Electro-superlubric springs for continuously tunable resonators and oscillators

Zhanghui Wu^{1,2,6} , Xuanyu Huang^{2,3,4,6} , Xiaojian Xiang^{1,2,5} & Quanshui Zheng^{1,2,3,4,5}

Resonators and resonator-based oscillators are used in most electronics systems and they are classified as either mechanical or electrical, with fixed or difficult-to-tune resonant frequencies. Here, we propose an electro-superlubric spring, whose restoring force between two contacting sliding solid surfaces in the structural superlubric state is linearly dependent on the sliding displacement from the balanced position. We use theoretical analysis and finite element methods to study the restoring force and stability. The stiffness of this electro-superlubric spring is proportional to the square of the applied electric bias, facilitating continuous tuning from zero to several megahertz or gigahertz for the microscale or nanoscale resonators, respectively. Furthermore, we propose an electro-superlubric oscillator that is easily operated by varying a pair of harmonic voltages. The resonant frequency, resonant amplitude, quality factor, and maximum resonant speed can be continuously tuned via the applied voltage and bias. These results indicate significant potential in the applications of electro-superlubric resonators and oscillators.

¹Department of Engineering Mechanics, Tsinghua University, 100084 Beijing, China. ²Center for Nano and Micro Mechanics, Tsinghua University, 10084 Beijing, China. ³Department of Mechanical Engineering, Tsinghua University, 10084 Beijing, China. ⁴State Key Lab of Tribology, Tsinghua University, 10084 Beijing, China. ⁵Institute of Superlubricity Technology, Research Institute of Tsinghua University in Shenzhen, 518057 Shenzhen, China. ⁶These authors contributed equally: Zhanghui Wu, Xuanyu Huang. ✉email: wuzh18@mails.tsinghua.edu.cn; huangxua20@mails.tsinghua.edu.cn; zhengqs@tsinghua.edu.cn

Microscale resonators, which can be classified into either electric or acoustic type^{1,2}, have attracted considerable research interest due to their broad applications in electromechanical filters³, gyroscopes⁴, resonant accelerometers⁵ and energy harvesting^{6–8}. All electric resonators, like L-C resonators, are based on the electronic oscillations in the circuit. Acoustic resonators, which mainly contain microelectromechanical system (MEMS), nanoelectromechanical system (NEMS), and quartz resonators, are based on the interaction of mechanical motion and thermal or acoustic phonons¹. Generally, electric resonators have better performance on the integration with IC process than acoustic resonators, while acoustic resonators have much higher quality factor (Q-factor) and lower energy dissipation than electric acoustic resonators, and thus acoustic resonators are widely used in many applications, including Si MEMS clock oscillators in Apple iPhone¹.

Large-range tunability of the resonant frequency is among the most desirable functions of resonators. For instance, with emerging wireless communication systems, an extreme resonant frequency tunability enables a single resonator to cover a wide range frequency; such a resonator could replace a series of frequency-fixed resonators, which will reduce costs as well as the size of the device, thereby offering unprecedented configurability^{9,10}. Furthermore, extreme resonant frequency tunability promises an extremely wide range of harvesting energy⁶, as the resonance frequency of such a device will be able to adapt to the external energy excitation frequency, which means that much more energy can be efficiently harvested than a fixed or a small-range resonant frequency resonator. There are several techniques for tuning the resonant frequencies of resonators, including piezoelectric^{6,11–13}, electrothermal^{3,14–17} and electrostatic^{18–21} techniques. Although these existing resonators can tune a certain range of frequencies around the natural resonant frequency, the tuning range is not extreme and the tuning techniques contain additional structures, which is inconvenient for integrated fabrication.

Structural superlubricity (SSL) refers to a state of ultralow friction and zero wear between two contacting solid surfaces^{22–24}. Since its first realisation in microscale graphite²², the SSL state has been observed in contact between microscale graphite and hBN, DLC (diamond like carbon), Au, etc.^{25–27}, at high speeds (up to 294 ms^{−1})²⁸ and for long ranges of sliding (>100 km)²⁵.

In this paper we propose an electro-superlubric spring (ESL-spring), whose restoring force between two contacting sliding solid surfaces in the structural superlubric state is linearly dependent on the sliding displacement from the balanced position. We show that the stiffness of this ESL-spring is proportional to the square of the applied electric bias, which provides a highly effective solution to realise extreme tunable range. Furthermore, we propose an ESL-oscillator that is easily operated by varying a pair of harmonic voltages, indicating significant potential in the applications of ESL-resonators and oscillators.

Results

The principle of ESL-spring. Figure 1a, c illustrate the side and top views of the elementary structure of the ESL-spring, respectively. We consider an elementary setup, where a rectangular electrode (SLIDER) of length, L , and width, W , and a significantly larger substrate are in contact in the SSL state. The substrate is covered or coated by a dielectric thin film of thickness, d , and relative permittivity, ϵ_r , on the structure of two rectangular electrodes (Electrodes 1 and 2) of the same width W and a larger and thicker dielectric material. Electrodes 1 and 2 are placed in a line, namely, on the x -axis (Fig. 1c), and separated by a distance s . In practice, we can transfer a microscale graphite flake, which is conductive, with the self-retracting motion property²⁹ to an

atomically smooth hexagonal boron nitride (hBN) film, which is a dielectric for forming a robust SSL contact²⁷. Silicon dioxide (SiO₂), which is an insulator and can be grown directly via chemical vapour deposition, provides another choice of dielectric thin film.

The SLIDER moves on the substrate (STATOR) in the x -direction in the range of $|x| < L_c/2$. By applying a constant electric bias V_b to Electrodes 1 and 2, we demonstrate that the slider will meet a linear restoring force, $F = -kx$, where x is the displacement from the balanced position. The slider is located on the upper right over the middle of Electrodes 1 and 2, and $L_c = L - s$ is the overlapping length of SLIDER with Electrodes 1 and 2. Hence, the setup shown in Fig. 1a, c corresponds to a linear spring in the displacement range of $|x| < L_c/2$. To demonstrate the above result, as shown in the equivalent circuit diagram in Fig. 1b, we note that the slider forms capacitances $C_1 = C_0(1 - \xi)/2$ and $C_2 = C_0(1 + \xi)/2$, with Electrodes 1 and 2, respectively. Here, $C_0 = C_1 + C_2 = \epsilon_0 \epsilon_r L_c W/d$ is the total capacitor, $\xi = 2x/L_c \in (-1, 1)$ is the relative displacement, and ϵ_0 is the permittivity of vacuum. As detailed in the Supplementary Note 1, the equivalent potential energy, U , of the entire system can be calculated as:

$$U = -\frac{1}{2} \frac{C_1 C_2}{C_1 + C_2} V_b^2 = -(1 - \xi^2) \frac{C_0}{8} V_b^2. \quad (1)$$

Substituting (1) into the relationship $F = -\partial U/\partial x$, we obtain the restoring force $F = -kx$ with the following ESL-spring coefficient:

$$k = \frac{C_0}{L_c^2} V_b^2 = \frac{\epsilon_0 \epsilon_r W}{L_c d} V_b^2. \quad (2)$$

Thus, we introduce a new spring type, called an ESL-spring, with a continuously tunable spring coefficient by changing the applied voltage, V_b .

To validate the above theoretical relationships, we carried out a finite element simulation on the electric and potential fields, equivalent potential energy, and driving force as functions of the displacement, x , as detailed in the Supplementary Note 2. We assume $L = W = 4\mu\text{m}$, $d = 100\text{nm}$, $s = 1\mu\text{m}$, and $V_b = 40\text{V}$, and set the thicknesses of the slider and substrate at 200nm and 4μm, respectively. Using SiO₂ as the dielectric ($\epsilon_r = 3.9$), we obtain the corresponding simulation results on the potential field (left panel) and electric field (right panel) for five symmetrical displacements of the ESL-spring, as depicted in Fig. 1d. The electric field (potential difference) is mainly concentrated in the overlap area between the slider and Electrodes 1 and 2, which means that the electrostatic energy is mainly composed of capacitors C_1 and C_2 . Figure 1e, f show the simulation results (solid lines) of the dimensionless equivalent potential energy, U/U_0 , and driving force, F/F_0 , with respect to four different width-to-thickness ratios, W/d (with the same $W = 4\mu\text{m}$), where $U_0 = C_0 V_b^2/8$ and $F_0 = C_0 V_b^2/2L_c$. The linear relationship $F = -kx$ (black dashed line) is excellent at $W/d > 40$ because edge effects decrease with increasing W/d . This condition is very easy to achieve through micro-nano-fabrication technology in the SSL state. Further, the linear relationship holds only within the range of $|x| < L_c/2$.

However, nonzero frictions in SSL contacts are inevitable, although they are ultralow (by definition). In ambient environments, typical friction forces affecting a sliding 4-μm-sized graphite flake on substrates in the SSL contacts were measured to be in the range of 0 ~ 1.0μN^{22,23,27,30}. Recently, it was revealed that these frictional forces are mainly caused by interactions between the edges of the flake and adsorptions of the substrates and can be mostly removed through an annealing process³¹. In comparison, the maximum ESL-spring force is equal to $F_{\text{max}} = \epsilon_0 \epsilon_r W V_{\text{cr}}^2/2d$ at $\xi = \pm 1$ whenever V_b takes its maximum allowed voltage, $V_{\text{cr}} = E_{\text{cr}}d$, before the electric breakdown of the

Fig. 1 Elementary structure, driving force of ESL-spring, and its numerical validation. **a, c** Illustration of ESL-spring viewed from side and top, respectively, where key variables and parameters are indicated. **b** Equivalent circuit diagram of ESL-spring. **d-f** are the two-dimensional FEM simulation results for the potential and electric fields as functions of sliding displacement of the SLIDER (the upper electrode), with $L = W = 4\ \mu\text{m}$, $H = 200\ \text{nm}$, $s = 1\ \mu\text{m}$, and $V_b = 40\ \text{V}$, and taking silicon oxide as Dielectrics 1 and 2 ($\epsilon_r = 3.9$). **d** Changes in the potential field (left panel) and electric field (right panel) with five symmetrical displacements for $d = 100\ \text{nm}$. **e, f** Dimensionless equivalent potential energy and driving force as functions of the relative sliding displacement $\xi = 2x/L_c$ with respect to four different width-thickness ratios, W/d ; the approximation accuracy is excellent for $W/d > 40$.

dielectric film, where E_{cr} is the breakdown strength of the dielectric film. According to the theory and experiments of the solid dielectric breakdown³², when the thicknesses of dielectric films are thinner than $\sim 4\ \mu\text{m}$, E_{cr} and the relative permittivity, ϵ_r , for most dielectric films can be approximately correlated to a unified form of $E_{\text{cr}} \approx E_0/\sqrt{\epsilon_r}$ with $E_0 = 2.24 \times 10^9\ \text{V m}^{-1}$. Therefore, the maximum allowed ESL-spring force can be written as $F_{\text{max}} \approx \epsilon_0 E_0^2 W d / 2$, which is interestingly independent of the dielectric film. For example, if we choose $W = 4\ \mu\text{m}$ and $d = 100\ \text{nm}$, we can obtain $F_{\text{max}} \approx 8.9\ \mu\text{N}$, which is significantly larger than $1\ \mu\text{N}$. The corresponding maximum allowed voltage for BaTiO_3 is $V_{\text{max}} = E_{\text{cr}} d \approx 7.1\ \text{V}$, which has a normal value of $\epsilon_r = 1,000$ as the dielectric film. The above analysis verifies that the ESL-spring is not only physically but also materially feasible. Furthermore, there will be a normal electrostatic force while applying a constant electric bias V_b to Electrodes 1 and 2. However, this would not generate enough additional friction to affect the operation of the ESL-resonator due to the extremely low friction coefficient of ~ 0.001 in the SSL contact state (as detailed in Supplementary Note 8).

Using the ESL-spring as a resonator, we obtain the (circular) resonance frequency as follows:

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{\epsilon_0 \epsilon_r}{\rho L L_c H d}} V_b, \quad (3)$$

where m , ρ and H denote the mass, mass density and thickness of the slider, respectively. By taking the maximum allowed voltage, V_{cr} , before the dielectric breakdown and choosing, for instance, $H = L/10$ and $s = L/2$, we can obtain the maximum achievable resonance frequency ω_{max} in the form $\omega_{\text{max}} = v_0 (d/L^3)^{1/2}$, where $v_0 = E_0 \sqrt{20 \epsilon_0 / \rho} \approx 628\ \text{ms}^{-1}$. Therefore, the resonant frequency

can be up to $\sim 25\ \text{MHz}$ in the microscale ($L = 4\ \mu\text{m}$, $d = 100\ \text{nm}$) and $\sim 2.5\ \text{GHz}$ in the nanoscale ($L = 40\ \text{nm}$, $d = 1\ \text{nm}$). Since the tunable frequency range of ESL-spring resonator is much smaller than the natural frequency of the graphite flake ($1.7\ \text{GHz}$ for micro scale and $170\ \text{GHz}$ for nano scale), we ignored the deformation of graphite in the above analysis (see Supplementary Note 7).

The tunable range comparison between different types of resonators. As demonstrated above, one of the most remarkable features of the ESL-resonator is its widely tunable range of resonant frequencies¹⁰. For comparison, Fig. 2a summarises some representative tunable ranges reported in the literature and the tunable range of the ESL-resonator. To better comprehend the advantage of the tunability of the ESL-resonator, we also plot the relative tuning ranges of $(\omega_{\text{max}} - \omega_{\text{min}})/\omega_{\text{max}}$ in Fig. 2b, where ω_{max} and ω_{min} denote the maximum and minimum tunable resonant frequencies, respectively. These comparison results demonstrate the superiority of the ESL-resonator in terms of tunability.

The stability of ESL-resonators. In the setup of the ESL-resonator shown in Fig. 1a, c, there is no mechanical constraint to the motions of the slider in either the lateral direction (i.e., the y -axis) or in the xOy rotational plane. The introduction of such a mechanical constraint is undesirable, as it not only increases the structural complexity and cost but also induces additional and often significant friction. Therefore, a fundamental question to ask is whether the kinetics of such a free-constrained slider are stable.

Hereafter, we theoretically demonstrate the kinetic robustness of the ESL-resonator. Figure 3a illustrates the top view of the ESL-resonator; the slider moves to a deviating position characterised by a coordinate in the (x, y, θ) -phase space with $y \neq 0$ and/or $\theta \neq 0$.

Fig. 2 Comparison between the ESL-resonator and conventional resonators. **a** Tunable resonant ranges for five piezoelectric resonators^{6,11-13}, six electrostatic resonators¹⁸⁻²¹, five thermoelectric resonators^{3,14-17} and that of the ESL-resonator (for $W = L = 4\mu\text{m}$, $s = 2\mu\text{m}$, $d = 100\text{nm}$, $\epsilon_r = 3.9$, $\rho = 2.16\text{gcm}^{-3}$, $H = 50\text{nm}$.) The solid lines of the ranges are given according to their respective theoretical or experimental relations, such as Eq. (3), or the literature^{3,6,11-21}. **b** A comparison of the relative frequency tuning ranges of $(\omega_{\max} - \omega_{\min})/\omega_{\max}$. Details in the Supplementary Note 3.

Fig. 3 Stability analysis of the ESL-resonator. **a** Illustration of the top view of the ESL-resonator. **b-d** Normalised energy surfaces in the $\{\eta, \gamma\}$ region for $\xi = 0.0, 0.4$ and 0.8 in **b**, **c**, and **d**, respectively. For better visualisation of the results, we have plotted the contour lines of the surfaces in **b-d**. Two typical size ratios of $W/L = 0.5$ and $s/L = 0.2$ are used.

To simplify the kinetic stability analysis, we introduce three dimensionless parameters, $\xi = 2x/L_c$, $\eta = y/W$, and $\gamma = 2\theta/\pi$, which are all constrained in the range of $(-1, 1)$. The two capacitors denoted by C_1 and C_2 , formed between the slider and Electrodes 1 and 2, respectively, can be expressed as $C_1 = \epsilon_0 \epsilon_r A_1/d$ and $C_2 = \epsilon_0 \epsilon_r A_2/d$, where $A_1(\xi, \eta, \gamma)$ and $A_2(\xi, \eta, \gamma)$ are the overlapping areas between the slider and Electrodes 1 and 2, respectively. We can express the equivalent potential energy, U , of the system in the form of the first relationship in Eq. (1), yielding:

$$U = g(\xi, \eta, \gamma) \frac{C_0}{8} V_b^2, \quad (4)$$

where the function $g(\xi, \eta, \gamma)$ represents the normalised energy that degenerates $g(\xi, 0, 0) = \xi^2 - 1$ and is determined by A_1 and A_2 . Mathematically, whenever the normalised energy $g(\xi, \eta, \gamma)$ is at a minimum at $(\xi, 0, 0)$, the kinetic system of the slider is stable at $(\xi, 0, 0)$.

We use numerical methods to calculate $g(\xi, \eta, \gamma)$ for different ξ, η , and γ , all in the range of $(-1, 1)$, as detailed in the Supplementary Note 4. Figure 3b-d show the normalised energy surfaces of $g = g(\xi_0, \eta, \gamma)$ for $\xi_0 = 0.0, 0.4, 0.8$, respectively, and

two typical size ratios $W/L = 0.5$ and $s/L = 0.2$. We can clearly see that the energy is always at its minimum at $\eta = \gamma = 0$, as the slider is non-rotationally moved along the x -direction. This means that the slider suffers restraints on the y and θ directions while moving. Hence, the kinetics of such a free-constrained slider are stable. The above-characterised motion robustness is valid for other values of ξ_0 , W/L and s/L , when $|\xi_0|$ is not too large, as explained in the Supplementary Note 4.

The application of ESL-resonators to ESL-oscillators. To apply the above-mentioned ESL-resonator as an oscillator, a major challenge is to apply a stimulating load without affecting the SSL state. To solve this problem, we propose another elementary setup, as illustrated in Fig. 4a, where the STATOR is formed by inserting a middle electrode (Electrode m) with length a , width W , and separation distance $s/2$ between Electrodes 1 and 2. We shall further show that when choosing the pair of voltages V_1 and V_2 between Electrode m and Electrodes 1 and 2, respectively, in the alternative forms $V_1 = V_0 \sin \omega_e t + V_b/2$, $V_2 = V_0 \sin \omega_e t - V_b/2$, the setup will become an oscillator stimulated by the harmonic voltage $V_0 \sin \omega_e t$ with a resonant

Fig. 4 Schematic representation of the ESL-oscillator. **a** Illustration of an elementary ESL-oscillator, where the two alternative voltages V_1 and V_2 are applied to the left and right electrodes, respectively. **b** Equivalent circuit diagram of the ESL-oscillator. **c** Relationship of the oscillating amplitude, A_{os} , versus V_0 . In this case, the slider displacement must be less than $(L_c - a)/2 = 1\mu\text{m}$, which implies an excitation voltage of $V_0 \leq 3\text{mV}$. **d, e** Relationship of the maximum velocity in the oscillation, v_{os} , and Q-factor versus the tunable electric bias, V_b , for $V_0 = 2\text{mV}$ in **d**. The parameters used in the calculations are: $\epsilon_r = 3.9$, $W = L = 4\mu\text{m}$, $s = 1\mu\text{m}$, $a = 1\mu\text{m}$, $H = 200\text{nm}$, $d = 20\text{nm}$, $\rho = 2.16\text{gcm}^{-3}$, and the damping is $c = 1.2 \times 10^{-11}\text{kgs}^{-1}$ ^{33,34}. Details are provided in the Supplementary Note 5.

frequency that is continuously tuned by changing the bias potential V_b . The slider will form capacitors C_1 , C_m and C_2 with Electrodes 1, m and 2, respectively, and the equivalent circuit diagram is shown in Fig. 4b. As detailed in the Supplementary Note 5, the dynamic equation of the slider can be expressed as $m\ddot{x} + c\dot{x} + kx = F_{ex} \sin \omega_e t$, where m is the mass of the slider, c is the mechanical damping (much larger than ohmic loss and dielectric loss; see Supplementary Note 6), and k is the spring coefficient, which has exactly the same form as in Eq. (2),

$$F_{ex} = \frac{W\epsilon_0\epsilon_r}{L_c d} a V_0 V_b, \quad (5)$$

denoting the excitation force with $L_c = L - s$.

Based on a physically meaningful understanding of Eq. (5), we can reform it to $F_{ex} = Q_m E_{ho}$, where $Q_m = C_m V_0$ can represent an equivalent charge of the middle capacitor $C_m = \epsilon_0\epsilon_r W a / d$ excited by the voltage V_0 , and $E_{ho} = V_b / L_c$ can correspond to an equivalent horizontal electric field formed by the bias voltage, V_b . This understanding endows the physical picture, where the force F_{ex} is electrostatically applied to the charge Q_m in the electric field E_{ho} .

By solving the dynamic equation, we can obtain the resonance amplitude, A_{os} ; velocity, v_{os} ; and Q-factor, Q , as:

$$\begin{aligned} A_{os} &= \frac{F_{ex}}{\omega_0 c} = \frac{1}{c_1} \sqrt{\frac{\epsilon_0\epsilon_r \rho L H}{d L_c}} a V_0, \\ v_{os} &= \omega_0 A_{os} = \frac{\epsilon_0\epsilon_r}{c_1 d L_c} a V_0 V_b, \\ Q &= \frac{\omega_0}{c/m} = \frac{1}{c_1} \sqrt{\frac{\epsilon_0\epsilon_r \rho L H}{d L_c}} V_b, \end{aligned} \quad (6)$$

where $c_1 = c/W$ represents the damping per unit width. To intuitively display the performance of the ESL-oscillator, we plot the above performance parameters for $\epsilon_r = 3.9$, $W = L = 4\mu\text{m}$, $s = 1\mu\text{m}$, $a = 1\mu\text{m}$, $H = 200\text{nm}$, $d = 20\text{nm}$, $\rho = 2.16\text{gcm}^{-3}$, and $c = 1.2 \times 10^{-11}\text{kgs}^{-1}$ ^{33,34}, as shown in Fig. 4c–e; the damping coefficient, c , is estimated in the Supplementary Note 5.

The spring coefficient, k , is independent of V_0 , and the electric excitation force, F_{ex} , depends on the product aV_0 . However, different choices of a affect the maximum allowed oscillation amplitude, $|x| < (L_c - a)/2$. The Q-factor increases with decreasing d in the form $\sim d^{-1/2}$, which implies a remarkable advantage of using nanometre-thick dielectric films to separate the SLIDER and underlying electrodes.

The advantages of ESL-resonators and ESL-oscillators. We point out two additional remarkable advantages of ESL-resonators and ESL-oscillators. First, MEMS resonators constitute a type of mechanical resonator that has a compact size, high sensitivity, high resolution, and low cost [1,2]. Because most MEMS resonators are based on cantilevers, the allowed maximum relative displacement of the cantilevers is relatively small, typically less than 1/10 the length of the cantilever owing to the pull-in phenomenon^{35–37}. This largely restricts the scope of MEMS applications in some important devices, such as energy-harvesting devices⁶ and micro-gyroscopes. In comparison, the ESL-oscillator allows significantly larger relative sliding displacements. In fact, the maximum allowed displacement, $x_{\max}$, is equal to $(L_c - a)/2$, which is of the same order as the ESL-oscillator size. This unique property promises applications of ESL-oscillators with better performance capabilities than those of MEMS in energy-harvesting devices⁶ and micro-gyroscopes (as detailed in Supplementary Note 9).

Second, the restoring force of the ESL-spring is strictly linear, which results in good harmonic vibration dynamic characteristics of ESL-oscillators at any voltage. In contrast, the additional restoring forces of the conventional tunable resonators and oscillators are often nonlinear^{6,18}, which leads to non-harmonic vibrations and often affects the performance of the resonators and oscillators.

Conclusions

In summary, using structural superlubricity, a recently introduced technology, we proposed one type of linear spring, called an electro-superlubric spring (ESL-spring), which possesses a unique feature of continuously tunable spring stiffness by alteration of the applied bias voltage. This suggests the future development of ESL-resonators and ESL-oscillators with continuously tunable resonant frequencies in the range from zero to several MHz for microscale ESL-resonators and from zero to several GHz for nanoscale ESL-resonators. Moreover, the relative displacement amplitudes of ESL-resonators can largely exceed those of mechanical ones. Therefore, the commercialisation of ESL-resonators can be expected to be realised in the near future, which will lead to their broad application. However, the experimental results are not yet accomplished; this remains to be addressed in future studies.

Methods

Finite element simulations of the ESL-spring. In order to verify the theoretical model of ESL-spring in Supplementary Note 1 and take into account the influence of edge effect, we develop a finite element model to simulate the structure in Fig. 1. At the boundary, a large area of air is set outside, as shown in Supplementary Fig. 2a, while the zero-charge boundary condition is set on the outside boundary of air (See Supplementary Eq. (S13)). Simultaneously, all electrodes are equipotential; and the voltage of Electrode 1 is set to 0 V, and the voltage of Electrode 2 is set to 40 V, which is equivalent to applying a constant voltage $V_b = 40$ V. For the Electrode 0, we give it a zero-total charge condition, as Supplementary Equation (S14) shows. The whole region is solved by the Maxwell equation in electrostatic field, as details in Supplementary Eq. (S15).

Calculating the overlap area of Slider and Electrodes. Here, we need to use a numerical method to calculate $A_1(\xi, \eta, \gamma)$ and $A_2(\xi, \eta, \gamma)$. Firstly, we divide the area of Electrode 0 into $N \times N$ (here $N = 500$) elements uniformly, as illustrated in Supplementary Fig. 3. The position of each element is discretized as a point $(\frac{x_0^{(i,j)}L}{2}, \frac{y_0^{(i,j)}W}{2})$, in which $x_0^{(i,j)}$ and $y_0^{(i,j)}$ is defined as Supplementary Eq. (S16). Then, when Electrode 0 produces the displacement (ξ, η, γ) , we can calculate (i, j) -th element's position in the global coordinate system x - O - y using Supplementary Eq. (S17). Next, we just need to judge each element whether it is in the area where Electrode1&2 are located, to determine whether it belongs to the overlap area A_1 and A_2 by Supplementary Eq. (S18). Then we can use Supplementary Eq. (S19) to obtain the overlap area.

Data availability

The data that support the plots within this paper and other finding of this study are available from the corresponding authors upon reasonable request.

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Author contributions

Z.W. and X. H. designed and analysed the resonators. Z.W., X.H. and X.X. discussed and proposed the idea. Q.Z. supervised the project. Z.W., X.H. and Q.Z. wrote the paper.

Competing interests

The authors declare no competing interests.

Additional information

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Correspondence and requests for materials should be addressed to Zhanghui Wu, Xuanyu Huang or Quanshui Zheng.

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