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Dynamical onset of light-induced unconventional superconductivity—a Yukawa-Sachdev-Ye-Kitaev study

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Lukas Grunwald[1,2,3], **Giacomo Passetti**[1,2] & **Dante M. Kennes**[1,2,3]

Using light irradiation to manipulate quantum materials has opened up avenues for transiently inducing superconductivity in some systems. Despite experimental confirmation across various compounds, the mechanism behind the dynamic formation of Cooper pairs remains highly debated, in part due to the strong electronic correlations at play, which pose challenges for theoretical investigations relying on perturbative or phenomenological approaches. Here, we investigate the dynamical onset of superconductivity in the strongly correlated, yet exactly solvable Yukawa-Sachdev-Ye-Kitaev model. Analyzing dynamical protocols motivated by theoretical mechanisms proposed for light-induced superconductivity, that is light-induced cooling and the dressing of Hamiltonian parameters, we investigate the exact relaxation resulting out of undercooling and interaction quenches. While, in contrast to BCS theory, it is not possible for superconductivity to emerge following interaction quenches across the superconducting phase transition, we find that the dynamical relaxation of undercooled states universally leads to superconductivity. Despite the strong correlations, the emerging order parameter dynamics are well captured by a coarse grained Ginzburg-Landau theory. Our study provides an integral stepping stone towards exploring light-induced superconductivity in strongly correlated systems in a theoretically controlled way.

The irradiation of matter with light has emerged as a powerful tool for controlling quantum materials[1–5]. Amongst the plethora of tantalizing light-induced phenomena, one of the most striking instances is light-induced superconductivity, in which superconductor-like behavior is induced after photo-excitation at temperatures larger than the equilibrium transition temperature[6]. This effect has been experimentally observed in many systems including cuprates[7–12], iron-based superconductors[13], fullerides[14–16] and organic superconductors[17], but the underlying mechanism leading to the dynamical formation of Cooper pairs is still highly debated.

The theoretical proposals can be broadly classified into two groups. One class of interpretations relies on a light-induced modification of Hamiltonian parameters (dressed Hamiltonians), leading to static and dynamic changes of the free energy landscape[18–38], while the second group attributes the emergent superconducting behavior, to effective light-induced cooling (dressed statistics)[39–46]. Both schemes might be distinguishable by the dynamical formation of superconductivity following the irradiation with light, calling for a detailed analysis of the real-time dynamics.

Strong electronic correlations, especially in cuprates and iron-based superconductors, make systematic theoretical investigations challenging. In these systems, superconductivity is believed to emerge out of a strongly correlated non-Fermi liquid (nFl) normal state[47–50], and studies of the real-time emergence of the superconducting condensate typically have to rely on perturbative approximations or purely phenomenological models. It is hence highly desirable to investigate the dynamical onset of light-induced superconductivity in a strongly correlated, yet theoretically controllable system, to gain intuition and assess the validity of common approximations.

In this work, we investigate the dynamical onset of superconductivity in the Yukawa–Sachdev–Ye–Kitaev (YSYK) model[51–62], which is a solvable toy model for strongly correlated superconductivity. It presents a fermion–boson generalization of the fermionic Sachdev–Ye–Kitaev (SYK) model[63–65] and provides the, to the best of our knowledge, unprecedented possibility to study the interplay of strong correlations and superconductivity in non-equilibrium without any approximations. The YSYK model consists of dispersionless bosons, randomly interacting with spinful

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[1]Institut für Theorie der Statistischen Physik, RWTH Aachen University, 52056 Aachen, Germany. [2]JARA-Fundamentals of Future Information Technology, 52056 Aachen, Germany. [3]Max Planck Institute for the Structure and Dynamics of Matter, Center for Free-Electron Laser Science (CFEL), Luruper Chaussee 149, 22761 Hamburg, Germany. e-mail: lukas.grunwald@mpsd.mpg.de; dante.kennes@mpsd.mpg.de

fermions and hosts an unconventional superconducting state that emerges out of a non-Fermi liquid normal state, reminiscent of the situation observed in cuprates and iron-based superconductors[47–50].

Extending previous non-equilibrium SYK studies[66–83], to the YSYK model by include bosons and superconductivity, we investigate two dynamical protocols motivated by the theoretical proposals for mechanisms of light-induced superconductivity (Fig. 1) by solving the exact Kadanoff–Baym equations numerically.

First, we consider (quasi-static) light-induced modifications of Hamiltonian parameters, that we model as interaction quenches across the superconducting phase transition. We find, that in contrast to BCS theory[84] and previous studies in the Hubbard model[85–89], it is not possible to induce superconductivity via interaction quenches in the YSYK model due to excessive heating of the nFl normal state. The light-induced cooling on the other hand, which we model as the relaxation of undercooled normal states to the superconducting ground state, is found to universally lead to superconductivity at late times. Providing a full characterization of the non-equilibrium relaxation, we identify an overdamped and oscillatory relaxation regime and demonstrate that a coarse-grained Ginzburg Landau theory can be effectively determined from microscopics to describe the emerging order parameter dynamic despite the strong correlations.

Results and discussion

Model

The YSYK model describes N fermions randomly interacting with N dispersionless bosons and is defined by the Hamiltonian[51–53,56]

$$H=\frac{1}{2}\sum_{k=1}^{N}\left(\pi_k^2+\omega_0^2\phi_k^2\right)+\frac{1}{N}\sum_{ijk}^{N}\sum_{\alpha=\pm}g_{ij,k}\phi_k\,c_{i\alpha}^{\dagger}c_{j\alpha}. \tag{1}$$

Here $c_{i\alpha}^{(\dagger)}$ are fermionic ladder operators, while the canonical boson fields ϕ_k and their conjugate momentum π_k satisfy $[\phi_k,\pi_{k'}]=i\delta_{kk'}$. The electron–boson vertices $g_{ij,k}$ are uncorrelated, random all-to-all couplings that are sampled from a Gaussian-orthogonal ensemble (GOE)[90] with mean $\overline{g_{ij,k}}=0$ and variance $\overline{g_{ij,k}g_{ij,k}^{*}}=g^2$.

The normal state at low temperatures is a strongly interacting quantum critical non-Fermi liquid (nFl) with power-law fermionic $G(\omega)\sim|\omega|^{2\Delta-1}$ and bosonic $D(\omega)\sim|\omega|^{1-4\Delta}$ spectral functions and universal $\Delta\approx 0.42$[51,54]. Allowing for pairing (U(1) symmetry breaking), superconductivity emerges for all coupling strengths g. It supersedes the quantum critical nFl and is the final low-temperature state. The superconducting condensate is a strongly interacting Cooper-pair fluid, unlike the non-interacting pair fluid in BCS theory[91]. The YSYK model hence provides a paradigmatic example of strong coupling unconventional superconductivity.

Non-equilibrium formalism

In the thermodynamic limit $N\to\infty$ the model becomes exactly solvable due to the vanishing of vertex corrections. This allows the derivation of a closed system of self-consistent equations for fermion, boson, and anomalous Green's functions (GF) on the Keldysh Contour, that we integrate numerically to obtain the exact GF's of the model for a given non-equilibrium protocol (see Methods).

After the full integration, the dynamics of the s-wave superconducting order parameter

$$\Delta(t)=\frac{1}{N}\sum_i\overline{|\langle c_{i\uparrow}(t)c_{i\downarrow}(t)\rangle|}, \tag{2}$$

can be directly extracted from the GF's and provides a measure for the pairing strength of the system. Using generalized fluctuation–dissipation relations (gFDR), we can further extract effective temperatures and non-equilibrium occupation functions. To that end, we transform the GF's to center of mass coordinate $\mathcal{T}=(t+t')/2$ and relative coordinate $\tau=t-t'$ and Fourier transforms the latter (Wigner transformation). Writing the gFDR

$$G^{\mathrm{K}}(\mathcal{T},\omega)=2i(1+2n(\mathcal{T},\omega))\,\mathrm{Im}\,G^{\mathrm{R}}(\mathcal{T},\omega), \tag{3}$$

with $G^{\mathrm{R/K}}$ the normal state electronic GF's, defines the electronic non-equilibrium occupation function $n(\mathcal{T},\omega)$, which in thermal equilibrium is a Fermi-distribution $n_{\mathrm{f}}=(e^{\beta\omega}+1)^{-1}$. In non-equilibrium, we can extract an effective temperature $T_{\mathrm{eff}}(\mathcal{T})$ from Eq. (3), provided that $n(\mathcal{T},\omega)$ resembles n_{f} at low energies[74].

To study the dynamical transition from a normal into a superconducting state, we provide an explicit U(1) symmetry breaking at $t=0$, which is implemented as

$$\mathcal{H}(t)=H_{\mathrm{YSYK}}+\alpha\theta(t)\sum_i\left(c_{i\uparrow}c_{i\downarrow}+\mathrm{h.c.}\right). \tag{4}$$

The symmetry-breaking strength α, provides an infinitesimal seed to start the relaxation process and is chosen so as to not affect the final state after relaxation.

Dressed Hamiltonians

First, we consider light-induced modifications of Hamiltonian parameters, that we model as interaction quenches across the superconducting phase transition. The quenches are implemented as a time-dependent fermion–boson coupling

$$H_{\mathrm{int}}\sim f(t)g_{ij,k}\phi_k\,c_{i\alpha}^{\dagger}c_{j\alpha},\quad \text{with}\, f(t)=g_{\mathrm{i}}\theta(-t)+g_{\mathrm{f}}\theta(t).$$

Starting from a normal state at g_{i} with temperature $T_{\mathrm{i}}>T_{\mathrm{c}}(g_{\mathrm{i}})$ we quench into the superconducting phase $g_{\mathrm{i}}\to g_{\mathrm{f}}$ such that $T_{\mathrm{i}}<T_{\mathrm{c}}(g_{\mathrm{f}})$. The exact dynamics following the interaction quench are illustrated in Fig. 2.

Shortly after the quench, the system is non-thermal and observables show oscillatory dynamics (Fig. 2c, with amplitudes that increase in the strong coupling regime, similar to observations made for quenches in the Holstein model[26]. At larger times, the system universally relaxes towards a thermalized equilibrium state. The final temperature is determined by matching the instant energy density $\epsilon(t)=\langle H_{\mathrm{YSYK}}(t)\rangle$ with that of a thermal ensemble $\langle\ldots\rangle_{T_{\mathrm{inst}}}$ by solving the self-consistency relation $\epsilon(t)=\langle H(t)\rangle_{T_{\mathrm{inst}}(t)}$, hence defining the instant temperature $T_{\mathrm{inst}}(t)$. For the current protocol $T_{\mathrm{inst}}(t)$ is constant for $t\gtrsim 0$, respectively, and agrees with T_{eff} in the long time limit (Fig. 2c), indicating thermalization[74]. This we confirm by also analyzing other thermodynamic observables and non-equilibrium occupation functions. The final temperature after the quench $T_{\mathrm{f}}=T_{\mathrm{eff}}(t\to\infty)$ is determined by a power law $(T_{\mathrm{i}}-T_{\mathrm{f}})\sim\omega_0(g_{\mathrm{f}}-g_{\mathrm{i}})^{x(g_{\mathrm{i}})}$ with weakly g_{i} dependent exponent. We universally find $T_{\mathrm{f}}>T_{\mathrm{c}}(g_{\mathrm{f}})$ (Fig. 2a), showing that no (thermal) superconductivity remains in the long time limit.

Indeed, we generally find that it is not possible to induce superconductivity by interaction quenches in the YSYK model, neither transiently nor at late times. The dynamics of the superconducting order parameter scales as $\Delta(t)\sim\mathcal{O}(\alpha)$ for all times (Fig. 2b), so that for the physical limit $\alpha\to 0$, no superconducting fluctuations remain. Instead, the non-Fermi liquid normal state of the YSYK models rapidly heats following the interaction quench, prohibiting the formation of cooper pairs. This rapid heating of the nFl state is consistent with observations made in the electronic SYK model[74]. These results remain true when considering quenches with finite, linear interaction ramps, which we discuss in Supplementary Note 3.

This is distinct from BCS theory, where one finds a coherently oscillating, yet finite order parameter following interaction quenches[84]. Further, it deviates from observations made for similar protocols in Hubbard models, treated perturbatively or with DMFT, where a transiently ordered state can emerge following an interaction quench[85–89].

Dressed statistics

Next, we consider the emergence of superconductivity due to light-induced cooling. This we model as the dynamical relaxation from an undercooled

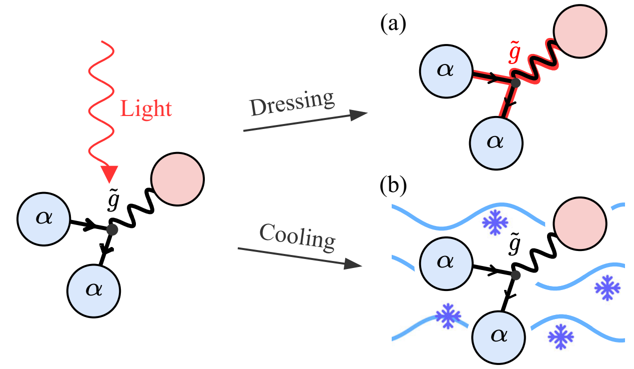

Fig. 1 | Cartoon of light irradiated YSYK model. Irradiation of the Yukawa–Sachdev–Ye–Kitaev (YSYK) model consisting of fermions with spin-α (blue) and bosons (red), randomly interacting with strength $\tilde{g}$, could dynamically induce superconductivity due to two effective mechanisms. **a** Dressing of coupling constants; here modeled as interaction quenches. **b** Cooling of the system; here modeled as the relaxation of an undercooled normal state into the superconducting ground state.

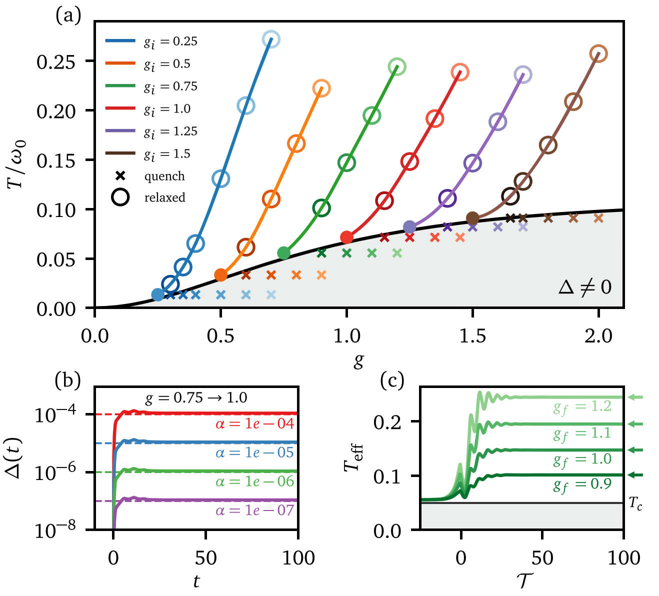

Fig. 2 | Quenches across superconducting transition for various interaction strengths g and dimensionless temperatures T/ω_0. **a** Starting from a normal state $T_i > T_c(g_i)$ (full circles) we quench into the superconducting phase (shaded gray) such that $T_i < T_c(g_f)$ (crosses). In the long time limit, the system always heats to $T_{\rm eff}(\infty) > T_c(g_f)$ (open circles), prohibiting the emergence of superconductivity. **b** Order parameter $\Delta(t)$ [Eq. (2)] for various symmetry-breaking strengths α. We observe $\Delta(t) \sim \mathcal{O}(\alpha)$, showing that no transient superconductivity emerges. **c** Effective Temperature $T_{\rm eff}$ extracted from generalized fluctuation–dissipation relation [Eq. (3)] for $g_i = 0.75$, together with $T_{\rm inst}$ (color-matched arrow), illustrating thermalization at late times.

YSYK normal state, into the superconducting ground state, following an explicit U(1) symmetry breaking [Eq. (4)]. The protocol approximates the quench-coupling of the YSYK model to a cool $T < T_c$, external bath, under the assumption that the cooling of the system is much faster than the relaxation into the ordered phase. An undercooled state, that is an unordered $\Delta = 0$ state with $T < T_c$, is unstable to such symmetry-breaking perturbations and its exact relaxation dynamics are shown in Fig. 3. Similar to the quench protocol above, we find that the system universally thermalizes in the long time limit, but now with $T_{\rm eff}(\infty) < T_c$, leading to emergent superconducting states.

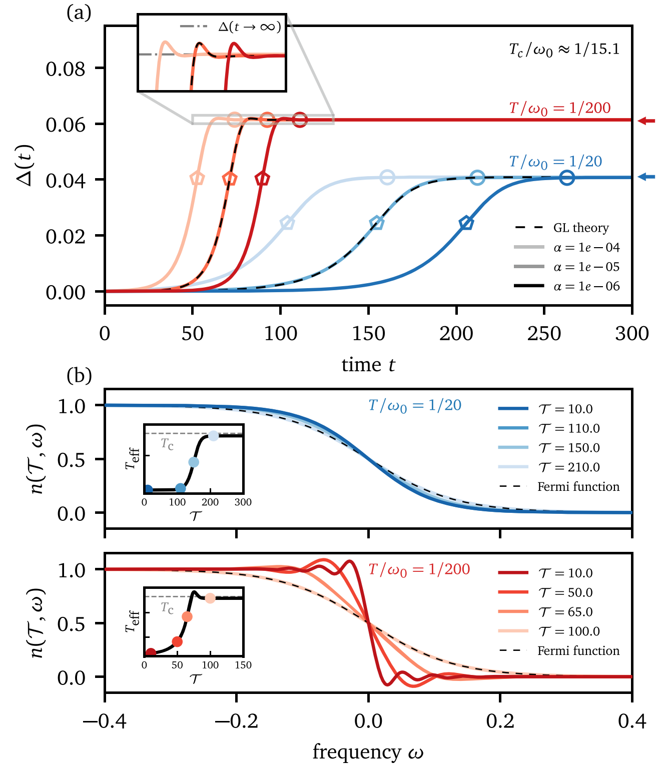

Fig. 3 | Dynamical relaxation from undercooled normal state into superconductivity at interaction strength $g = 1$. **a** Order parameter $\Delta(t)$ [Eq. (2)] for different symmetry-breaking strengths α [Eq. (4)] and dimensionless temperatures T/ω_0. Close to the critical temperature T_c (blue) the dynamics is overdamped while for $T \ll T_c$ (red) it becomes underdamped and oscillatory. Both regimes are well described by Ginzburg–Landau theory (dashed lines). The color-matched arrows mark the thermal order parameter Δ evaluated at $T_{\rm eff}(\infty)$, indicating thermalization. **b** Non-equilibrium occupation function $n(\mathcal{T}, \omega)$ [Eq. (3)] for different center mass times $\mathcal{T}$ indicated as dots in the effective temperature inset. Close to T_c the system is locally thermal and follows a hydrodynamic time evolution. For $T \ll T_c$ the early times are non-thermal and only at late times does the system thermalize.

The exact dynamics of the superconducting order parameter in shown in Fig. 3a. While the thermalization time t_* (circles) shows a strong dependence on α, the overall dynamics as well as the final state $\Delta(t > t_*)$ are independent of the symmetry-breaking strength. The latter agrees with the thermal gap function evaluated for $T_{\rm eff}(t \to \infty)$ (color-matched arrows), further illustrating thermalization at late times. Supplementary Note 4 discusses the dependence of the dynamics on the symmetry-breaking strength in more detail.

The early time dynamics ($t < \tilde{t}_*$; pentagon), are universally determined by an exponentially growing order parameter

$$\Delta(t) \sim e^{\Gamma(T)t} \tag{5}$$

with α independent relaxation rate $\Gamma(T)$, while at later times ($t > \tilde{t}_*$) the dynamics in the weak ($T \lesssim T_c$) and strong ($T \ll T_c$) undercooling regime is qualitatively different. Close to T_c it is underdamped, in the sense that $\delta(t) = \Delta(t) - \Delta(\infty)$ has no zero crossing, while for $T \ll T_c$ we observe oscillatory dynamics (Fig. 3a; inset). This difference can be understood intuitively. Close to T_c, the superconducting and normal states do not deviate strongly such that the ‘effective force’, driving the relaxation towards the ordered state is small compared to internal relaxation rates. Locally the system is in thermal equilibrium and the time evolution is hydrodynamic, with non-equilibrium occupation functions resembling Fermi-distributions (Fig. 3b; upper panel). For $T \ll T_c$ the ‘effective force’ becomes large compared to the internal relaxation rate so that the system is non-thermal at

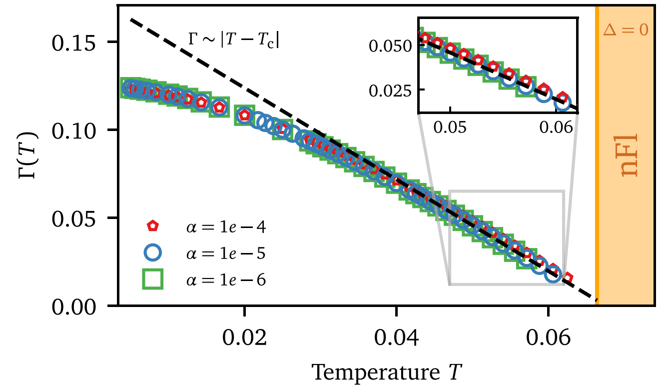

Fig. 4 | Relaxation rate $\Gamma(T)$ of superconducting order parameter from the early time dynamics $\Delta(t) \sim e^{\Gamma(T)t}$ at interaction strength $g = 1$. Γ extracted at different symmetry breaking strengths α (different shapes) becomes indistinguishable for $\alpha < 10^{-5}$ illustrating the α independence. Close to the critical temperature T_c we find $\Gamma \sim |T - T_c|$ in agreement with the overdamped Ginzburg–Landau prediction (dashed line). Deep within the superconducting phase $T \ll T_c$ the rate saturates. Note $\lim_{T \to T_c} \Gamma(T) = 0$, indicating the critical slowing down.

short and intermediate times and only thermalizes in the long time limit (Fig. 3b; lower panel).

The superconducting order parameter dynamics can be reproduced by time-dependent Ginzburg–Landau theory (GL) with $F_{GL} = c_1\Delta^2 + c_2\Delta^4$ and dynamical equation

$$\eta\partial_t^2\Delta(t) + \delta\partial_t\Delta(t) = -\frac{\partial F_{GL}}{\partial\Delta}. \tag{6}$$

For weak undercooling, the dynamics are described by overdamped GL ($\eta \to 0$), while the oscillatory case requires both finite η, δ to reproduce the exact data (dashed lines in Fig. 3a). BCS theory corresponds to the limit $\delta \to 0$, implying that the interactions effectively introduce damping for the order parameter dynamics. In Supplementary Note 5 we analyze the parameter dependence of the GL theory in more detail. The possibility of a full description in terms of Ginzburg–Landau theory further implies, that the emerging order parameter dynamics of the transition from nFl to superconductivity resembles that of a conventional superconductor. The nFl nature of the normal state only enters as numerical parameters in Eq. (6) and does not modify the phenomenology.

The temperature dependence of the relaxation rate $\Gamma(T)$ is a defining characteristic of the relaxation process. For $T \lesssim T_c$ it agrees well with the overdamped GL prediction $\Gamma(T) \sim |T - T_c|$, while it saturates for $T \ll T_c$ (Fig. 4). The reason for the different regimes is again intuitively understood via the 'effective force' argument. Close to T_c the speed of the relaxation dynamics is limited by the force, while for $T \ll T_c$ internal relaxation processes determine the relaxation timescale.

Generically, the relaxation rate vanishes as $\Gamma(T) \sim |T - T_c|^\gamma$ with critical exponent γ, a phenomenon known as critical slowing down[91,92]. From our exact dynamics, we find $\gamma = 1$, which despite the strong correlations and nFl normal state, agrees with time-dependent GL theory.

While the general structure of the relaxation is similar for different couplings g, we find that the dynamics are faster at strong coupling $\partial_g\Gamma(T) > 0$, but like the critical temperature saturates for $g \gtrsim 2$[51]. Further, the oscillatory order parameter dynamics is most pronounced for $g \lesssim 1$, while the amplitude of the oscillations becomes strongly suppressed at stronger interactions. We discuss these features in Supplementary Note 6.

Conclusion

We studied the dynamical onset of superconductivity in a paradigmatic, solvable toy model for strongly correlated superconductivity by analyzing two dynamical protocols. These are motivated by the mechanism of light-induced superconductivity; that is the dressing of Hamiltonian parameters and light-induced cooling.

Modeling the (quasi-static) dressing of Hamiltonian parameters, as interaction quenches across the superconducting phase transition we found that no superconductivity is induced due to excessive heating of the non-Fermi liquid normal state. This contrasts results obtained for BCS theory and DMFT studies in the Hubbard model and highlights how strong correlations can spoil the dynamic formation of order.

Imitating light-induced cooling by analyzing the dynamical relaxation from an undercooled YSYK normal state, we universally observed superconductivity at late times. Depending on the strength of the undercooling, we found overdamped or oscillatory relaxation dynamics towards these states. The resulting order parameter dynamics could be reproduced by time-dependent Ginzburg–Landau theory implying an analog phenomenology to the onset of superconductivity in a conventional superconductor, despite the strong correlations.

An exciting extension of our work is the investigation of periodically and parametrically driven YSYK models, as this protocol has been suggested to be able to induce superconductivity[19–21,27]. Our studies of the driven YSYK model revealed that driving leads to excessive heating, prohibiting the formation of superconductivity, but the additional coupling to a thermal bath[67,76–78,82] might mitigate this effect and lead to Floquet steady-states[93,94] that could host non-thermal light-induced superconductivity. This promises to further enhance our understanding of the non-equilibrium superconductivity in strongly correlated systems.

Note Added: After the completion of this work we learned about an independent study of the non-equilibrium dynamics of the YSYK normal state, following a quench coupling to an external reservoir[95]. This complements our work on the dynamical emergence of light-induced superconductivity.

Methods

In the thermodynamic limit $N \to \infty$ the model becomes self-averaging, which in this context means that each disorder realization is equivalent to the averaged theory, as well as that all Green's functions are diagonal in the site indices and the same on all sites. Further, it becomes exactly solvable due to the vanishing of vertex corrections. This allows the derivation of a closed system of self-consistent equations for fermion, boson, and anomalous Green's functions (GF) on the Keldysh contour. Focusing on s-wave superconductivity, we introduce Nambu vectors $\psi_i = (c_{i\uparrow}, c_{i\downarrow}^\dagger)^T$ and the disorder-averaged Nambu–Keldysh $\mathbf{G}(t,t')$ and bosonic $D(t,t')$ GFs defined by

$$\mathbf{G}(t,t') = -i/N\sum_i \overline{\left\langle T_C\left[\psi_i(t)\psi_i^\dagger(t')\right]\right\rangle}, \tag{7}$$

$$D(t,t') = -i/N\sum_k \overline{\left\langle T_C\left[\phi_k(t)\phi_k(t')\right]\right\rangle}, \tag{8}$$

where T_C denotes contour-time ordering along the Keldysh contour C[96]. The time evolution of Green's functions is governed by the Kadanoff–Baym equations (KBe)

$$\mathbf{G}(t,t') = \mathbf{G}_0(t,t') + \int_C \mathrm{d}1\mathrm{d}2\,\mathbf{G}_0(t,1)\mathbf{\Sigma}(1,2)\mathbf{G}(2,t'), \tag{9}$$

$$D(t,t') = D_0(t,t') + \int_C \mathrm{d}1\mathrm{d}2\,D_0(t,1)\Pi(1,2)D(2,t') \tag{10}$$

with exact fermionic and bosonic self-energies

$$\mathbf{\Sigma}(t,t') = ig^2D(t,t')\left[\sigma_3\mathbf{G}(t,t')\sigma_3\right], \tag{11}$$

$$\Pi(t,t') = -ig^2\mathrm{tr}\left[\sigma_3\mathbf{G}(t,t')\sigma_3\mathbf{G}(t',t)\right], \tag{12}$$

which we derive in Supplementary Note 1 using a path integral approach. The quench protocol with $g_{ijk} \to g_{ijk}f(t)$ modifies the self-energies as $g^2 \to g^2 f(t) f(t')$. Equations (9)–(12) represent the exact solution of the YSYK model, and they are structurally similar to the famous Migdal–Eliashberg equations of superconductivity[97], but on the Keldysh contour. Providing initial conditions in the lower quadrant of the two-time plane $(t, t' < 0)$ by solving the equilibrium equations via a fixed point iteration, we start the non-equilibrium protocol at $t = 0$ and follow the exact evolution of the system by integrating the resulting equations numerically using a predictor-corrector scheme; see Supplementary Note 2 for details.

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Data availability

The data that supports the findings of this study is available in a public repository[98].

Code availability

The code is available from the corresponding authors upon reasonable request.

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Author contributions

D.K. conceived the idea and supervised the project jointly with G.P. L.G. performed the theoretical and numerical analysis. All authors contributed ideas and worked on preparing the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

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Correspondence and requests for materials should be addressed to Lukas Grunwald or Dante M. Kennes.

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