



Wide-band negative permeability of nonlinear metamaterials

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We propose a novel way to achieve an exceptionally wide frequency range where metamaterial possesses negative effective permeability. This can be achieved by employing a nonlinear response of metamaterials. We demonstrate that, with an appropriate design, a frequency band exceeding 100% is available for a range of signal amplitudes. Our proposal provides a significant improvement over the linear approach, opening a road towards broadband negative refraction and its applications.

The quest for negative refraction has brought the initial inspiration to the entire area of metamaterials research^{1,2}, and served as one of the main driving factors in this field over the last decade^{3,4}. Achieving a negative index in homogenisable materials normally involves making their effective permittivity and permeability simultaneously negative. In natural materials, however, the frequency ranges where such parameters can be observed, are drastically remote and cannot be brought together with a mere mixing. The problem is deemed to be solved with artificial metamaterials, where the frequency of negative permittivity can be brought down by using wire media which effectively mimic a low-frequency plasma^{5,6}, while the negative permeability can be reached with resonant particles or structures such that a localised magnetic response leads to effective magnetic properties⁷.

Remarkably, with all the designs so far available, the frequency range of negative permeability is relatively narrow, as imposed by the resonant nature of the engineered response. There is, in fact, a solid physical argument⁸ behind that restriction: the fundamental Kramers–Kronig relations that govern the frequency behaviour of any generalised susceptibility, imply that there is no way to have negative (and thus strongly dispersive) real part of permeability in a wide range, except if the imaginary part is large, imposing a discouraging dominance of dissipation. This problem is inevitable as long we are dealing with a linear system.

Here we demonstrate that nonlinearity offers a way out of these restrictions. As we show below, it is possible to design nonlinear metamaterials where the nonlinear feedback intervenes the linear parameters so as to enable a considerable extension of the frequency range where effective permeability is negative (Fig. 1). With our approach, negative permeability may readily span a 100% frequency band, offering great opportunities of ultra-wide-band negative refraction. At the same time, such metamaterials are much easier to implement as compared to the suggestion of connecting transistors to metamaterial elements⁹, which would require extensive circuitry within the structure and would be more prone to noise¹⁰.

Our proposal is enabled by nonlinear metamaterials because their properties depend on the intensity of propagating waves. In particular, the very frequency of resonance is not fixed any more, being dependent on the amplitude of the electromagnetic fields^{11–14}. But the negative range of permeability is found right above the resonance of metamaterial. Accordingly, as soon as the resonance floats away from the original position in response to the propagating wave, the range of negative permeability also moves. Therefore, to design the self-tunable negative response we need to make sure that this shift follows the changing frequency of a propagating wave.

The shift of the resonance frequency with a change of signal frequency, required for the wide-band negative permeability, is naturally achieved through the resonance response of metamaterial: the same feature which puts restrictions over the linear properties, turns into an advantage in a nonlinear system. Indeed, when the signal frequency is scanned across the resonance, the excitation of the resonator — all the voltages and currents — dramatically change in amplitude, automatically providing the required effect.

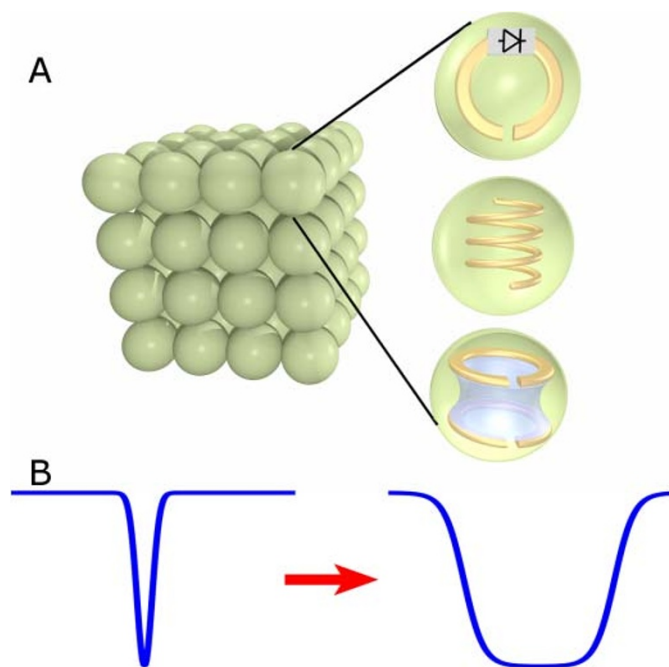


Figure 1 | Conceptual representation of the self-adjustable resonance. By adding nonlinearity to metamaterial building blocks, a dynamic extension of the resonance band can be achieved, which provides negative permeability in a wide frequency range.

Before we proceed to the particulars of design and the details of operation, let us give a few examples on how much the resonant frequency can typically be changed in nonlinear metamaterials so far available. In Table 1, we present the data obtained from the analysis of different nonlinearity types, theoretically or experimentally reported. The span of resonance shift reflected in Table 1, suggests how much the range of negative permeability can be extended when the nonlinear system changes its properties depending on the incident frequency.

Results

Let us demonstrate a practical example of a wide-band permeability, using the case of recently proposed magnetoelastic metamaterials¹⁵. In such metamaterials, an extra degree of freedom is provided to allow for mechanical compression. This way, the lattice constant varies in response to the incident waves, leading to nonlinear mutual interaction between the elements. This is an unusual mechanism for providing nonlinearity, as opposed to the standard approach of achieving that on the level of individual elements^{11–14}. At the same time, this example provides qualitatively the same nonlinear response as a metamaterial made of nonlinear split-ring resonators with self-focusing Kerr-type dielectric inclusions¹¹, so the overall analysis is fairly general.

The essence of magnetoelastic metamaterials is, that the currents induced in resonators by external fields, result in attractive forces between the elements and cause their displacement from the original

positions until balanced with the elastic repulsion. However, this alters the lattice constant and therefore affects the effective impedance of each element through the collective effect of mutual interaction¹⁷. Mathematically, this provides a coupling between the otherwise linear impedance equation, and the equation of the force balance (see *Methods*). The system can be numerically solved, yielding, generally, a nonlinear and bistable dependence of the lattice constant and currents on the incident field. As a consequence, magnetisation is also nonlinear, while the resonance frequency varies depending on the amplitude and frequency of the signal.

For a wide range of the amplitudes of incident fields, the nonlinear coupled equations provide two stable solutions, so magnetoelastic metamaterials demonstrate bistable behaviour¹⁵. Transition between the two states occurs with a change of amplitude or frequency of the signal. In particular, if the frequency of the signal is brought above the original resonance, metamaterial enters a range where there is no bistability. If the frequency then gets lower, transition to the second state occurs “automatically” and metamaterial remains in this state unless the frequency drops below certain threshold. This method to arrive at another state is quite useful as it helps to avoid large variation of power which would have been necessary to trigger such transition at a fixed frequency.

In Fig. 2 we show how the resonance frequency in a particular metamaterial (see *Methods* for the detailed parameters) is changing in response to varying signal frequency, and how this affects the effective permeability.

In the first state, metamaterial remains close to the initial, non-compressed state, and little variation of the resonance frequency is observed (except for the close vicinity of the linear resonance; see “state-1” in Fig. 2a). Thus, altogether, the effective permeability μ_1 is not much different from one observed in a linear metamaterial except for some peculiarities in a close vicinity to the initial resonance, as shown in Fig. 2b.

The second state is characterised by a strong and dynamic compression, and the resonance frequency is essentially driven by the signal frequency (“state-2” in Fig. 2a). Situation here is such that when the resonance is approached from above, the increasing currents cause compression which, in turn, shifts the resonance to lower frequencies; and any further decrease shifts it further down. The resonance, therefore, is effectively pushed by the signal and this may occur across a large frequency span, until the metamaterial is compressed to a technological limit. In this way, the equilibrium corresponds to certain ratio between the exciting fields and induced currents. As a result, the effective permeability μ_2 (Fig. 2c) remains approximately the same as long as the resonance is being moved. Under certain conditions, the permeability can be “trapped” at such a point, that the real part is negative but the imaginary part is quite low, as shown in our example. Thus, the negative permeability with low losses can be observed in an astonishingly wide frequency range, exceeding even a 100% bandwidth.

It is important to emphasise that the bistability observed in such systems, does not imply that the state is switched depending on increase or decrease of frequency. The switch only occurs at the specific points (depending on the signal intensity); however within the range of bistability the frequency can be changed up and down while the metamaterial remains in the state it currently is. In

Table 1 | Typical range of resonance frequency variation. Resonance shift and the required signal amplitudes, reported for various nonlinear metamaterials with nonlinear resonance

Nonlinearity type	Resonance shift, %	Operating field, A/m	Refs.
Resonators with varactors	30	0.1	[12–14]
Resonators in Kerr media	20	1.6	[11]
Magnetoelastic metamaterials	45	1	[15]
Spiral resonators	3	2	[16]

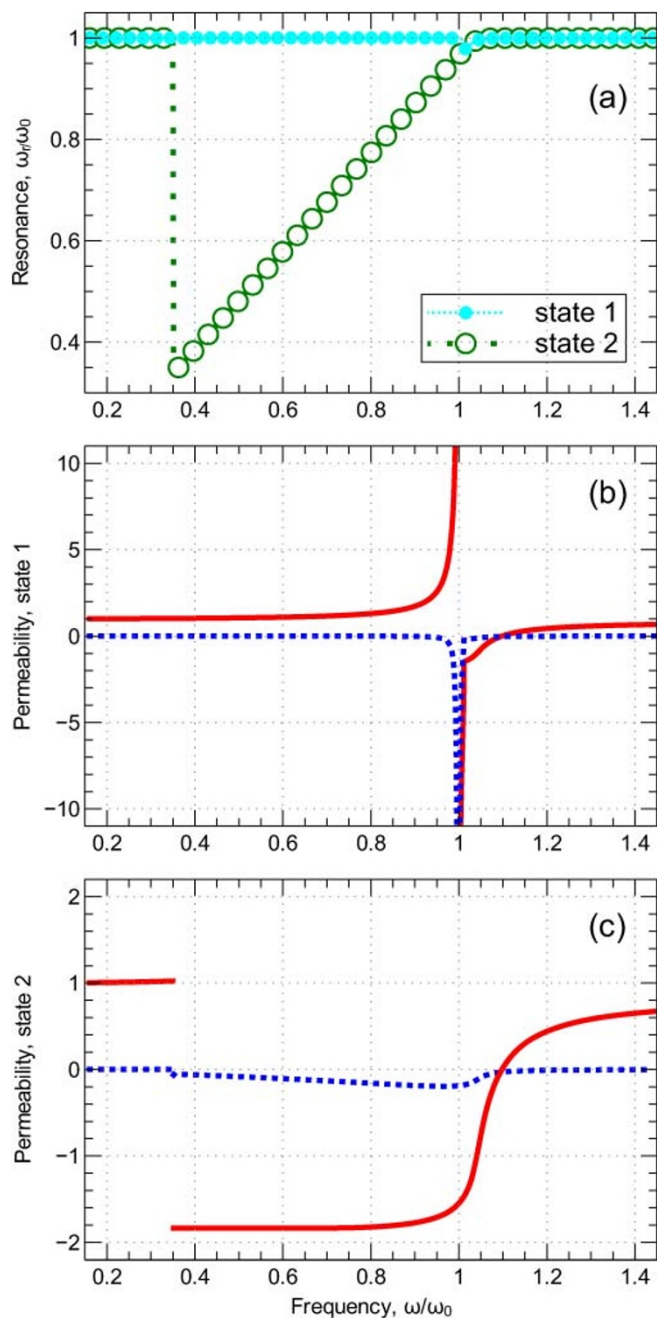


Figure 2 | Self-adjustable resonance and effective permeability of nonlinear metamaterial. (a) Frequency dependence of the resonance frequency in magnetoelastic metamaterial with bistable behaviour, observed with an amplitude of the incident field of 1 A/m. Small filled circles and large empty circles correspond to the two different states of bistability, “state-1” and “state-2”. (b, c) Effective permeability which corresponds to state-1 (b) and state-2 (c) regimes. Real parts are shown in solid red and imaginary parts in dotted blue. The frequency is normalised to that of the initial (linear) resonance of the entire metamaterial, ω_0 .

particular, the state of μ_2 is stable unless the signal frequency drops below the threshold, and it can be reached again by bringing the frequency above the bifurcation point.

Clearly, this exotic behaviour relies on the nonlinear response and is therefore intensity-dependent. At very low amplitudes, no bistability is available and the second state does not exist. On the other hand, very large amplitudes destroy the desirable operation, because, although the resonance is still moved, the permeability is “trapped”

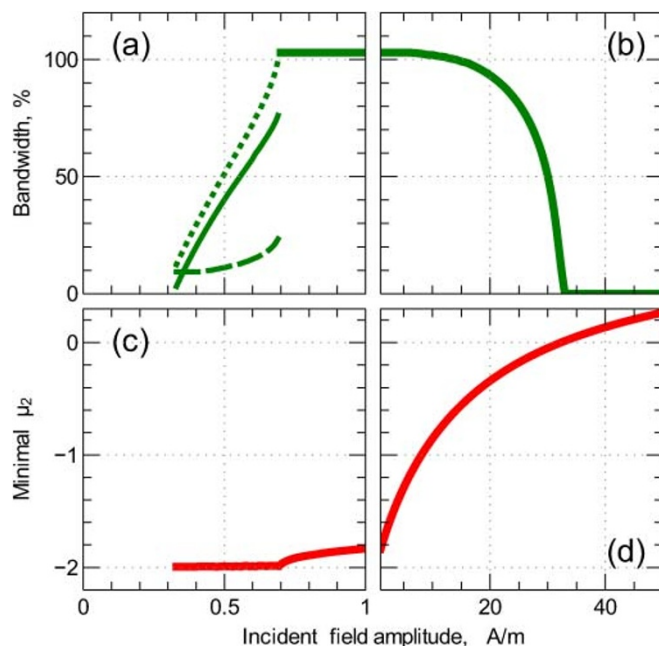


Figure 3 | Frequency range of negative permeability. (a, b) Relative frequency band of the negative real part of the effective permeability at lower (a) and higher (b) incident amplitudes. (c, d) The corresponding magnitude of the “trapped” effective permeability. In panel (a), where an additional bistable state is available, the solid line corresponds to the extra band which is detached from resonance, the dashed line — to the band near the resonance, and the dotted line is their sum.

at values far from resonance, and at some stage it is no longer negative when trapped. Therefore, with increasing power the negative band decreases while the value of permeability increases. We illustrate these features in Fig. 3. In our example (see *Methods*), which is tuned for a good performance for the amplitudes of magnetic field of about 1 A/m, interesting phenomena occur above 0.3 A/m, first with the appearance of the split second state of the bistable operation¹⁵, which is, however, not convenient for applications. Above 0.7 A/m, a continuous band is available, having a maximal frequency range and a maximal magnitude of negative permeability. With an increase of field amplitude, the absolute value of permeability decreases slowly, while the bandwidth remains almost the same until relatively high intensities, so that it is still of the order of 50% when the permeability turns positive (above 30 A/m field amplitude). Therefore, a wide-band operation with negative permeability values is available with the amplitudes of the incident waves spanning more than one order of magnitude.

Discussion

As we have demonstrated above, nonlinear metamaterials introduce a novel way towards achieving negative permeability across an extremely wide frequency span, and that it can be observed within a fair range of amplitudes of the incident signal. This approach opens new horizons for the design and development of materials with negative refraction. Our proposal lifts away the restrictions imposed in linear systems, and therefore, we believe, it will trigger an enthusiastic search for new phenomena and applications.

We emphasize that the above analysis corresponds to a steady-state operation of metamaterial, when the signal frequency is gradually changed in a broad range. Typical response time of such metamaterial is defined by the nature of nonlinearity and may vary significantly. For the magnetoelastic metamaterials, for example, it is the characteristic time of mechanical response (which was of the order of 25 ms in our example), while for varactor-based nonlinearity it is much faster



(from μs to few ns) and corresponds to carriers relaxation time in semiconductors. Propagation of shorter pulses or signals with sharp temporal profiles would invoke a rich variety of complex dynamic effects, which are the subject of future research.

Methods

We assume a system of resonators with radius r , arranged anisotropically in a lattice with dimensionless (normalised to r) parameters a (in the planes of resonators) and b (in axial direction). Generally, the response of such a nonlinear system to an external magnetic field H of an incident wave is governed by the nonlinear impedance equation:

$$\left(R - i\omega L + \frac{i}{\omega C} - i\omega\mu_0 r \Sigma(a, b)\right) \cdot I = i\omega\mu_0 \pi r^2 H, \quad (1)$$

where, in principle, the parameters of resonators (radius r , resistance R , self-inductance L and capacitance C), as well as the lattice parameter b and therefore the lattice sum¹⁷ Σ , may be functions of the induced current I , depending on the mechanism of non-linearity introduced in particular metamaterial.

The effective permeability of the bulk metamaterial can be calculated through macroscopic averaging as explained in detail in Ref. 17. Through solving Eq. (1), magnetisation of the metamaterial follows as $M = \pi r I / (a^2 b)$ and the effective permeability can be found as

$$\mu_{\text{eff}} = \left(H + \frac{2}{3}M\right) / \left(H - \frac{1}{3}M\right). \quad (2)$$

In the linear case, it has a standard resonant shape. The resonance frequency, however, differs significantly¹⁷ from the resonance of an individual element and equals

$$\omega_0 = \frac{1}{\sqrt{LC \left[1 + \frac{\mu_0 r}{L} \left(\Sigma + \frac{\pi^2}{3a^2 b}\right)\right]}}, \quad (3)$$

being, generally, dependent on the induced currents through b , Σ , r , L and C values.

For the case of magnetoelastic nonlinearity, an explicit dependence of L and C on the current is absent, while the thermal expansion which affects these characteristics through the nonlinearity of r , can be neglected. Then the nonlinearity is reflected by variations of b (and thus Σ), which is found by solving the transcendent equation (1) together with the equation of the force balance¹⁵,

$$\frac{\pi\mu_0 I^2}{4b^2} = kr(b_0 - b), \quad (4)$$

where we have used an approximation for the attractive force valid for closely positioned resonators; k is the stiffness coefficient, and b_0 corresponds to the initial (zero power) equilibrium condition.

For our illustrative calculations, we took circular resonators with radius $r = 3$ mm (wire radius 0.03 mm), resonating individually at 1 GHz with a quality factor of 100. They were arranged in a lattice with $a = 4$ and $b_0 = 0.3$, while the limiting value of b for maximal compression was 0.03; the stiffness coefficient was taken to be $k = 0.7$ mN/m. The system of equations (1) and (4) is solved numerically, and the resonance frequency and the permeability are then found with equations (3) and (2).

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Author contributions

Theoretical consideration was carried out by M.L. All the authors have analysed and discussed the results. The manuscript was written by M.L., Y.S.K. and I.V.S. and the figures prepared by I.V.S. and M.L.

Additional information

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