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Energy dissipation and error probability in fault-tolerant binary switching

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The potential energy profile of an ideal binary switch is a symmetric double well. Switching between the wells without energy dissipation requires time-modulating the height of the potential barrier separating the wells and tilting the profile towards the desired well *at the precise juncture* when the barrier disappears. This, however, demands perfect timing synchronization and is therefore fault-intolerant even in the absence of noise. A fault-tolerant strategy that requires no time modulation of the barrier (and hence no timing synchronization) switches by tilting the profile by an amount at least equal to the barrier height and dissipates at least that amount of energy in abrupt switching. Here, we present a third strategy that requires a time modulated barrier but *no timing synchronization*. It is therefore fault-tolerant, error-free in the absence of thermal noise, and yet it dissipates arbitrarily small energy in a noise-free environment since an arbitrarily small tilt is required for slow switching. This case is exemplified with stress induced switching of a shape-anisotropic single-domain soft nanomagnet dipole-coupled to a hard magnet. When thermal noise is present, we show analytically that the minimum energy dissipated to switch in this scheme is $\sim 2kT\ln(1/p)$ [p = switching error probability].

The fundamental limits of energy dissipation in computing^{1–7} are best understood by exploring the minimal energy dissipated to toggle a binary switch from one state to the other. The potential energy profile of such a (bistable) switch is a *symmetric* double well as shown in the far left sketch of Fig. 1(a), with the two degenerate minima corresponding to the two stable states. One well-known scheme for switching between the two states entails modulating the potential barrier height between the wells periodically in time. As the barrier is gradually eroded, the symmetric double well profile is first tilted towards the initial state to keep its potential energy constant. Just when the barrier is completely eroded and the well becomes monostable, it is translated horizontally in state space. The barrier then re-emerges on the opposite side of the well and therefore the system switches^{1,2}. This scheme results in vanishing dissipation because the system never acquires kinetic energy and is ideally error-free in the absence of thermal noise. However, it also requires perfect timing synchronization between barrier modulation and the translation in order to switch without error. That makes it fault-intolerant even in the absence of thermal noise.

Another scheme³ that is dissipative (even in the absence of noise) but fault-tolerant is shown in Fig. 1(b). Here, the potential barrier is never modulated and hence no synchronization between two events is required. Whenever switching is desired, the potential profile is simply tilted abruptly towards the desired well in such a way that the tilt is at least equal to the maximum barrier height. This ensures that the system will definitely switch to the desired well in the absence of noise (error-free switching), but the system now gains kinetic energy equal to the amount of tilt, which is dissipated when the system relaxes to the new ground state (desired well). Of course the energy dissipation could have been made to approach zero with adiabatic switching carried out by tilting the potential profile gradually in numerous infinitesimally small steps¹⁶, but we do not consider that here because of the issues mentioned in ref. 3. Clearly, error-resilience has been purchased with dissipation – a trade-off that is well-known in the context of the Fredkin billiard ball model of computation⁷.

In this article, we propose a new scheme that captures the best of both worlds. The height of the potential barrier separating the wells is modulated in time, but an arbitrarily small tilt towards the desired final state is maintained in the potential profile *all the time* as shown in Fig. 1(c). At the instant when the barrier disappears, the system automatically switches to the final state, which is the minimum energy state, as long as there is no noise to disrupt the switching. The switching may take very long (and the barrier modulation may have to be very slow) when the tilt is vanishingly small, but it occurs with 100% probability (error-free) and ideally dissipates an amount of energy equal to the degree of tilt, which approaches zero. Thus, *error-free* and *fault-tolerant* binary switching can take place with *vanishingly small energy dissipation* in a noise-free environment, which makes this scheme unique. At

Figure 1 | Switching strategies involving. (a) Infinitely precise synchronization but near zero energy dissipation². (b) No synchronization needed but energy dissipation greater than or equal to energy barrier³. (c) The new strategy proposed: no synchronization and arbitrarily small energy dissipation at 0 K.

non-zero temperatures T , when thermal noise is present, switching will occur with some probability p ($p < 1$). We will show analytically that the minimum energy E_d dissipated in switching approaches $\sim 2kT \ln(1/p)$. The advantage of this (third) scheme over the first (non-dissipative but fault-intolerant) scheme is *error resilience and fault tolerance without energy dissipation at $T \rightarrow 0$ K*, and the advantage over the second scheme (fault-tolerant but dissipates at least the barrier energy) is the much lower energy-dissipation ($E_d \ll E_{\text{barrier}}$) without any additional error vulnerability at $T \rightarrow 0$ K.

The scheme that we propose here is also more practical to implement in certain scenarios, such as bit propagation in nanomagnetic logic (NML) built with nanomagnetic binary switches that are dipole-coupled to their nearest neighbors in long chains⁸. Such chains consist of a linear array of nanomagnets where the first nanomagnet's state is propagated through all ensuing magnets⁹ by Bennett clocking⁴. Thus, the first magnet's state is set *before* the energy barrier separating the two stable states of the second magnet is modulated. Hence, the “tilt” determined by the dipolar influence of the first magnet on the second is fixed and cannot be varied synchronously with the raising/lowering of the barrier in the second magnet. Therefore, neither the first, nor the second switching scheme, is suitable for NML, but the third is.

Results

We study the energy dissipation and error probability in the third scheme using a model nanomagnetic system both in the absence ($T = 0$ K) and presence ($T = 4.2$ K, 77 K and 300 K) of thermal noise. We need three ingredients for this purpose: (1) A bistable switch with a symmetric double well potential profile; (2) A method to introduce a tilt in the symmetric potential profile of the switch; and (3) A method to modulate the height of the potential barrier separating the two energy minima (the global minima and the local minima in the presence of the tilt) which will allow the system initially in the local minimum to switch to the global minimum.

Bistable switch. The bistable switch is implemented with a single-domain shape-anisotropic magnetostrictive nanomagnet shown in Fig. 2 (a). It is delineated on top of a piezoelectric layer, forming a 2-phase multiferroic where the magnetostrictive and piezoelectric layers are elastically coupled. The magnetostrictive nanomagnet is shaped like an elliptical cylinder. In the relaxed (unstrained) condition, and in the absence of dipole coupling with any other magnet, the magnetization vector of this magnet will point (with equal likelihood) in one of the two directions along the major axis of the ellipse, which is the easy axis of magnetization. The two-dimensional potential energy profile of such a magnet is a symmetric double well. In other words, if we assume the magnetization vector to always lie in the plane of the magnet (x-y plane in Fig. 2(a)) and plot the potential energy as a function of the angle ϕ subtended by the magnetization vector with the minor axis of the ellipse (see Fig. 2(a)), then it will be a symmetric double well whose two degenerate minima are located at $\phi = \pm 90^\circ$. Therefore, this system replicates the ideal binary switch.

The tilt. If the magnet is now placed close to an elliptical hard magnet that has a fixed magnetization orientation, then dipole coupling between the two will break the symmetry of the potential profile and introduce a slight “tilt”, making one energy minimum slightly lower than the other. The lower minimum (i.e. the global minimum) will correspond to anti-parallel ordering (i.e. the two magnets will have mutually anti-parallel magnetizations) provided the line joining the centers of the magnets is parallel to the (common) minor axis of the ellipses. The potential profile of the magnet in the presence of dipole coupling is shown in Figs. 2(d) and 2(e) [solid lines] for two different strengths of the coupling. Note that the tilt increases with increasing strength of dipole coupling (decreasing separation between the magnets). Thus, dipole coupling offers the means to cause the permanent tilt in the potential profile of the binary switch.

Figure 2 | (a) An elliptical multiferroic nanomagnet consisting of a piezoelectric layer in intimate contact with a magnetostrictive layer. (b) A 2-magnet system comprising a hard magnet with large shape anisotropy and a soft multiferroic magnet with smaller shape anisotropy whose shape anisotropy energy barrier is modulated with stress. (c) Energy dissipated in flipping (switching) the magnetization of the second nanomagnet as a function of the center-to-center separation between the two magnets (R). (d) The energy profile of the multiferroic nanomagnet discussed in the text in the relaxed (unstressed) state and the critically stressed ($\sigma = \sigma_c$) state for large dipole coupling, $R = 150$ nm. (e) The energy profile of the multiferroic nanomagnet discussed in the text in the relaxed (unstressed) state and the critically stressed state ($\sigma = \sigma_c$) for small dipole coupling, $R = 400$ nm.

Method to depress the energy barrier between the global and local energy minima. Application of a voltage across the piezoelectric layer generates strain in it via the d_{31} coupling, which is transferred to the magnetostrictive layer. This will generate uniaxial mechanical stress in the soft magnet along the major axis if mechanical clamps are placed on the magnet's sides to prevent expansion/contraction along the minor axis. This uniaxial stress will depress the energy barrier between the global and local minima provided the product of the magnet's magnetostrictive constant and stress (compressive stress is negative and tensile positive) is negative¹⁰ [see, also, Equation (S2) of the supplementary material]. Therefore, this dipole-coupled multiferroic nanomagnet has all the ingredients to illustrate the switching scheme.

We will evaluate the energy dissipation and error probability associated with this switching scheme under the following scenario: Consider the dipole-coupled magnet pair and assume that initially the hard magnet was magnetized “up” and the soft magnet was magnetized “down” so that the system was in the ground state (antiparallel ordering). Next, an external agent flips the magnetization of

the hard magnet down, thus temporarily making the ordering parallel as shown in Fig. 2(b). This is an excited state for the soft magnet which is now stuck in the local minimum of its potential profile. One would expect that eventually the soft magnet will spontaneously flip up to reach the global energy minimum state (in the process dissipating some energy), but this *cannot* happen with reasonable probability because the soft magnet has to overcome the energy barrier between the local and global minima, which is several times larger than kT . Thermionic emission over the barrier and tunneling through it are both impossible in a reasonable time. However, an applied uniaxial stress along the easy axis can depress this energy barrier and make the switching possible. *This is an exact implementation of the switching scheme shown in Fig. 1(c).* All we have to do is study the probability of switching under these conditions and estimate the associated energy dissipation.

Model formulation. Switching of the soft magnet from the local to the global energy minimum under stress and in the presence of dipole coupling is studied in the manner of Ref. 11–13 (based on

the solution of the Landau-Lifshitz-Gilbert equation). The magnetization dynamics of a single domain nanomagnet under the influence of an effective field $\vec{H}_{eff}$ can be described by the Landau-Lifshitz-Gilbert (LLG) equation:

$$\frac{d\vec{M}(t)}{dt} = -\frac{\nu}{1+\alpha^2} \vec{M}(t) \times \vec{H}_{eff}(t) - \frac{\alpha\nu}{M_s(1+\alpha^2)} [\vec{M}(t) \times (\vec{M}(t) \times \vec{H}_{eff}(t))], \quad (1)$$

where $\vec{H}_{eff}$ is the effective magnetic field on the nanomagnet, which is the partial derivative of its total potential energy (U) with respect to its magnetization ($\vec{M}$), ν is the gyromagnetic ratio (a universal constant), M_s is the saturation magnetization of the magnetostrictive layer and α is the Gilbert damping factor (a material constant) associated with internal dissipation in the magnet when its magnetization rotates. The effective magnetic field is given by

$$\vec{H}_{eff}(t) = -\frac{1}{\mu_0\Omega} \frac{\partial U(t)}{\partial \vec{M}(t)} = -\frac{1}{\mu_0 M_s \Omega} \nabla_{\vec{M}} U(t), \quad (2)$$

where Ω is the volume of any nanomagnet and $U(t)$ is the total potential energy of the soft magnet, which can be written as

$$U(t) = E_{shape-anisotropy}(t) + E_{stress-anisotropy}(t) + E_{dipole}(t). \quad (3)$$

Here E_{dipole} is the dipole-dipole interaction energy due to dipole coupling with the hard magnet. This term is evaluated via the standard expression for two magnetic dipoles separated with a center-to-center distance of R as shown in Fig. 2(b). A detailed description of the formulation and expression for the energy terms are given in Section A of the supplementary material accompanying this article.

While the $\vec{H}_{eff}(t)$ evaluated from Equations (2) and (3) is adequate to describe the magnetization dynamics at $T = 0$ K, we will add an additional contribution due to thermal noise at $T > 0$ K¹⁷. The procedure for finding this random field at a given temperature is described in in Ref. 17 and is summarized below.

The field due to thermal noise is:

$$\vec{h}(t) = h_x(t) \hat{e}_x + h_y(t) \hat{e}_y + h_z(t) \hat{e}_z, \quad (4)$$

where each component $h_x(t)$, $h_y(t)$ and $h_z(t)$ is assumed to be isotropic, uncorrelated with other components, and described by a Gaussian distribution given by:

$$h_i(t) = \sqrt{2\alpha kT/\gamma\mu_0 M_s \Omega(\Delta t)} * G(0,1). \quad (5)$$

Here, T = temperature, k = Boltzmann constant and Δt is related to the inverse of the attempt frequency for thermal torque to rotate the magnetization and $G(0,1)$ is a Gaussian distribution with zero mean and unit standard deviation.

If we replace $\vec{H}_{eff}(t)$ in Equation (1) with $\vec{H}_{eff}(t) + \vec{h}(t)$, the resulting equation is called the stochastic Landau-Lifshitz-Gilbert equation (s-LLG) that governs the magnetization dynamics in the presence of random thermal noise. This equation yields the switching error probabilities as described in the following.

In order to find how the magnetization of the soft magnet evolves in time at $T > 0$ K under the influence of torques caused by its own shape anisotropy, dipole coupling, stress, and thermal noise, we carry out 100,000 simulations of switching trajectories governed by the s-LLG equation. The initial magnetization orientation for each trajectory is picked from a thermal Boltzmann distribution around the easy axis parallel to the orientation of the hard magnet ($\theta = 90^\circ$, $\phi = -90^\circ$), which is the initial state of the soft magnet as shown in Fig. 2(b). This distribution is found first by solving the s-LLG equation in the absence of stress for several trajectories (simulations are carried out for a very long time and the final orientation of the

magnetization vector is noted for each trajectory to determine the distribution). We then start the simulation of the switching trajectory in accordance with the s-LLG equation and sample the magnetization states at fixed time intervals to determine the temporal evolution of the magnetization vector. This yields the magnetization dynamics in the presence of noise. If the magnetization vector ends up close to the initial orientation (within 5°), we terminate the simulation of the trajectory and conclude that switching did not occur (failure), whereas if the magnetization ends up close (within 5°) to the orientation anti-parallel to the magnetization of the hard magnet ($\theta = 90^\circ$, $\phi = 90^\circ$), we again terminate the simulation and conclude that switching took place successfully. The probability of failure (error-probability p) is the fraction of the 100,000 switching trajectories that results in switching failure. The duration of the simulation is the switching time τ for the corresponding trajectory.

The energy dissipated during switching can be calculated as

$$E_d(\tau) = \int_0^\tau -\left(\frac{dE_d(t)}{dt}\right) dt = -\mu_0 \int_\Omega \vec{H}_{eff}(t) \cdot \frac{d\vec{M}(t)}{dt} d\Omega \quad (6)$$

$$= \int_0^\tau \frac{\alpha\mu_0\nu\Omega}{(1+\alpha^2)M_s} |\vec{H}_{eff}(t) \times \vec{M}(t)|^2 dt.$$

We also calculate the difference in the dipole coupling energy between the parallel and anti-parallel configurations of the two magnets, which is the value of the “tilt”, for different separations between the magnets. An expression for this quantity (E_{tilt}) is given in Equation (S4) of Appendix B of the supplementary material. In Fig. 3, we plot the tilt energy and the energy dissipated in switching (calculated from Equation (6) at $T = 0$ K) as a function of the separation between the magnets. The two curves are indistinguishable, showing that the *energy dissipation is equal to the tilt* ($E_d(\tau) = E_{tilt}$), as we expected. Although this equality may not strictly hold at $T > 0$ K, we tacitly assume its validity at all temperatures and always set the energy dissipation equal to the tilt energy.

Switching in the absence of thermal noise. We ignore thermal noise (assuming $T = 0$ K) and solve Equation (1) to obtain the time evolution of the magnetization vector [$\vec{M}(t)$ vs. t]. Switching is deemed to be completed when the magnetization vector reaches within 5° of the orientation anti-parallel to the magnetization of the hard magnet. We obtain the time-evolution (or magnetization dynamics) for various values of applied stress and compute the associated energy dissipation. Switching is always successful in this case ($p = 0$) since there is no thermal noise to disrupt switching.

The critical stress σ_c is the value of stress at which the energy barrier separating the local and global minima of the soft magnet’s potential profile is completely eroded. An expression for this critical stress is given in Equation (S3) in Section B of the supplementary material. For the chosen magnet dimensions (see Section C of the supplementary material), the shape-anisotropy energy barrier (the energy barrier separating the two degenerate energy minima in the absence of stress and dipole coupling) is 0.83 eV and $\sigma_c = 3.145$ MPa. The critical stress can be induced by a voltage of only ~ 15 mV across the piezoelectric layer.

In Figs. 2(d) and 2(e) we show the potential energy profiles under critical stress (note that the broken lines [critical stress] and solid lines [no stress] touch at the location of the erstwhile energy barrier peak, the local minimum has become the global maximum and the global minimum remains the global minimum. Note also that the potential has become monostable and not bistable under critical stress). We assume that the critical stress is applied abruptly to instantaneously lower the barrier, and we solve Equation (1) to track $\vec{M}(t)$ in time until switching is completed and the magnetization vector assumes an orientation nearly anti-parallel to that of hard

Figure 3 | Comparison of the energy dissipated and the tilt energy (which is the difference between the dipole coupling energies in the parallel and anti-parallel configurations of the two magnets) as a function of the separation between the two magnets at $T = 0$ K. The two curves are indistinguishable showing that these two energies are roughly equal.

magnet's magnetization [steady-state is reached in about 100 ns for the weakest dipole coupling considered (600 nm center-to-center separation between magnets) that results in dipole coupling energy of 0.01 eV]. We evaluate the energy dissipation during the switching process from Equation (6). The energy dissipation is plotted as a function of center-to-center separation R between the two magnets (or the dipole coupling strength) in Fig. 2(c). Note that the dissipation asymptotically approaches zero, indicating that fault-tolerant and error-free switching with vanishing dissipation is possible at 0 K when no thermal noise is present.

We also calculate the difference in the dipole coupling energy between the parallel and anti-parallel configurations of the two magnets, which is the value of the “tilt”, for different separations between the magnets. An expression for this quantity is given in Equation (S4) of Appendix B of the supplementary material. In Fig. 3, we plot the tilt energy and the energy dissipated in switching as a function of the separation between the magnets. The two curves are indistinguishable, showing that the energy dissipation is equal to the tilt, as we expected.

We point out that it is critical to apply no more than the critical stress σ_c so that the barrier is just removed. The reason for this is that the barrier needs to be “eroded”, but not “inverted”. Excess stress ($\sigma > \sigma_c$) will invert the barrier, creating a potential well where the peak of the potential barrier was (at $\theta = 90^\circ$, $\phi = 0^\circ$), which will then become the global energy minimum. If this happens, the potential profile of the magnet becomes monostable, but with the stable state (energy minimum) at the wrong location which will make the magnetization vector settle along the hard axis (minor axis of the ellipse) and not switch to the correct orientation along the easy axis (major axis). Fortunately, there is no “criticality” associated with this, i.e. we do not need extreme accuracy in stress generation. If we overshoot and generate excess stress, then as we time-modulate the stress, it passes through the critical value at some point in time. At that point, switching to the correct state will occur if the stress modulation is so slow that the system persists in the critical stress state much longer than the switching time. However, this process may entail some additional energy dissipation due to the excess stress creating an energy minimum at the hard axis lower than the desired energy minimum at the easy axis, which is unnecessary. If we want to avoid this excess dissipation, we do have to control the stress amplitude

precisely, but it is far easier to do that than to control timing synchronization accurately. We also point out that generating and time-modulating the stress do not have to dissipate any energy since the voltage over the piezoelectric that causes the stress can be produced with a resonant resistor-capacitor-inductor circuit that dissipates no energy.

Switching in the presence of thermal noise. Next, we will consider switching with critical stress in the presence of thermal noise and derive an analytical relationship between the energy dissipated E_d and the error probability p under the following assumptions:

- (i) We will still assume that the energy dissipated is equal to the tilt energy ($E_d = E_{\text{tilt}}$). This was verified for the $T = 0$ K case and assumed to hold for $T \neq 0$ K case.
- (ii) We will also assume that the E_{dipole} vs. ϕ relationship is linear when critical stress is applied. The dipole coupling energy (E_{dipole}) actually contains a $\sin(\phi)$ term (see Section A of the supplementary material), so that the linear assumption is not valid for large angles ϕ , but without this assumption, we cannot derive an analytical relation between E_d and p .
- (iii) We apply the critical stress for a long enough duration that an equilibrium (Boltzmann) distribution of the magnetization orientation is attained prior to withdrawal of stress. We consider the distribution in ϕ -space only, and not in (θ, ϕ) -space, since we assume the magnetization vector to be constrained to the plane of the magnet under equilibrium.
- (iv) We also assume that when stress is finally withdrawn (barrier restored), it is withdrawn abruptly, so the magnetizations that were in the $[-\pi/2, 0]$ interval of ϕ would eventually settle in the “down” state, while those in the $[0, \pi/2]$ interval would finally settle in the “up” state. Extensive stochastic-LLG analysis has shown that dynamic effects during restoration of the barrier typically make the error probability p larger than what would be estimated from this assumption. Nevertheless, this picture gives an estimate of the “minimum” energy that must be dissipated to limit p to a certain value. Thus, the utility of this analysis is to estimate a lower bound for the energy dissipation associated with a certain error probability. This turns out to be larger than the Landauer limit of $kT \ln(2)$ ¹ attainable only with extremely complex time modulations of the barrier.

Case I: analytical model. Assuming that the energy of the down state ($\phi = -90^\circ$) is E_1 and that of the up state ($\phi = +90^\circ$) is E_2 , ($E_1 > E_2$) and further assuming a linear relation between the tilt energy $E_d = E_1 - E_2$ and ϕ , as shown in Fig. 4(a), one can write the probability distribution function in ϕ -space (assuming Boltzmann statistics) as:

$$\rho(\phi) = A e^{\frac{-E_{\text{dipole}}}{kT}} \approx A e^{\frac{-(E_1 - E_2)(\frac{\pi}{2} - \phi)}{\pi kT}} = A e^{\frac{-E_d(\frac{\pi}{2} - \phi)}{\pi kT}} \quad (7)$$

where $A = \frac{E_d}{kT} \left(\frac{1}{e^{E_d/kT} - 1} \right)$.

The probability that the magnetization is oriented between $\phi = -90^\circ$ and 0° , just before the barrier is raised, is the error probability p and can be found as:

$$p = \int_{-\pi/2}^0 \rho(\phi) d\phi = \frac{e^{\frac{E_d}{2kT}} - 1}{e^{\frac{E_d}{kT}} - 1} \approx e^{\frac{-E_d}{2kT}} \text{ for } E_d \geq 4kT. \quad (8)$$

Equation (9) can be recast as:

$$E_d \approx 2kT \ln(1/p) \text{ for } E_d \geq 4kT. \quad (9)$$

This gives the lower bound on the energy dissipation in this switching scheme.

Some of the trade-offs between dissipation and error probability were discussed in Refs. 14, 15, but without deriving any analytical

expression for the energy dissipated as a function of error probability by looking at magnetization distributions in ϕ -space. We also differ from Ref. 16 as we do not allow any energy recovery scheme. Note that Equation (9) is counter-intuitive. Intuitively, one would expect that if Boltzmann statistics holds, then the relative probability of being in state E_1 (wrong state) with respect to E_2 (correct state) would be $e^{\frac{-(E_1-E_2)}{kT}}$ and hence the static error probability would be $e^{\frac{-(E_1-E_2)}{kT}}$ which would result in $E_d = E_1 - E_2 = E_{\text{tilt}} = kT \ln(1/p_{\text{static}})$. Equating this with Equation (10), we get that $p = \sqrt{p_{\text{static}}}$ (and not $p = p_{\text{static}}$) when we switch with critical stress. Thus, the dynamic error probability p is greater than the static error probability p_{static} .

Case II: semi-analytical model. We account for the sinusoidal dependence of dipole coupling energy on ϕ , instead of tacitly assuming a linear dependence. The probability distribution is now $\rho(\phi) = Ae^{\frac{(E_1-E_2)\sin(\phi)}{2kT}}$ and has to be numerically integrated to find A and p unlike CASE I where an analytical result could be derived. When $\log(1/p)$ is plotted against $E_d = E_{\text{tilt}}$ (see Fig. 4 (b)) it turns out to be linear and can be approximated by an analytical result for $E_d > 0.1$ eV (or 4 kT at room temperature)

$$\Delta E_d / \Delta \log(1/p) = 1.9kT \quad \text{for } E_d \geq 4kT. \quad (10)$$

This estimate is slightly less conservative than Equation (9) since the sinusoidal energy profile results in higher energy for the undesirable orientation (and less energy for the desired orientation) than a linear energy profile.

Case III: stochastic LLG simulations in the presence of thermal noise. Finally, we study the energy dissipation vs. dynamic error probability using stochastic LLG simulation. The LLG approach is based on the macro-spin model where the magnet is assumed to be single-domain and coherent rotation of the magnetization is assumed⁸. There are slight differences between the LLG results and results from micro-magnetic simulations (which allow for incoherent rotation) when switching is carried out with an external magnetic field, but these differences are too slight to be perceptible. Therefore, we tacitly assume the macro-spin model to be valid.

We run extensive stochastic LLG analysis in the presence of thermal noise in the manner of Ref. 17. The results of this simulation are summarized in Fig. 4 (b) where the E_d vs. p is compared to the analytical estimate.

Figure 4 | Energy dissipated vs. minimum dynamic switching error. (a) Schematic that shows the effect of dipole coupling (tilt/asymmetry) and stress on the dynamic switching error. Blue thick line: approximate energy profile under critical stress when the barrier is just eroded but not inverted, red dotted line: corresponding probability distribution function. Clearly, higher dipole coupling (tilt) will produce a more favorable probability distribution at the expense of dissipating more energy. (b) Energy dissipated vs. dynamic switching error at various temperatures. The analytical bound is compared to estimates from stochastic Landau-Lifshitz-Gilbert simulations of switching trajectories. (c) Schematic that shows the effect of out-of-plane excursion of the magnetization vector on the switching probability. In this particular case, out-of-plane orientation above the plane aids the switching and below the plane impedes switching.

The stochastic LLG results show a higher error probability p for a given energy dissipation than what the quasi-analytical model predicts. The disagreement increases at higher E_d (or lower p). This disagreement is due to the out-of-plane excursion of the magnetization vector during switching (as shown in Fig. 4 (c)) that was completely ignored in the quasi-analytical model. If the out-of-plane motion lifts the magnetization vector above the plane of the magnet, the out-of-plane component generates an additional precessional torque $\vec{M}_{\text{out-of-plane}}(t) \times \vec{H}_{\text{eff}}(t)$ that helps to rotate the magnetization vector in the correct direction and aids switching (thereby decreasing p). On the other hand, if the out-of-plane motion dips the magnetization vector below the plane of the magnet, then the resulting precessional torque tends to rotate the magnetization vector in the wrong direction and impedes switching (thereby increasing p). This effect is completely absent in the quasi-analytical model and is the source of the disagreement between the two models.

At a given temperature, increasing dipole coupling strength initially helps at low E_d (low tilt) but increasing the tilt does not continue to decrease error probability since the effect of the out-of-plane motion gradually takes over and dominates. As the temperature increases, thermal kicks generated by noise steer the magnetization vector farther out-of-plane and hence the disagreement between the stochastic-LLG result and the quasi-analytical result increases.

Discussion

In this article, we have shown that in the absence of thermal fluctuations, we can switch a binary switch with 100% probability and arbitrarily small energy dissipation without requiring accurate timing synchronization in modulating the barrier. We may need some precision in barrier height control (precisely applying the critical stress) but that is far less demanding than accurate time synchronization. When thermal fluctuations are present, there is a trade-off between energy dissipated and the dynamic switching error probability. For the special case of switching a multiferroic magnet with critical stress, we have derived an analytical relationship between the minimum energy dissipated and the dynamic switching probability. This turns out to be larger than the Landauer bound of $kT \ln(2)^1$, but the latter can only be achieved with exquisite engineering design involving extremely complex modulations of the barrier and precise timing sequences.

Methods

The LLG (Landau-Lifshitz-Gilbert) differential equation (1) was solved using Euler method with time step of 1 ps. For thermal effects the stochastic magnetization distribution was modeled by adding an effective field due to random thermal noise (see supplement for details) during each 1 ps step of the LLG simulation. Thus, the magnetization evolved differently each time and 100,000 simulations of the magnetization trajectory were carried out to evaluate the switching error.

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Author contributions

J.A. and S.B. conceived the idea, M.S.F. ran all the simulations. All authors contributed to discussion and interpretation of results and writing the paper.

Additional information

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Corrigendum: Energy dissipation and error probability in fault-tolerant binary switching

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In the original paper our stochastic simulations had a systematic error that lead to an incorrect error probability in Figure 4 (b). The corrected version of this figure is Fig. 1 of this corrigendum. The actual error probability turns out to be smaller than what we had reported earlier. However, the new plots convey the same physics and overall message; hence, the key message of this paper and our conclusions remain unchanged.

We plot the simulation results for the switching error probability vs. dipole energy for the 4.2, 77 and 300 K cases up to the point where the dipole coupling (tilt in profile) is still low enough that the error probability exceeds $\sim 10^{-6}$ and 10 million simulations suffice to capture the statistics accurately. (NOTE: Errors of 10^{-5} and 10^{-6} approximately correspond to 11.5 and 13.8 respectively on the y-axis). The leveling off of switching error probability with increasing dipole energy at 300 K is still observed, albeit at a much smaller switching error probability value. The error probabilities up to which we perform simulations are too high for the leveling off behavior to set in at 4.2 and 77 K.

Figure 1. Energy dissipated vs. the natural logarithm of the inverse dynamic switching error probability at various temperatures. The analytical bound is compared with estimates from stochastic Landau-Lifshitz-Gilbert simulations of switching trajectories. The approximate slope of the analytical line for the 4.2 K case is mentioned to clarify it is not completely vertical.

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