Observation of interaction-induced phenomena of relativistic quantum mechanics

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Relativistic quantum mechanics has been developed for nearly a century to characterize the high-energy physics in quantum domain, and various intriguing phenomena without low-energy counterparts have been revealed. Recently, with the discovery of Dirac cone in graphene, quantum materials and their classical analogies provide the second approach to exhibit the relativistic wave equation, making large amounts of theoretical predications become reality in the lab. Here, we experimentally demonstrate a third way to get into the relativistic physics. Based on the extended one-dimensional Bose-Hubbard model, we show that two strongly correlated bosons can exhibit Dirac-like phenomena, including the Zitterbewegung and Klein tunneling, in the presence of giant on-site and nearest-neighbor interactions. By mapping eigenstates of two correlated bosons to modes of designed circuit lattices, the interaction-induced Zitterbewegung and Klein tunneling are verified by measuring the voltage dynamics. Our finding not only demonstrates a way to exhibit the relativistic physics, but also provides a flexible platform to further investigate many interesting phenomena related to the particle interaction in experiments.

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In 1928, the Dirac equation was proposed to describe the motion of fermionic particles in the relativistic region, and various interesting effects without low-energy counterparts were predicted afterwards¹. The two most famous phenomena are Zitterbewegung², referring to the rapid trembling motion of a free Dirac electron, and Klein tunneling³, where a below-barrier Dirac electron can pass through the large potential step without the exponential damping. Although these intriguing phenomena were already proposed for high-energy electrons, the experimental observation of relativistic effects is still an intractable challenge for particle physics. Because the trembling motion of a free relativistic electron has an extremely small amplitude (in the order of the Compton wavelength $\sim 10^{-12}$ m) and a high frequency ($\sim 10^{21}$ Hz). Meanwhile, the extremely high fields are required to accelerate the particle and a steep potential step should be constructed to observe the Klein tunneling.

Recently, the non-relativistic electrons in graphene monolayer were found to obey the massless relativistic dispersion known as the Dirac cone, leading to the realization of analogies to Zitterbewegung and Klein tunneling in low-energy condensed-matter physics and atom optics^{4–19}. Except for the quantum platform, many classical systems have also been proposed to mimic the conical singularity of energy bands in graphene, and the classical wave analogs of relativistic phenomena have been fulfilled latterly^{20–28}. These direct observations of relativistic effects in low-energy systems not only prove the predicted relativistic effects in high-energy physics, but also promote many applications in the field of signal processing, super-collimated beams, and communications.

In this work, we experimentally demonstrate a third way to get into the relativistic region relying on the particle interaction. Based on the extended one-dimensional Bose–Hubbard model, it is shown that two strongly correlated bosons (called doublons) obeying the massive Dirac equation can exhibit some relativistic effects, including the Zitterbewegung and Klein tunneling. By mapping eigenstates of two correlated bosons to modes of designed circuit lattices, these interaction-induced relativistic effects could be observed and manipulated in the designed electric circuits with extremely high degrees of freedom. Our proposal provides a useful laboratory tool to investigate and visualize many interesting effects related to the particle interaction, and possesses a great potential in the field of intergraded circuit design and electronic signal control.

Results

The theory of simulating interaction-induced relativistic effects by electric circuits. We consider a pair of correlated bosons hopping on a one-dimensional (1D) lattice where both the on-site and nearest-neighbor interactions exist. In this case, the system can be described by the extended version of the 1D Bose–Hubbard Hamiltonian as

$$H = -J \sum_l (a_l^\dagger a_{l+1} + a_{l+1}^\dagger a_l) + 0.5U \sum_l n_l(n_l - 1) + U' \sum_l n_l n_{l+1}, \quad (1)$$

where $a_l^\dagger$ (a_l) and $n_l = a_l^\dagger a_l$ are the creation (annihilation) and particle number operators for the boson at the l th site, respectively. J is the single-particle hopping rate between adjacent sites, and U (U') defines the on-site (nearest-neighbor) interaction energy. The two-boson solution can be expanded in the Fock space as

$$|\psi\rangle = \frac{1}{\sqrt{2}} \sum_{m,n=1}^N c_{mn} a_m^\dagger a_n^\dagger |0\rangle \quad (2)$$

Here, c_{mn} is the probability amplitude for the case with one boson at the site m and the other at the site n . Due to the bosonic nature, the probability amplitude should satisfy the relation of $c_{mn} = c_{nm}$. Substituting Eqs. (1) and (2) into the Schrödinger

equation $H|\psi\rangle = \varepsilon|\psi\rangle$, we obtain the eigen-equation with respect to c_{mn} as

$$\varepsilon c_{mn} = -J[c_{m(n-1)} + c_{m(n+1)} + c_{(m-1)n} + c_{(m+1)n}] + U\delta_{mn}c_{mn} + U'\delta_{m(n\pm 1)}c_{mn} \quad (3)$$

It is worthy to note that the above eigen-equation of the 1D two-boson system can be regarded as the eigen-equation describing a single particle in the 2D lattice^{29–33}, in which the evolution of two bosons in Fock space is mapped into the hopping dynamics of a particle on the 2D lattice.

Based on the similarity between circuit Laplacian and lattice Hamiltonian^{34–50}, electric circuits can be used as an extremely flexible platform to fulfill the mapped 2D lattice. The designed 2D circuit lattice is plotted in Fig. 1a. The capacitor C is used to connect a pair of adjacent circuit nodes. The inductor $L_g()$ is selected to connect each node to the ground. And, capacitors C_U and $C_{U'}$ are used to ground nodes on the main ($m = n$, red nodes) and first two lateral ($m = n + 1$ or $m = n - 1$, blue nodes) diagonals, respectively. The top insets present the detailed ground setting of circuit nodes enclosed by frames with consistent colors. Through the appropriate setting of grounding and connecting, the circuit eigen-equation can be derived as

$$\left(\frac{f_0^2}{f^2} - 4\right)V_{mn} = -V_{m(n+1)} - V_{m(n-1)} - V_{(m+1)n} - V_{(m-1)n} + \left(\frac{C_U}{C}\delta_{mn} + \frac{C_{U'}}{C}\delta_{m\pm 1,n}\right)V_{mn}, \quad (4)$$

where f is the eigen-frequency ($f_0 = 1/2\pi\sqrt{CL_g}$) of the designed circuit and V_{mn} represents the voltage at the circuit node (m, n). The details for the derivation of circuit eigen-equations are provided in Supplementary Note 1.

It is clearly shown that the eigen-equation of the designed electric circuit possesses the same form as the 1D two-boson model where the eigen-energy (ε) of two bosons is directly related to the eigen-frequency (f) of the designed circuit as $\varepsilon = f_0^2/f^2 - 4$ with other parameters being $J = 1$, $U = C_U/C$, and $U' = C_{U'}/C$. In such an analogy, the probability amplitude for the 1D two-boson model with one boson at the site m and the other at the site n is directly mapped to the voltage at the circuit node (m, n). Specifically, the circuit node on the main diagonal ($m = n$, red dots), first two lateral diagonals ($m = n \pm 1$, blue dots) and other sites ($|m - n| > 1$, black dots) correspond to the state of two bosons located at the same site, two nearest-neighbor sites and two distant sites, respectively, as illustrated by bottom insets in Fig. 1a. In this case, the coupling capacitor C along a certain direction corresponds to the hopping of one boson in the 1D two-body model. The grounding capacitor at the main ($m = n$, C_U) and first two lateral ($m = n \pm 1$, $C_{U'}$) diagonals can mimic the on-site and nearest-neighbor interactions of two bosons, respectively. With a good correspondence between the designed 2D circuit and the 1D two-body Bose–Hubbard model, the behavior of two correlated bosons in the 1D lattice can be effectively simulated by the designed 2D circuit.

Using the designed 2D circuit, it is very convenient to simulate the 1D two-boson system with extremely strong interactions, that is $C \ll C_U$, $C_{U'} (J \ll U, U')$. In the strong interaction region, the coupling between two-boson bound states (doublon), which correspond to two strongly correlated bosons locating at the same site or nearest-neighbor sites, and two-boson scattering states (two bosons are far away from each other without interactions) can be ignored. Hence, the effective model of doublons can be viewed as the binary superlattice where the one corresponds to the state of two bosons locating at the same node c_{nn} and the other corresponds to two bosons at adjacent nodes $\frac{1}{\sqrt{2}}(c_{n,n+1} + c_{n+1,n})$. In this case, the dispersion of circuit binary

Fig. 1 The designed electric circuit for simulating interaction-induced relativistic effects. **a** The top view of the designed 2D circuit. The red, blue and black nodes represent the circuit nodes corresponding to the probability amplitude of two-bosons in the same site, nearest neighbor sites and distant sites, respectively, as illustrated by three bottom insets. The top insets present the grounding setting of different circuit nodes framed by box with the same color. **b** The Dirac-like dispersion curves of designed circuit (in the unit of f_0) for the two-boson bound states. The circuit parameters are chosen as $C_U/C = 15$ and $C_{U'}/C = 11-15$. C_U and $C_{U'}$ are grounding capacitors on the main and first lateral diagonals.

superlattice of doublons reads explicitly as $f_{\pm}(k)^2 = f_0^2 \{ (C_U + C_{U'})/2C + 4 \pm \sqrt{[(C_U - C_{U'})/2C]^2 + 8\cos^2(ka)} \}^{-1}$ with k and a being the Bloch vector and lattice period (detailed derivations are given in Supplementary Note 2). Five sub-plots in Fig. 1b show the dispersion curves with $C_U/C = 15$ and $C_{U'}/C = 11-15$, respectively. It is clearly shown that the dispersion around $k = \pm 0.5\pi/a$ is similar to the case for a 1D freely moving Dirac particle with an effective mass, which could be easily controlled by the grounding capacitor $C_{U'}$ with a fixed value of C_U .

The exact analog between the circuit eigen-equation of doublons (around $k = 0.5\pi/a$) and the 1D massive Dirac equation can be obtained by setting the voltage at diagonals ($m = n$ and $m = n \pm 1$) as $V_m = (-1)^n \phi_n$, $V_{n(n+1)} + V_{n(n+1)} = -i\sqrt{2}(-1)^n \phi_n$, and considering n as a continuous variable. In such a case, the two-component voltage signal $V_n = (\phi_n, \phi_n)^T$ should satisfy the 1D (spinless) Dirac equation as

$$EV_n = i\sqrt{2}\sigma_x \frac{\partial}{\partial n} V_n + m\sigma_z V_n, \quad (5)$$

where σ_x and σ_z are Pauli matrices. The corresponding eigen-energy E and effective mass m are written as $E = \left(\frac{f_0^2}{f_2^2} - 4 - \frac{C_U + C_{U'}}{2C} \right)$ and $m = \frac{C_U - C_{U'}}{2C}$, respectively. With the consistency of eigen-equation between circuit counterpart of doublons and the 1D spinless Dirac equation, we expect that the designed electric circuit could act as a

classical simulator for the interaction-induced relativistic effects, including Zitterbewegung and Klein tunneling, of doublons.

Observing the interaction-induced Zitterbewegung in electric circuits. In this part, we investigate the interaction-induced Zitterbewegung effect of doublons based on the designed electric circuit with 31×31 circuit nodes (corresponding to the 1D two-boson model with $N = 31$ lattice sites). Here, the value of C , C_U and L_g are taken as 1, 15 nF and 3.3 μ H (the same parameters of these elements are used below), respectively, indicating the strong on-site interaction between two bosons ($C_U \gg C$).

Firstly, to observe the Zitterbewegung of doublons in our designed circuits, we perform the time-domain simulation of voltage dynamics using the LTSpice software, where nine circuit nodes on diagonals are simultaneously excited to introduce an initial wave packet of doublons. To match the required phase distribution for exhibiting Dirac-like behavior (around $k = \pm 0.5\pi/a$), the input voltage at nine circuit nodes are set as $V_{17,17} = V_0 e^{i\pi}$, $V_{16,17} = V_0 e^{i\pi/2}$, $V_{17,16} = V_0 e^{i\pi/2}$, $V_{16,16} = V_0$, $V_{16,15} = V_0 e^{-i\pi/2}$, $V_{15,16} = V_0 e^{-i\pi/2}$, $V_{15,15} = V_0 e^{-i\pi}$, $V_{1,1} = V_0 e^{-i3\pi/2}$ and $V_{31,31} = V_0 e^{i3\pi/2}$ ($V_0 = 4$ V), respectively. In addition, it is worthy to note that the oscillation frequency and amplitude of the input packet are both related to the Dirac-mass term, which depends on the unbalanced on-site and nearest-neighbor interactions of doublons ($m = \frac{C_U - C_{U'}}{2C}$) in the designed circuit simulator. In this case, the effective Dirac mass can be easily tuned by changing the grounding capacitor $C_{U'}$ on the

Fig. 2 The results for observing interaction-induced Zitterbewegung of doublons in electric circuits. **a** The red, blue, and green lines correspond to simulated results of the center mass ($\langle n \rangle$) for the input voltage-packet as a function of time with $C_U = 11, 12$ and 13 nF, respectively. **b** The left chart plots the photograph image of the fabricated circuit, and the corresponding enlarged view is presented in right charts. The red, blue, and green circles enclose the grounding capacitor C_U , C_U , on the main and first lateral diagonals and grounding inductor L_g , respectively. The value of C , C_U , and L_g are taken as $1, 15$ nF and 3.3 μ H, respectively. **c** The measured results of the center mass in the time domain are presented by red, blue and green lines with $C_U = 11, 12$ and 13 nF, respectively.

first two lateral diagonals, giving us an extremely flexible freedom to investigate the Zitterbewegung effect with the altered mass term. It is noted that the excitation frequency should locate near the massive Dirac point. In this case, we set $f = 0.2294$ and $f_0 = 0.6356$ MHz, that locates at the band edge for the positive-energy branch of doublons ($E = U$). When the excitation frequency is deviated from the massive Dirac point, the Zitterbewegung of doublons is gradually disappeared (see Supplementary Note 3 for details).

The red, blue, and green lines in Fig. 2a present the simulated envelope curve of the center mass ($\langle n \rangle$) for the input voltage-packet with $C_U = 11, 12$ and 13 nF, respectively. Here, the center mass of voltage-packet is defined as $\langle n \rangle = \{\sum_{n < N} [(2n-1)V_{nn}^2 + 2nV_{n(n+1)}^2 + 2nV_{(n+1)n}^2] + (2N-1)V_{NN}^2\} / \{\sum_{n < N} [V_{nn}^2 + V_{n(n+1)}^2 + V_{(n+1)n}^2] + V_{NN}^2\}$, corresponding to center mass for the probability of doublons. The time for calculations is up to 55μ s. After 55μ s, the circuit gradually goes into the steady-state without remarkable trembling of $\langle n \rangle$. It is clearly shown that the trembling motion of $\langle n \rangle$

is observed. With C_U being decreased, the oscillation frequency gets increased, and the oscillation amplitude is decreased. These phenomena are consistent with the dynamics of probability amplitudes c_{mn} determined by the 1D Bose-Hubbard model (see Supplementary Note 4 for details). Moreover, we also calculate the dynamics of voltage with weak interaction strengths and find that the Zitterbewegung effect is disappeared (see Supplementary Note 5 for details), indicating only two strongly correlated bosons can exhibit the trembling motion. This is due to the fact that only two strongly correlated bosons can form a bound state, which sustains Dirac-like dispersion with both positive and negative energy bands, as shown in Fig. 1b. In this case, the Zitterbewegung could be realized by the interference between the effective positive and negative energy states of doublons.

To experimentally demonstrate the interaction-induced Zitterbewegung of doublons, we fabricated the designed electric circuit. The top view of the fabricated circuit is presented at the left chart of Fig. 2b, and enlarged views of the front and back sides are

shown in the right chart. Here, a single printed circuit board (PCB), which contains 31×31 circuit nodes, is applied. The adjacent nodes are connected through the capacitor C (framed by black circles) and the grounding capacitors C_U and $C_{U'}$ on the main and first lateral diagonals ($m=n$ and $m=n \pm 1$) are enclosed by the red and blue circles in the front side of the sample. The grounding inductor L_g is marked by the yellow block as shown in the back side of the circuit. Additionally, the tolerance of the circuit elements is only 1% to avoid the detuning of circuit responses, and the quality factor of inductors is relatively high to avoid the influence of losses. Details of the sample fabrication is provided in the “Methods” section.

We measure the temporal dynamics of three fabricated electric circuits with $C_U = 11, 12$ and 13 nF, respectively, where nine circuit nodes are simultaneously excited to inject an initial wave-packet of doublons (the same to the case used in simulations). Details of the experimental measurements are also provided in the “Methods” section. The red, blue, and green lines in Fig. 2c display the measured envelope curve of center mass ($\langle n \rangle$) in the time-domain. Comparing Fig. 2c with Fig. 2a, it is found that the agreement between the experimental results and numerical simulations is very well, meaning that the interaction-induced Zitterbewegung of doublons has been demonstrated experimentally.

Observing the interaction-induced Klein tunneling in electric circuits. Except for the Zitterbewegung effect, another interesting phenomenon rooted in the Dirac equation is the Klein tunneling. In this part, we focus on the observation of Klein tunneling of doublons by designed electric circuits. To investigate the tunneling behaviors of two correlated bosons, the potential step P_l should be added to the 1D extended Bose–Hubbard model with $P_l = \Delta(P_l = 0)$ in the range of $l \geq (N+1)/2$ ($l < (N+1)/2$). In this case, the eigen-equation of two correlated bosons becomes:

$$\begin{aligned} \varepsilon_{mn} = & -J[c_{m(n-1)} + c_{m(n+1)} + c_{(m-1)n} + c_{(m+1)n}] \\ & + [U\delta_{mn} + U'\delta_{m\pm 1,n} + P_m + P_n]c_{mn} \end{aligned} \quad (6)$$

It is noted that the potential step could also be mapped to the position-dependent onsite potential in the associated 2D lattice. For the circuit simulator, such a requirement can be easily realized by adding position-dependent grounding capacitors. In this case, the circuit eigen-equation is written as

$$\begin{aligned} \left(\frac{f_0^2}{f^2} - 4\right)V_{mn} = & -V_{m(n+1)} - V_{m(n-1)} - V_{(m+1)n} - V_{(m-1)n} \\ & + \left(\frac{C_U}{C}\delta_{mn} + \frac{C_{U'}}{C}\delta_{m\pm 1,n} + \frac{C_{P_m}}{C} + \frac{C_{P_n}}{C}\right)V_{mn} \end{aligned} \quad (7)$$

where C_p is the grounding capacitor related to the position-dependent potential step. Figure 3a shows the image of the fabricated circuit (31×31) sustaining the potential step. The dark brown frames in the front side of the sample enclose the grounding capacitors $C_p = 2\Delta C$ related to the potential step in the first quadrant ($n > 16, m > 16$). And, the light brown frames mark the grounding capacitors $C_p = \Delta C$, which correspond to the potential step in the second ($n \leq 16, m > 16$) and third ($n > 16, m \leq 16$) quadrants.

At first, we perform the simulation of voltage tunneling on three diagonals ($n = m, m \pm 1$) using LTspice to observe the Klein tunneling of doublons. The value of $C(L_g)$, C_U and $C_{U'}$ are taken as 1 nF ($3.3 \mu\text{H}$), 15 and 12 nF, respectively. The excitation frequency and position are set as $f = 0.6195$ MHz (locating at the effective positive-energy band of doublons $E = U + 1$) and ($n = 9, m = 9$), respectively. The red line in Fig. 3b displays the variation of normalized transmissivity of doublons, which is defined as $|V_{20,20}|^2/|V_{9,9}|^2$, with $\Delta = 0.5, \Delta = 1.5, \Delta = 2.35$ and $\Delta = 5$. In

addition, Fig. 3c–g show the calculated circuit dispersions of doublons with $\Delta = 0.0, \Delta = 0.5, \Delta = 1.5, \Delta = 2.35$, and $\Delta = 5$, respectively. Here, the excitation frequency is set as $f = 0.2236$ and $f_0 = 0.6195$ MHz, that is around the massive Dirac point (marked by the dash line in Fig. 3c). If the excitation frequency is far away from the massive Dirac point, the Klein tunneling of doublons is disappeared (see Supplementary Note 3 for numerical results with different excitation frequencies).

It is found that the voltage signal can pass through the interface of barrier ($n = 16$) when the potential step is relatively low ($\Delta = 0.5$). This is consistent with the fact that there is still a large overlap of the excited minibands between two regions, as shown in Fig. 3c and d. As the potential step being increased ($\Delta = 1.5$), the voltage signal is nearly fully reflected from the potential barrier. Because there is no overlap of the eigen-band between two regions, as plotted in Fig. 3c and e. If the barrier height is further increased ($\Delta = 2.35$), the effective positive-energy band of doublons in the $n < 16$ region gets overlapped with the effective negative-energy band in the region of $n \geq 16$ (as shown in Fig. 3c and f). Hence, the voltage signal can go through the potential step again, that corresponds to the Klein tunneling of a massive relativistic particle. Note that, since we are dealing with Klein tunneling of a massive particle, the transmitted voltage from the potential step is not complete³. Finally, when the potential step becomes sufficiently large ($\Delta = 5$), there is no overlap between energy band in two regions (as shown in Fig. 3c and g) and the voltage signal is fully reflected. The tunneling behavior of designed circuit simulators is consistent with that of the probability amplitudes c_{mn} determined by the 1D Bose–Hubbard model (see Supplementary Note 4 for details). For comparison, in Supplementary Note 4, we also calculate the dynamics of voltage in the designed circuit with the weak interaction strength. It is found that the weakly interacting bosons are scattered off by the potential barrier, and no Klein-like tunneling appears. Hence, we note that only two strongly correlated bosons can penetrate the high barrier with an assistance of the inter-band tunneling resulting from the Dirac-like dispersion of doublons.

To further prove realization of Klein tunneling of doublons, we measure the transmissivity of four fabricated circuit with different values of the potential step ($\Delta = 0.5, \Delta = 1.5, \Delta = 2.35$ and $\Delta = 5$), as plotted by green dots in Fig. 3b. It is clearly shown that the measured transmissivity is consistent with numerical simulations, which manifests the fulfillment of simulating interaction-induced Klein tunneling of doublons. The slight deviation of the measured result is mainly due to the lossy and disorder effects existed in the circuit (see Supplementary Note 6 for details about the influence of disorder effects). It is worthy to note that the unity transmittance should appear around the gapless Dirac point. To exhibit such a novel effect, we tune the circuit simulator to the massless domain by setting the same value of the on-site and nearest-neighbor interaction energies ($C_U = C_{U'} = 15$ nF). In this case, we find that the nearly total transmission could appear with different potential steps when the excitation frequency is around the Dirac point (see Supplementary Note 7 for detailed results).

Conclusion

With the advantage of diversity and flexibility for circuit elements, except for the above designed LC circuit with matched stationary eigen-equations to the 1D two-boson system, we can design another kind of electric circuit, which is based on resistances and capacitances, to precisely match the time-dependent Schrödinger equation of two correlated bosons. In this case, the Dirac-like phenomena of doublons can also be observed in the designed RC circuit with consistent time evolution behaviors. See detailed results in Supplementary Note 8.

In conclusion, we have experimentally demonstrated that electric circuits can be used as a flexible simulator to investigate the

Fig. 3 The results for observing interaction-induced Klein tunneling of doublons in electric circuits. **a** The image of the fabricated circuit for simulating the Klein tunneling of doublons. The dark and light brown frames in the front side of the sample enclose the grounding capacitors related to the potential step in the first and second/third quadrants, respectively. The black frames in the front side of the sample mark C. The red, blue and yellow circles enclose the grounding capacitor C_U , C_L , on the main and first lateral diagonals and grounding inductor L_g , respectively. The value of C_U , C_L , and L_g are taken as 1, 15, 12 nF and 3.3 μ H, respectively. **b** The calculated and measured transmissivities with $\Delta = 0.5$, $\Delta = 1.5$, $\Delta = 2.35$ and $\Delta = 5$, respectively. The error bar, which corresponds to the difference for the measured transmission at many times, is much smaller than the corresponding data points. **c–g** show the calculated circuit dispersions of doublons with $\Delta = 0.0$, $\Delta = 0.5$, $\Delta = 1.5$, $\Delta = 2.35$ and $\Delta = 5$, respectively, and the dash lines mark the excitation frequency.

interaction-induced relativistic effects of doublons, whose dispersion contains two minibands being analogous to the positive- and negative-energy branches of the 1D Dirac equation. Using the exact mapping of two correlated bosons to modes of designed circuit lattices, both the Zitterbewegung and Klein tunneling of doublons have been observed in experiments by time-domain measurements. Because circuit networks possess remarkable advantages of being versatile and reconfigurable, the relativistic effects of doublons with various effective masses and potential steps could be easily fulfilled with suitable position-dependent grounding. By extending the current scheme to the system with a larger number of bosons, some other relativistic phenomena, such as the Weyl physics in 3D, may also be simulated. Our proposal provides a flexible simulator to investigate and visualize many interesting phenomena related to the particle interaction, and could be used in applications of the electronic signal control, such as the design of logic devices and switches as well as the electronic energy harvesting.

Methods

Sample fabrications and circuit signal measurements. We exploit the electric circuits by using PADS program software, where the PCB composition, stack up layout, internal layer and grounding design are suitably engineered. Here, the well-designed PCB possesses totally eight layers to arrange the intralayer and interlayer site-couplings. It is worthy to note that the ground layer should be placed in the gap between any two layers to avoid their coupling. Moreover, all PCB traces have a relatively large width (0.5 mm) to reduce the parasitic inductance and the spacing between electronic devices is also large enough (1.0 mm) to avert spurious inductive coupling. The SMP connectors are welded on the PCB nodes for the signal input. To ensure the realization of Zitterbewegung and Klein tunneling of

doublons in electric circuits, both the tolerance of circuit elements and series resistance of inductors should be as low as possible. For this purpose, we use WK6500B impedance analyzer to select circuit elements with high accuracy (the disorder strength is only 1%) and low losses.

As for the measurement of Zitterbewegung, we use the signal generator (NI PXI-5404) with eight output ports to act as the current source for exciting nine circuit nodes with a constant amplitude (4 V) and node-dependent initial phases. One output of the signal generator (the initial phase is set to 0) is directly connected to one end of the oscilloscope (Agilent Technologies Infiniivision DSO7104B) to make sure an accurate start time. The scanning speed of oscilloscope is set as 10 ms/s. Considering that there are four circuit nodes on diagonals possessing the same initial phase, the remained seven outputs of the current source are used to excite nine circuit nodes to introduce an initial wave packet of doublons. The measured voltage signals are in the range from 0 to 100 μ s in the time-domain, where the 0 μ s is defined as the time for the simultaneously signal injection and measurement.

In the experiment of Klein tunneling, the DG1022Z signal generator is used for the excitation, where the excitation node is selected in the third quadrant at the main diagonal ($m = 9$, $n = 9$). Similar to the case of Zitterbewegung, the voltage on the circuit node in the first quadrant ($m = 20$, $n = 20$) is measured by the oscilloscope before the reflected signals arrived. The measured transmissivity is normalized by the case without potential steps.

Data availability

All data files are available from the corresponding author upon reasonable request. Source data are provided with this paper.

Code availability

The code that supports the plots within this paper are available from the corresponding author upon reasonable request.

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References

- Greiner, W. *Relativistic Quantum Mechanics* (Springer Verlag, 1990).
- Huang, K. On the zitterbewegung of the Dirac electron. *Am. J. Phys.* **20**, 479 (1952).
- Klein, O. Die Reflexion von Elektronen an einem Potentialsprung nach der relativistischen Dynamik von Dirac. *Z. Phys.* **53**, 157 (1929).
- Novoselov, K. S. et al. Two-dimensional gas of massless Dirac fermions in graphene. *Nature* **438**, 197 (2005).
- Zhou, S. Y. et al. First direct observation of Dirac fermions in graphite. *Nat. Phys.* **2**, 595 (2006).
- Katsnelson, M. I., Novoselov, K. S. & Geim, A. K. Chiral tunnelling and the Klein paradox in graphene. *Nat. Phys.* **2**, 620 (2006).
- Cserti, J. & David, G. Unified description of Zitterbewegung for spintronic, graphene, and superconducting systems. *Phys. Rev. B* **74**, 172305 (2006).
- Lamata, L., León, J., Schätz, T. & Solano, E. Dirac equation and quantum relativistic effects in a single trapped ion. *Phys. Rev. Lett.* **98**, 253005 (2007).
- Rusin, T. M. & Zawadzki, W. Transient Zitterbewegung of charge carriers in mono- and bilayer graphene, and carbon nanotubes. *Phys. Rev. B* **76**, 195439 (2007).
- Vaishnav, J. Y. & Clark, C. W. Observing Zitterbewegung with ultracold atoms. *Phys. Rev. Lett.* **100**, 153002 (2008).
- Beenakker, C. W. J. Colloquium: Andreev reflection and Klein tunneling in graphene. *Rev. Mod. Phys.* **80**, 1337 (2008).
- Castro Neto, A. H. et al. The electronic properties of graphene. *Rev. Mod. Phys.* **81**, 109 (2009).
- Young, A. F. & Kim, P. Quantum interference and Klein tunneling in graphene heterojunctions. *Nat. Phys.* **5**, 222 (2009).
- Stander, N., Huard, B. & Goldhaber-Gordon, D. Evidence for Klein tunneling in graphene junctions. *Phys. Rev. Lett.* **102**, 026807 (2009).
- Steele, G. A., Gotz, G. & Kouwenhoven, L. P. Tunable few-electron double quantum dots and Klein tunnelling in ultraclean carbon nanotubes. *Nat. Nanotechnol.* **4**, 363 (2009).
- Gerritsma, R. et al. Quantum simulation of the Dirac equation. *Nature* **463**, 68–71 (2010).
- Greenaway, M. et al. Resonant tunnelling between the chiral Landau states of twisted graphene lattices. *Nat. Phys.* **11**, 1057–1062 (2015).
- Kaminer, I. et al. Self-accelerating Dirac particles and prolonging the lifetime of relativistic fermions. *Nat. Phys.* **11**, 261–267 (2015).
- Perconte, D. et al. Tunable Klein-like tunnelling of high-temperature superconducting pairs into graphene. *Nat. Phys.* **14**, 25–29 (2018).
- Zhang, X. Observing Zitterbewegung for photons near the Dirac Point of a two-dimensional photonic crystal. *Phys. Rev. Lett.* **100**, 113903 (2008).
- Zhang, X. & Liu, Z. Extremal transmission and beating effect of acoustic waves in two-dimensional sonic crystals. *Phys. Rev. Lett.* **101**, 264303 (2008).
- Wang, L.-G., Wang, Z.-G., Zhang, J.-X. & Zhu, S.-Y. Realization of Dirac point with double cones in optics. *Opt. Lett.* **34**, 1510 (2009).
- Dreisow, F. et al. Classical simulation of relativistic Zitterbewegung in photonic lattices. *Phys. Rev. Lett.* **105**, 143902 (2010).
- Bahat-Treidel, O. et al. Klein tunneling in deformed honeycomb lattices. *Phys. Rev. Lett.* **104**, 063901 (2010).
- Longhi, S. Photonic analog of Zitterbewegung in binary waveguide arrays. *Opt. Lett.* **35**, 235 (2010).
- Longhi, S. Klein tunneling in binary photonic superlattices. *Phys. Rev. B* **81**, 075102 (2010).
- Ni, X. et al. Spin- and valley-polarized one-way Klein tunneling in photonic topological insulators. *Sci. Adv.* **4**, eaap8802 (2018).
- Jiang, X. et al. Direct observation of Klein tunneling in phononic crystals. *Science* **370**, 1447 (2020).
- Longhi, S. Optical realization of the two-site Bose–Hubbard model in waveguide lattices. *J. Phys. B* **44**, 051001 (2011).
- Krimer, D. O. & Khomeriki, R. Realization of discrete quantum billiards in a two-dimensional optical lattice. *Phys. Rev. A* **84**, 041807 (2011).
- Longhi, S. & Della Valle, G. Klein tunneling of two correlated bosons. *Eur. Phys. J. B* **86**, 231 (2013).
- Corrielli, G. et al. Fractional Bloch oscillations in photonic lattices. *Nat. Commun.* **4**, 1555 (2013).
- Mukherjee, S. et al. Observation of pair tunneling and coherent destruction of tunneling in arrays of optical waveguides. *Phys. Rev. A* **94**, 053853 (2016).
- Albert, V. V., Glazman, L. I. & Jiang, L. Topological properties of linear circuit lattices. *Phys. Rev. Lett.* **114**, 173902 (2015).
- Ning, J., Owens, C., Sommer, A., Schuster, D. & Simon, J. Time and site resolved dynamics in a topological circuit. *Phys. Rev. X* **5**, 021031 (2015).
- Lee, C. et al. Topoelectrical circuits. *Commun. Phys.* **1**, 39 (2018).
- Imhof, S. B. et al. Topoelectrical-circuit realization of topological corner modes. *Nat. Phys.* **14**, 925 (2018).
- Helbig, T. et al. Band structure engineering and reconstruction in electric circuit networks. *Phys. Rev. B* **99**, 161114(R) (2019).
- Hofmann, T., Helbig, T., Lee, C., Greiter, M. & Thomale, R. Chiral voltage propagation and calibration in a topoelectrical Chern circuit. *Phys. Rev. Lett.* **122**, 247702 (2019).
- Bao, J. et al. Topoelectrical circuit octupole insulator with topologically protected corner states. *Phys. Rev. B* **100**, 201406(R) (2019).
- Ezawa, M. Electric circuit simulations of n th-Chern-number insulators in $2n$ -dimensional space and their non-Hermitian generalizations for arbitrary n . *Phys. Rev. B* **100**, 075423 (2019).
- Li, L., Lee, C. & Gong, J. Emergence and full 3D-imaging of nodal boundary Seifert surfaces in 4D topological matter. *Commun. Phys.* **2**, 135 (2019).
- Yu, R., Zhao, Y. & Schnuder, A. P. 4D spinless topological insulator in a periodic electric circuit. *Natl. Sci. Rev.* **7**, nwaa065 (2020).
- Zhang, W. et al. Topoelectrical-circuit realization of a four-dimensional hexadecapole insulator. *Phys. Rev. B* **102**, 100102(R) (2020).
- Helbig, T. et al. Generalized bulk–boundary correspondence in non-Hermitian topoelectrical circuits. *Nat. Phys.* **16**, 747 (2020).
- Wang, Y., Price, H. M., Zhang, B. & Chong, Y. D. Circuit implementation of a four-dimensional topological insulator. *Nat. Commun.* **11**, 2356 (2020).
- Olekhnko, N. et al. Topological edge states of interacting photon pairs realized in a topoelectrical circuit. *Nat. Commun.* **11**, 1436 (2020).
- Song, L., Yang, H., Cao, Y. & Yan, P. Realization of the square-root higher-order topological insulator in electric circuits. *Nano Lett.* **20**, 7566 (2020).
- Zhang, W. et al. Experimental observation of higher-order topological Anderson insulators. *Phys. Rev. Lett.* **126**, 146802 (2021).
- Liu, S. et al. Non-Hermitian skin effect in a non-Hermitian electrical circuit. *Research* **2021**, 5608038 (2021).

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Author contributions

W.X.Z. and H.Y. finished the theoretical scheme and designed the experiments. H.Y., W.J.H., X.G.Z., N.S. and F.X.D. finished experiments under the supervision of H.J.S. and X.D.Z. W.X.Z. and X.D.Z. wrote the manuscript. X.D.Z. initiated and designed this research project.

Competing interests

The authors declare no competing interests.

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