon_{g,eq} = 1 + i\sigma_g\eta_0/(k_0\Delta), \quad (1)$$

where σ_g represents the conductivity of graphene, $\eta_0 \approx 377 \Omega$ is the impedance of air, and $k_0 = 2\pi/\lambda_0$ is the wavenumber in vacuum. With Maxwell's equation, the dispersion of TM-polarized guided waves in graphene can be expressed as¹⁴

$$\frac{\varepsilon_{r1}}{\sqrt{k_{GP}^2 - (\varepsilon_{r1}\omega^2/c^2)}} + \frac{\varepsilon_{r2}}{\sqrt{k_{GP}^2 - (\varepsilon_{r2}\omega^2/c^2)}} + \frac{i\sigma_g}{\omega\varepsilon_0} = 0, \quad (2)$$

where k_{GP} is the wavenumber of the guided modes, ε_{r1} and ε_{r2} are the dielectric permittivities of air and spacer, respectively.

Within the random-phase approximation, the dynamic optical response of graphene can be derived from the Kubo's formula consisting

of the interband and intraband contributions: $\sigma_g = \sigma_{inter} + \sigma_{intra}$ ³³. In the THz and infrared ranges, the intraband transition of electrons dominates. On condition that $\mu_c \gg k_B T$, where k_B is the Boltzmann's constant, the surface conductivity of graphene can be simplified to a Drude-like form^{25,34}

$$\sigma_g = \frac{ie^2\mu_c}{\pi\hbar^2(\omega + i\tau^{-1})}, \quad (3)$$

here τ is the momentum relaxation time, e and $\hbar$ are the electron charge and reduced Planck's constant, respectively. The carrier relaxation time τ determined by the carrier mobility μ in graphene as $\tau = \mu\mu_c/(eV_f^2)$ ³⁵. Recently, it has been reported that the carrier mobility of $\mu = 8000 \text{ cm}^2/(\text{V}\cdot\text{s})$ of graphene can be obtained by mechanical cleavage of bulk graphite and then transferred to SiO_2/Si substrate³⁶, and $\mu = 230000 \text{ cm}^2/(\text{V}\cdot\text{s})$ can be experimentally achieved in high-quality suspended graphene³⁷. When the chemical potential μ_c is 0.15 eV with a certain gate voltage³⁸, the above carrier mobilities correspond to the relaxation times of $\tau = 0.12 \text{ ps}$ and $\tau = 3.45 \text{ ps}$, respectively. Here, we choose $\tau = 0.5 \text{ ps}$. This value follows the ballistic transport features of graphene, whose mean free path was measured to be up to 500 nm at room temperature¹⁶. It should be noted that the carrier relaxation time τ can affect the imaginary part of the effective mode index of the graphene, which may result in the distortion of images and the deteriorated focusing effect. In practice, therefore, the carrier mobility and relaxation time should be appropriately tuned to reduce the distortion of images and the deteriorated focusing effect. In our analysis, $\mu_c = \hbar V_f \sqrt{\pi n_s}$, where $V_f = 10^6 \text{ m/s}$ is the Fermi velocity and n_s is the sheet doping of graphene³². In the proposed Selfoc lens, n_s can be controlled using an external bias voltage following

$$C_{ox} V_g = en_s, \quad (4)$$

where $C_{ox} = \varepsilon_{r2}\varepsilon_0/h$ is the gate capacitance, and V_g is the external bias voltage³². Based on the above equations, the effective mode indices n_{GP} ($=k_{GP}/k_0$) of GP waves for different thicknesses of dielectric spacer and bias voltages can be obtained. As shown in Fig. 2, it is found that n_{GP} can be adjusted by tuning the h and V_g . In the following discussions, we mainly employ the changing of h to control the effective mode index n_{GP} and manipulate the propagation of GP waves in

Figure 4 | Simulation results of the Selfoc lens, showing the amplitude of the y component of electric field (E_y) of GP waves. The plane GP waves are excited and propagate parallel (a) and obliquely (b) to the x -axis direction. Point source is illuminated at the position of $z = 0$ (c) and $z = 150 \text{ nm}$ (d). In the calculations, the frequency of the incident light is 40 THz and lateral profile of the effective mode index of graphene is the same as that in Fig. 3.

Figure 5 | (a) Lateral profiles of the effective mode index n_{GP} as a function of the incident frequencies. (b) Slope of the lateral profiles of n_{GP} (dn_{GP}/dz) for different incident frequencies. (c) Derivative of n_{GP} with respect to incident frequency (dn_{GP}/df). (d) Parameter ζ_{fre} as a function of the incident frequencies. In the calculations, we set $V_g = 20$ V, $\tau = 0.5$ ps, $T = 300$ K and the thickness of dielectric spacer is the same as that shown in Fig. 3.

graphene. The influences of bias voltage V_g on the performance of the proposed Selfoc lens are discussed in the Supplementary Materials.

It is well-known that a Selfoc lens, which is a graded-index (GRIN) lens, behaves essentially like a convex lens. It can transform a cylindrical wave to a plane wave³⁹. In general, the refractive index is highest in the optical axis and decreases with transverse distance from the axis. The following formula closely describes the refractive index of a GRIN lens^{39,40},

$$n(z) = n_0 \left(1 - \frac{1}{2} g^2 z^2 \right). \quad (5)$$

Here, $g = 2\sqrt{2\Delta n}/w$ is the gradient constant and $\Delta n = (n_0 - n_1)/n_0$ is the relative change in refractive index, n_0 and n_1 are the refractive indices at the positions of the optical axis (i.e., $z = 0$) and edge (i.e., $z = \pm \frac{1}{2}w$) of the graphene. When the effective mode index of the graphene n_{GP} is designed and changes following Eq. (5), the graphene can act as a Selfoc lens. As shown in Fig. 2(a), one can obtain n_{GP} varies following Eq. (5) by elaborately designing the thickness of the dielectric spacer. Here, the refractive indices n_0 and n_1 are chosen as 87.5 and 69, respectively. According to Eqs. (2)–(5), the lateral

profiles of $n(z)$ and $h(z)$ are shown in Fig. 3. It can be seen that the parabolic characteristic of h makes the index profile vary continuously from the optical axis to the periphery along the transverse direction. So when the GP waves are excited and propagate along the graphene sheet, they follow a cosinoidal path along the $+x$ -axis direction as shown in Fig. 1(b). The period of the cosinoidal path (“pitch” of the lens) is given by $P = 2\pi/g$, which is only determined by the gradient constant g (see Methods Section).

Numerical results. To demonstrate the performance of the Selfoc lens, we employ finite-difference time-domain (FDTD) method to calculate the field distributions for different kinds of incident sources. In the calculations, graphene is treated as an ultra-thin metallic film with the thickness of $\Delta = 1$ nm^{16,25,41}. The minimum mesh size equals 0.2 nm in the FDTD calculations, which is small enough to ensure the convergence of the results²⁵. For simplicity, in the simulation, graphene sheet is free-standing in vacuum with the spatially inhomogeneous conductivity patterns. Figures 4(a) and 4(c) demonstrate the transformation between the plane source and point source. When a plane (point) source is illuminated to the graphene sheet along the $+x$ -axis direction, the excited GP waves can be transformed to be a point (plane) source. As depicted in Fig. 4(b),

Figure 6 | Simulation results of the Selfoc lens, showing the amplitude of the y component of electric field (E_y). The incident frequencies are 35 THz (a)–(b) and 45 THz (c)–(d), respectively. In calculations, the lateral profile of h is the same as that in Fig. 3.

when the plane source propagates obliquely along $+x$ -axis direction, it can be focused and transformed to be a point source that locates out of the optical axis. When the incident source is point source at the position of $z = 150$ nm, it is evolved into an obliquely transmitted plane source at the exit of the lens shown in Fig. 4(d). In the calculations, the width of the lens is chosen as $w = 800$ nm and the gradient constant can be calculated as $g = 0.001626$. So the pitch of the lens is $P = 3864.2$ nm. Considering that the proposed Selfoc lens is a “ $P/4$ ” lens, the focal length is $d' = P/4 = 966.1$ nm. In Fig. 4, the focal length is $d_0 = 963.2$ nm, which agrees well with the analytical results. So we can conclude that the proposed structure can act as a Selfoc lens and can be employed to manipulate the GP waves that propagate along the graphene sheet. Moreover, this Selfoc lens can realize the transformation optics, focusing and collimating light. Especially, when the proposed GP lens is used as an optical coupler, the full-width at half-maximum (FWHM) of the output light can be flexibly tuned by changing the external bias voltage. Thus, the GP waves can be coupled into other subwavelength optical devices with different dimensions (see Supplementary Materials). It should be noted that in the proposed Selfoc lens, the effective mode index ranges from 69 to 87.5 for the incident frequency of $f = 40$ THz ($\lambda = 7.5$ μm). Thus, the wavelength of GP wave is $\lambda_{\text{spp}} = \lambda/69 = 108.7$ nm. It can be seen that the proposed graphene-based lens possesses excellent SPP confinement characteristics. Considering that the thickness of the dielectric spacer changes from 217 nm to 350 nm, the loss of the substrate can be neglected in the simulations because the excited GP wave is highly confined in the graphene.

The effective mode index of GP waves differs for varied incident frequencies, which means that graphene is a quite dispersive material. Therefore, it is necessary to analyze the performance of the proposed Selfoc lens for different incident frequencies. In Fig. 5(a), we calculate the lateral profiles of the effective mode index as a function of the incident frequencies while maintaining the dielectric spacer thickness unchanged. It can be seen that the effective mode index increases with the increase of the incident frequency, which confirms the dispersive characteristic of graphene. In addition, the slope of lateral profiles of effective mode index (dn_{GP}/dz) for different incident frequencies is calculated and shown in Fig. 5(b). For higher incident frequencies, the slope changes severely, indicating that the lateral profile of n_{GP} becomes steep and the radius of lateral profile

curvature decreases. In Fig. 5(c), we calculate the derivative dn_{GP}/df at different lateral positions. For a specific lateral position, the derivative is a constant, which implies that n_{GP} changes linearly with the incident frequency. We define a new parameter $\zeta_{\text{fre}} = n_{\text{GP}}(f, z)/n_{\text{GP}}(f_0, z)$ to describe the variation of n_{GP} at different positions, where $n_{\text{GP}}(f, z)$ is the effective mode index at the position of z for the incident frequency of f . Here, $f_0 (= 40$ THz) is the incident frequency which determines the original lateral profile of h in Fig. 3. Figure 5(d) shows the parameter ζ_{fre} for different incident frequencies. It is found that for a specific incident frequency, the parameter ζ_{fre} keeps unchanged at different lateral positions, which denotes that the relative change in effective mode index $\Delta n (= (n_0 - n_1)/n_0)$ is also unchanged. We can also conclude that the lateral profile of n_{GP} for different frequencies always has the form of Eq. (5), besides n_0 and n_1 have new values. Thus, though the incident frequency differs, the pitch of the proposed Selfoc lens is still determined by $P = 2\pi/g$ with $g = 2\sqrt{2\Delta n}/w$. Due to Δn retains constant for different incident frequencies, the pitches P and the focal length $d_0 (= P/4)$ are constant and the Selfoc lens is dispersionless. When the proposed Selfoc lens is designed and the dielectric spacer thickness h is fixed, the Selfoc lens can always focus light at the same position in the graphene sheet. This is because the increase of the incident frequency has two effects on the Selfoc lens: the increase of n_{GP} (as shown in Fig. 5(a)) and the steeper lateral profile of n_{GP} (as shown in Fig. 5(b)). The increase of n_{GP} results in the increase of the Selfoc lens pitch (see the Supplementary Materials), meanwhile the steeper lateral profile of n_{GP} decreases the Selfoc lens pitch³⁹. When these two effects are complementary, the pitch of the proposed Selfoc lens keeps unchanged and the GP lens is dispersionless. As a result of this important feature, the GP lens designed for $f_0 = 40$ THz also works perfect well for other incident frequencies.

To validate the above analyses, the field distributions are performed for different incident frequencies by using FDTD method. As shown in Figs. 6 (a) and 6(c), when the frequencies of the incident point source are chosen as 35 THz and 45 THz, the excited GP waves can be transformed to be plane sources. Moreover, when plane sources with the frequencies of 35 THz and 45 THz are illuminated, the GP waves can be focused and transformed to be point sources as presented in Figs. 6(b) and 6(d). It can be concluded that even though

Figure 7 | Image transfer in the graphene-based Selfoc lens. (a) Lateral profile of the input intensity of GP waves. (b) y component of electric field (E_y) of GP waves: the image of two point sources separated by a distance of $\lambda_0/30$ ($=250$ nm) is transferred through the Selfoc lens. (c) Lateral profile of the output intensity of GP waves. In calculations, the incident frequency of the two point sources is 40 THz.

the Selfoc lens is designed for 40 THz, it can still work well for other incident frequencies. In addition, the focal lengths for the incident frequencies of 35 THz and 45 THz are $d_1 = 963.1$ nm and $d_2 = 963.5$ nm, respectively. While for the frequency of 40 THz, the focal length is $d_0 = 963.2$ nm. It is found that the focal lengths for these three incident frequencies are nearly unchanged, which confirms that the proposed Selfoc lens is dispersionless. This important characteristic ensures that the Selfoc lens possesses a wide operating frequency regime, which is necessary in practical applications. It should be noted that the pitch of graphene-based Selfoc lens is also a constant for different external voltages, which is discussed in the Supplementary Materials.

As an important application, we demonstrate the image transfer in the graphene-based Selfoc lens. As depicted in Fig. 7 (b), two incident point sources are separated by a distance of $\lambda_0/30$ ($=250$ nm) and the excited guided GP waves can propagate along the graphene. The intensity of GP waves at the output interface of the image transfer system is presented in Fig. 7(c). Comparing with the input intensity of GP waves in Fig. 7(a), the image of the two point sources can be well resolved at the output interface with a separation of 250.4 nm. It is found that the separation between the output intensity peaks only has 0.16% change. Especially, the separation between input intensity peaks is only $\lambda_0/30$, which implies that such graphene-based lens can overcome the diffraction limit and realize image transfer on a monolayer graphene without distortion.

Comparing with the proposed graphene-based Selfoc lens, the employment of metal to realize the similar function is hardly possible. To get the same effective mode index as the graphene, the dimension

of the metallic devices has to be several micrometers⁴¹. The proposed graphene plasmonic lens possesses ultra-compact footprint of $1.05 \times 0.85 \mu\text{m}^2$, which is more suitable for ultra-compact optoelectronic devices. In addition, the characteristic of the output GP waves can be easily tuned by applying an external bias voltage (see the Supplementary Material). In contrast, it is difficult to tailor the characteristics of metallic plasmonic devices. Thus, such a graphene-based Selfoc lens is more suitable to manipulate energy flow of light at nanoscale and resolve subwavelength features.

Discussion

We have proposed and investigated the GP lens for effectively manipulating energy flow. The effective mode index of GP waves can be easily tuned by changing the dielectric spacer thickness or the external bias voltage. Based on this principle, the graphene-based Selfoc lens is elaborately designed and FDTD simulation is performed to characterize the optical properties of the Selfoc lens. It is found that the proposed Selfoc lens can be used to collimate and concentrate GP waves at subwavelength scale. We also demonstrate that the proposed Selfoc lens is dispersionless over a wide frequency range. As an important application of the graphene-based Selfoc lens, we demonstrate that the image of two point sources separated by a distance of $\lambda_0/25$ can be well resolved on the graphene. Comparing with metallic plasmonic devices, such a graphene-based Selfoc lens possesses ultra-compact dimension and flexible tunability, and hence is more suitable for manipulating energy flow at nanoscale and resolving subwavelength features.

Methods

Light propagation in GRIN media. In the proposed structure, the different substrate thicknesses result in the spatially inhomogeneous effective mode index across a flake of the graphene. The index profile varies continuously from the optical axis to the periphery along the transverse direction. In our studies, the mode index is designed and varies quadratically with the distance in the transverse direction and the graphene is regarded as Selfoc lens^{39,42}.

According to the Fermat's principle, the light equation for the light propagating in GRIN media can be expressed as³⁹,

$$\frac{d}{ds} \left(n \frac{d\vec{r}}{ds} \right) = \vec{\nabla} n, \quad (6)$$

where ds is a differential arc length along the ray trajectory, $\vec{r}$ is the position vector and $\vec{\nabla} n$ is the gradient of n . In the xz -plane, with considering the paraxial light, $ds \approx dx$ and Eq. (6) is:

$$n \frac{d^2 z}{dx^2} = \frac{\partial n}{\partial z} - \frac{\partial n}{\partial x} \frac{dz}{dx}. \quad (7)$$

The refractive index of the medium varies in the z -axis direction satisfies Eq. (5) and $\frac{1}{2} g^2 z^2 \ll 1$, thus Eq. (7) can be simplified as:

Figure 8 | Path of light propagating in GRIN media when the incident light is plane source (a) and point source (b).

$$\frac{d^2 z}{dx^2} = -g^2 z, \quad (8)$$

the general solution of Eq. (8) is

$$z = B \cos(gx) + C \sin(gx), \quad (9)$$

where B and C are constants determined by the initial conditions. According to Eq. (9), the slope of light line can be derived as,

$$K = \frac{dz}{dx} = -Bg \sin(gx) + Cg \cos(gx). \quad (10)$$

Here, the initial conditions are assumed as (K_0, z_0) , which means that the slope and position of the incident light is K_0 and z_0 , respectively. By solving Eqs. (9) and (10), the constants B and C are calculated as $B = z_0$, $C = K_0/g$, then Eq. (9) can be obtained as,

$$z = z_0 \cos(gx) + (K_0/g) \sin(gx). \quad (11)$$

When the incident light at the position of z_0 is plane wave parallel to the optical axis (i.e., the initial conditions are $K_0 = 0$ and $z = z_0$), according to Eq. (11), we can get the light equation

$$z = z_0 \cos(gx), \quad (12)$$

which means that the light propagating in GRIN media follows a cosinoidal path and the period (also called “pitch”) of the Selfoc lens is $P = 2\pi/g$.

As the examples, the path of the light propagating in GRIN media is shown in Fig. 8. It can be seen that when the proposed Selfoc lens is used as an optical coupler, the length of the lens should be $P/4$. When it is used to realize the image transfer, the length of the lens should be $P/2$.

When the effective mode index has an increment of Δn , by solving the above equations, the pitch of the Selfoc lens can be easily obtained as,

$$P' = \frac{2\pi}{g} \sqrt{\frac{n + \Delta n}{n_0}}. \quad (13)$$

It can be seen that the pitch of the Selfoc lens increases with the increase of effective mode index.

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Acknowledgments

This work was supported by the National Natural Science Foundation of China under Grants 10874239, 10604066, 61223007, and 11204368. We are grateful to Dr. Bingli Zhu, Dr. Jian Liang, Dr. Kai Liu and Dr. Zhaolu Wang for helpful suggestions.

Author contributions

G.W. conceived the idea, completed the numerical simulation, and wrote the manuscript text. X.L. discussed the design of the proposed structure and simulation results, and supervised the whole project. H.L. improved the manuscript presentation. C.Z. carried out the data analysis and prepared part figures. All authors discussed the results and substantially contributed to the manuscript.

Additional information

Supplementary information accompanies this paper at <http://www.nature.com/scientificreports>

Competing financial interests: The authors declare no competing financial interests.

How to cite this article: Wang, G.X., Liu, X.M., Lu, H. & Zeng, C. Graphene plasmonic lens for manipulating energy flow. *Sci. Rep.* **4**, 4073; DOI:10.1038/srep04073 (2014).

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