Towards communication in a curved spacetime geometry

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The current race in quantum communication – endeavouring to establish a global quantum network – must account for special and general relativistic effects. The well-studied general relativistic effects include Shapiro time-delay, gravitational lensing, and frame dragging which all are due to how a mass distribution alters geodesics. Here, we report how the curvature of spacetime geometry affects the propagation of information carriers along an arbitrary geodesic. An explicit expression for the distortion onto the carrier wavefunction in terms of the Riemann curvature is obtained. Furthermore, we investigate this distortion for anti de Sitter and Schwarzschild geometries. For instance, the spacetime curvature causes a 0.10 radian phase-shift for communication between Earth and the International Space Station on a monochromatic laser beam and quadrupole astigmatism; can cause a 12.2% cross-talk between structured modes traversing through the solar system. Our finding shows that this gravitational distortion is significant, and it needs to be either pre- or post-corrected at the sender or receiver to retrieve the information.

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Photons, electromagnetic waves, are widely used in classical and quantum communication since they do not possess electric charge or rest mass. However, a photon's traits, e.g. group and phase velocity, wavelength, linear and optical angular momentum, are modified inside or during propagation through a linear or a nonlinear medium. These traits are governed by Maxwell's equations, which are the relativistic quantum field theory of the $U(1)$ gauge connection. Understanding how these optical properties are altered upon propagation is a key element for any optical communication network. In optical communication, the sender and the receiver, namely Alice and Bob, use one or several internal photonic degrees of freedom, such as wavelength, polarisation, transverse mode or time-bins, to share information, including a ciphertext and the secret key to decrypt the ciphertext. The propagation, e.g. through fibre, air or underwater channels, causes these photonic degrees of freedom to be altered, and thus causes undesired errors on the shared information. Therefore, the alteration to those degrees of freedom for any communication channel needs to be considered and well examined. Sharing information with a longer range or with moving objects, e.g. satellites, airplanes, submarines^{1–5}, requires the optical beam to not only traverse through a medium, but in a few cases, also in the fabric of the spacetime geometry, where general relativistic effects manifest^{6–9}. Effects associated to the change of the geodesic due to a mass distribution, such as Shapiro time-delay¹⁰, gravitational lensing¹¹ and frame dragging¹², are well studied and observed^{13–16}.

We explore the propagation of relativistic wavepackets along an arbitrary null geodesic in a general curved spacetime geometry, and show how the curvature of the spacetime geometry distorts the wavepacket as it travels along the null geodesic. Different methods are used to tackle this study. For instance, the four-dimensional Klein-Gordon equation is approximated to a simple two-dimensional partial differential equation by ignoring all the multi-polar modes^{17,18}. In particular, in derivation of Eq. (7) from Eq. (6) in ref. ¹⁷, the second term on the left-hand side of Eq. (41) of ref. ¹⁹ has not been taken into account, therefore,^{17,18} cannot claim to reproduce all the effects of a curved spacetime geometry. In ref. ¹⁹, all the multi-polar ℓ modes are presented only at the level of the equations; however, the upper value of $\ell = 100$ on the multi-polar modes is considered to compute the solution. On the surface of the earth, a narrow beam with an initial width of 10 cm and a large value of Rayleigh range requires taking into account the contribution of multi-polar modes up to at least $\ell = 10^9$. So the solutions presented in ref. ¹⁹ do not take into consideration all the multi-polar modes required to calculate

the effects of the curvature at the vicinity of the Earth. Here, we present a computationally simple method to calculate the distortion by the curvature of any spacetime geometry on any localised wavepacket.

In flat spacetime geometry, the propagation of each polarisation of photon is isomorphic to the propagation of a massless scalar field. Based on our understanding of the Einstein equivalence principle, we expect that studying a massless scalar field theory also captures some features of a photon's propagation in curved spacetime geometry. Therefore, in this study, we consider the propagation of a massless scalar in the bulk of the paper. In Supplementary Note 2, we prove that in the Lorentz gauge, each linear polarisation of the photon in a curved spacetime geometry gets corrected as if it were a massless scalar field. It is reported that the Riemann tensor quantum mechanically alters the wavepacket propagating along a geodesic. The alteration operators depend on the geodesic and components of the Riemann tensor on the geodesic. The alteration is calculated for examples including the spacetime geometry around the Earth and the Sun.

Results and discussion

We start by considering a relativistic massless scalar field $\psi := \psi(x^\mu)$ that propagates in a curved spacetime geometry $x^\mu = (t, x^i) = (t, x, y, z)$ with an arbitrary Riemann curvature tensor R_{abcd} . The units are chosen such that the speed of light in vacuum and the reduced Planck constant are equal to one, i.e. $c = 1$ and $\hbar = 1$. Let us consider a localised wavefunction (information carrier) whose size is small compared to the curvature of the spacetime geometry. At the leading order, therefore, the carrier can be treated as a massless point-like particle that travels along a null geodesic γ , see Fig. 1a. We choose the local Fermi coordinates²⁰ along the geodesics in order to compute the quantum relativistic corrections. The metric's components in the Fermi coordinates can be expanded in terms of the components of the Riemann tensor R_{abcd} and its covariant derivatives evaluated on the geodesic, see Fig. 1b,c. The expansion of the metric up to quadratic order in the transverse coordinates of the geodesic of a massless particle γ is given by²¹,

$$ds^2 = 2dx^+ dx^- + \delta_{ab} dx^a dx^b - \left[R_{+\bar{a}+\bar{b}} x^{\bar{a}} x^{\bar{b}} (dx^+)^2 + \frac{4}{3} R_{+\bar{b}\bar{a}\bar{c}} x^{\bar{b}} x^{\bar{c}} (dx^+ dx^{\bar{a}}) + \frac{1}{3} R_{\bar{a}\bar{c}\bar{b}\bar{d}} x^{\bar{b}} x^{\bar{c}} (dx^{\bar{a}} dx^{\bar{b}}) \right] + \dots, \quad (1)$$

where $x^\pm = (x^3 \pm t)/\sqrt{2}$ represent the Dirac light-cone coordinates²² in the Fermi coordinates (x^+ is always tangent to

Fig. 1 Schematics of communication in a general curved spacetime geometry and proper chosen coordinates. **a** Two parties, namely Alice and Bob, communicate in a general curved spacetime geometry. Alice encodes her message in a sequence of information carriers and sends them to Bob. The information traverses through the spacetime over a geodesic γ . The physical traits of information are distorted by the curvature of the spacetime, causing errors to the field wavepacket. **b** The shown null geodesic γ . Close to γ , at the local coordinates, the metric is approximately pseudo-Euclidean. **c** The associated Fermi coordinates, the null-geodesic-path is mapped to x^+ .

the null geodesic Fig. 1b), δ_{ab} is the Kronecker delta, $\hat{y}$ is the tangent of the null geodesic and $a, b \in \{1, 2\}$, and $(x^a) = (x^-, x^a)$ and the curvature components are evaluated on γ . The tree-level action of a massless scalar field ψ in a general curved spacetime geometry is given by $S[\psi] = \frac{1}{2} \int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi$, where g is the determinant of the metric $g_{\mu\nu}$, and $\mu, \nu \in \{\pm, a\}$. Since the tree-level action is quadratic in terms of ψ , it is quantum mechanically exact, which can be verified by looking at its generating function, i.e. $Z[J] = \int \mathcal{D}\psi e^{-i(S[\psi] + \int d^4x \sqrt{-\det g} J \psi)} / \int \mathcal{D}\psi e^{-iS[\psi]} = \mathcal{D}\psi$ represents the integration over all field configurations and J is the source field. Its exact effective action, as defined by the Legendre transformation of $\ln Z$, coincides with the tree-level action, i.e. $\Gamma[\psi_c] = S[\psi_c]$. For a general action, ϕ_c resembles a "classical" field whose action is given by $\Gamma[\psi_c]$, while $\Gamma[\psi_c]$ encapsulates all the quantum loop corrections. The exact effective action includes both the classical and quantum effects. The classical effects are those that can be reproduced by motion of a point-like particle along the geodesic; the rest are quantum. The effective action of a free photon propagating in curved spacetime geometry coincides to the tree-level action, therefore, we omit the subscript c .

The massless scalar (quantum) field ψ obeys the (covariant) wave equation, $\square \psi = (-g)^{-\frac{1}{2}} \partial_\mu (-g)^{\frac{1}{2}} g^{\mu\nu} \partial_\nu \psi = 0$. In the Fermi coordinates, $g_{\mu\nu}$ can be viewed as a perturbation to the Minkowski metric, inducing expansion series for the inverse and determinant of the metric: $g^{\mu\nu} = \eta^{\mu\nu} + \varepsilon \delta g^{\mu\nu} + O(\varepsilon^2)$ and $\ln(\sqrt{-\det g}) = \varepsilon \delta g + O(\varepsilon^2)$. ε is the systematic perturbation parameter introduced to keep track of the perturbation series, which means that all the components of the Riemann tensor in Eq. (1) are multiplied with ε , and ε is treated as an infinitesimal parameter. At the end of the computation, we set $\varepsilon = 1$. This technique helps us to systematically perform perturbations for small curvatures. Utilising the perturbation gives,

$$\square \psi = \square^{(0)} \psi + \varepsilon (\partial_\mu (\delta g^{\mu\nu} \partial_\nu \psi) + \eta^{\mu\nu} \partial_\nu \delta g \partial_\mu \psi) + O(\varepsilon^2), \quad (2)$$

where $\square^{(0)} = \eta^{\mu\nu} \partial_\mu \partial_\nu = 2\partial_- \partial_+ + \nabla_\perp^2$ is the d'Alembert operator in the flat spacetime geometry, and $\nabla_\perp^2 = \partial_1^2 + \partial_2^2$. The perturbative nature of Eq. (2) seeks for a series expansion, $\psi = \psi^{(0)} + \varepsilon \psi^{(1)} + O(\varepsilon^2)$. Here, $\psi^{(0)}$ satisfies the scalar wave equation in the flat spacetime geometry $\square^{(0)} \psi^{(0)} = 0$, and the perturbed term to the wavefunction, $\psi^{(1)}$, yields,

$$\square^{(0)} \psi^{(1)} = -\partial_\mu (\delta g^{\mu\nu} \partial_\nu \psi^{(0)}) + \eta^{\mu\nu} \partial_\nu \delta g \partial_\mu \psi^{(0)}. \quad (3)$$

We assume the Fourier expansion in terms of x^- variable, $\psi^{(0)} = \int d\omega f_\omega^{(0)}(x^+, x^a) e^{i\omega x^-}$, where $f_\omega^{(0)}$ satisfies the paraxial equation $(2i\omega \partial_+ + \nabla_\perp^2) f_\omega^{(0)} = 0$. This implies that the solutions, given by the paraxial approximation in optics^{23,24}, are exact. The paraxial equation is isomorphic to the Schrödinger equation, and its solutions (the transverse and longitudinal parts) can be expressed in the form of Laguerre-Gauss (LG) modes (with an azimuthally symmetric intensity profile) or Hermite-Gaussian (HG) wavepackets²⁵. We consider a wavepacket wherein the field is slowly varying, and assume that the metric does not significantly change inside the wavepacket. Therefore, all derivatives of $\partial_\mu \psi^{(0)}$, except $\partial_- \psi^{(0)}$, are negligible, and the leading term in the right hand side of Eq. (3) is $\partial_- (\delta g^- - \partial_- \psi)$, where $\delta g^- = -g_{++}^{(1)} = R_{+\bar{a}+\bar{b}} x^{\bar{a}} x^{\bar{b}}$. Therefore, Eq. (3) reduces to,

$$\square^{(0)} \psi^{(1)} = -R_{+\bar{a}+\bar{b}} x^{\bar{a}} x^{\bar{b}} \partial_-^2 \psi^{(0)}. \quad (4)$$

Here, $\psi^{(1)} = \int d\omega f_\omega^{(1)}(x^+, x^a) e^{i\omega x^-}$ with $f_\omega^{(1)}$ being the correction to the structure function for the frequency of ω . We have found the solutions to Eq. (4)—see the Supplementary Note 1 for more detail on the derivation. The solution is,

$$\psi(x^\mu) = \int d\omega \sum_{p,\ell,n} c_{p,\ell,n}(\omega) e^{i\omega x^-} (1 + \varepsilon(\mathcal{O}^\omega + \mathcal{Q}_U^\omega + \mathcal{Q}_N^\omega)) f_\omega^{(0)},$$

where $c_{p,\ell,n}(\omega)$ are defined based on the initial and boundary conditions. The operators, $\mathcal{O}^\omega$, $\mathcal{Q}_U^\omega$ and $\mathcal{Q}_N^\omega$ encodes how the curvature of the spacetime geometry distorts the wavepacket. They are given by,

$$\begin{aligned} \mathcal{O}^\omega &= -\frac{i\omega}{2} \mathcal{G}_{ab} x^a x^b + \tilde{\mathcal{G}}_{ab} x^a \partial^b + \frac{i}{\omega} \tilde{\mathcal{G}}_{ab} \partial^a \partial^b, \\ \mathcal{Q}_U^\omega &= \frac{i}{\omega} \left(1 - \frac{\omega^2}{2} (x^-)^2 \right) \mathcal{G}_{--} - ix^- \omega x^a \mathcal{G}_{-a} + x^- \tilde{\mathcal{G}}_{-a} \partial^a, \\ \mathcal{Q}_N^\omega &= x^- \mathcal{G}_{--} + \mathcal{G}_{-a} x^a + \frac{2i}{\omega} \tilde{\mathcal{G}}_{-a} \partial^a, \end{aligned} \quad (5)$$

where $\mathcal{G}_{ab}$, $\tilde{\mathcal{G}}_{ab}$ and $\tilde{\mathcal{G}}_{ab}$ are integrals of the components of the Riemann tensor $R_{+\bar{a}+\bar{b}}$ evaluated on the geodesic—see Supplementary Note 1:

$$\mathcal{G}_{\bar{a}\bar{b}} = \int_0^\tau d\tau R_{+\bar{a}+\bar{b}}, \quad \tilde{\mathcal{G}}_{\bar{a}\bar{b}} = \int_0^\tau d\tau \mathcal{G}_{\bar{a}\bar{b}}, \quad \tilde{\mathcal{G}}_{\bar{a}\bar{b}} = \int_0^\tau d\tau \tilde{\mathcal{G}}_{\bar{a}\bar{b}}, \quad (6)$$

where τ is the affine parameter on the geodesic. The distortions provided by Eq. (5) is the solution to the exact quantum effective action and cannot be reproduced by motion of a point-like particle along a geodesic. They do not exist in flat spacetime geometry, so they manifest a set of quantum effects in curved spacetime geometry. A similar approach can be used to find the wavefunction of a massive scalar particle. The physical degrees of $U(1)$ gauge fields get corrected as if they were scalar fields, see Supplementary Note 2 and Supplementary Note 3. Supplementary Note 4 presents the operators in the Hilbert space that corresponds to these corrections. The following subsections show how these operators distort the physical information encoded in wavepackets traveling along couple of examples of null geodesics in the Solar system and around the Earth.

We now study the distortion operators in a couple of spacetime geometries, including de Sitter and Schwarzschild spacetime geometries. We first study the de Sitter and anti de Sitter spacetime geometries because their symmetry allows one to immediately write down the components of the Riemann tensor in Fermi coordinates evaluated on the geodesic.

de Sitter and anti de Sitter spacetime geometries. The de Sitter and anti de Sitter space-times are maximally symmetric and the Riemann tensor at any given event in the spacetime in any coordinates, including the Fermi coordinates, is given by $R_{\mu\nu\mu'\nu'} = \Lambda(g_{\mu\mu'} g_{\nu\nu'} - g_{\mu\nu} g_{\nu'\mu'})$. The value of Λ determines different geometries: $\Lambda > 0$ represents the de Sitter spacetime geometry; $\Lambda < 0$ represents the anti de Sitter spacetime geometry; and $\Lambda = 0$ is the Minkowski spacetime geometry. $R_{+\bar{a}+\bar{b}} = \Lambda$ is the only non-zero component for $R_{+\bar{a}+\bar{b}}$ evaluated on the geodesic, and thus the correction operators, Eq. (5) are,

$$\mathcal{O}^\omega = 0, \quad \mathcal{Q}_U^\omega = \frac{i}{\omega} \left(1 - \frac{\omega^2}{2} (x^-)^2 \right) \Lambda x^+, \quad \mathcal{Q}_N^\omega = \Lambda x^+ x^- . \quad (7)$$

Let us consider a Gaussian wavepacket with normal distribution for ω around ω_0 with the width of σ , i.e. $\psi_{\text{Alice}} = f^{(0)}(x^+, x^1, x^2) e^{i\omega_0 x^-} e^{-\frac{(x^-)^2}{2\sigma^2}}$. The validity of the perturbative solution demands that $|\Lambda| \ll \omega_0^2$, $|\Lambda| \ll \sigma^2$ and $\sigma \ll \omega_0$. The wavepacket after the propagation is $\psi_{\text{Bob}} \simeq (1 + \varepsilon \mathcal{Q}_U^{\omega_0} + \varepsilon \mathcal{Q}_N^{\omega_0}) e^{i\omega_0 x^-} f^{(0)} e^{-\frac{(x^-)^2}{2\sigma^2}}$.

$\mathcal{Q}_N^{\omega_0}$ and $\mathcal{Q}_U^{\omega_0}$ change the wavepacket amplitude and phase, respectively. The maximum of $|\mathcal{Q}_N^{\omega_0} \psi_{\text{Alice}}|$ occurs at $x^- = \pm 1/\sigma$. Requiring it to be smaller than 1 yields $T < \tau_A$ where $\tau_A = (\sigma \sqrt{e})/|\Lambda|$. τ_A is the maximum time that the wavepacket feels the curvature of the spacetime geometry and keeps its amplitude intact. $\mathcal{Q}_U^{\omega_0}$ alters the phase of the wavepacket. The maximum of $|\mathcal{Q}_U^{\omega_0} \psi_{\text{Alice}}|$ occurs at $x^- = \pm \sqrt{2}/\sigma$. Requiring it to be smaller than 1 results in $T < \tau_\varphi$ where $\tau_\varphi = (\sigma \tau_A \sqrt{e})/(2\omega_0)$.

τ_φ is the maximum time that a wavepacket can feel the curvature of the spacetime geometry and keep its phase intact. For $T > \tau_\varphi$, information about phase is lost at perturbation. This may point to a “gravitational decoherence”, and its possible consequence on anti de Sitter/conformal field theory correspondence²⁶ demands attention. τ_A represents the amount of time of interaction with the curvature that the wavepacket can keep its amplitude intact. We observe that $\tau_\varphi \ll \tau_A$. So the phase changes sooner than the change in the amplitude.

Schwarzschild spacetime geometry. We choose the standard spherical coordinates r, θ, φ where geodesics are extrema of,

$$\mathcal{L} = -\left(1 - \frac{m}{r}\right)\dot{t}^2 + \left(1 - \frac{m}{r}\right)^{-1} \dot{r}^2 + r^2(\dot{\theta}^2 + \sin^2\theta\dot{\varphi}^2), \quad (8)$$

and $m = 2GM_s$ is the Schwarzschild radius, M_s is the mass of the blackhole and G is the gravitational constant—the units are such that $m = 1$. The components of the Riemann tensor in the Fermi coordinates adapted to a general null geodesic of Schwarzschild spacetime geometry are derived in the Supplementary Note 5. Figure 2a shows several null geodesics that go very close to a blackhole. We first consider that the wavepacket propagates along the radial direction, Fig. 2b, where the only non-zero components of the Riemann tensor is $R_{+-+-} = -1/r^3$. This is the same component that appeared in the de Sitter spacetime geometry. The radial geodesic has $l = 0$, and its correction operators are $\mathcal{O}^\omega = 0$, $\mathcal{Q}_U^\omega = -\frac{i}{2\omega}\left(1 - \frac{(\omega x^-)^2}{2}\right)\left(\frac{1}{r_a^2} - \frac{1}{r^2}\right)$, and $\mathcal{Q}_N^\omega = -\frac{1}{2}\left(\frac{1}{r_a^2} - \frac{1}{r^2}\right)x^-$, where Alice is located at r_a . These correction terms do not contain derivatives of the spatial transverse coordinates. Thus, the Riemann tensor does not affect the spatial transverse profile of the wavepacket. This is due to the symmetry, as the radial geodesic inherits the static and spherical symmetry of the background. Figure 3a shows the amplitude and phase of a Gaussian time-bin signal. Figure 3b depicts the alternations in the amplitude

and the phase:

$$\begin{aligned} \delta\mathcal{A} &= -\frac{\varepsilon x^-}{2}\left(\frac{1}{r_a^2} - \frac{1}{r_b^2}\right)e^{-\frac{(\varepsilon x^-)^2}{2}} + O(\varepsilon^2) \\ \delta\chi &= \frac{i\omega_0}{4\sigma^2}\left(\frac{1}{r_a^2} - \frac{1}{r_b^2}\right)(\sigma x^-)^2 e^{-\frac{(\sigma x^-)^2}{2}} + O(\varepsilon^2), \end{aligned} \quad (9)$$

where r_b is the location of Bob. The maximum alteration to the amplitude and phase of the Gaussian wavepacket occur at $x^- = \pm 1/\sigma$ and $x^- = \pm\sqrt{2}/\sigma$, respectively. For a Gaussian wavepacket that propagates radially close to the Earth, the maximum alteration to the amplitude and phase are, respectively, $|\delta\mathcal{A}_{\max.}| = \frac{1}{2\sigma}\left(\frac{1}{r_a^2} - \frac{1}{r_b^2}\right)$ and $\delta\chi_{\max.} = \frac{\omega_0 m_\oplus}{2\sigma^2}\left(\frac{1}{r_a^2} - \frac{1}{r_b^2}\right)$, where $m_\oplus$ is the Schwarzschild radius of Earth.

For $\nu_0 = 456$ THz and $\Delta\nu = 1$ kHz, $\delta\chi_{\max.} = 0.10$ rad. Supplementary Note 8 provides further details on choosing these values.

As a final example, we examine the weak regime of gravity when the beam possesses well-defined transverse modes—see Fig. 2a. The wavepacket carrying a well-defined transverse mode traverses through space and reaches to the minimum distance of l to the central mass—here, we assume l is large. We now consider a specific wavepacket, a Hermite-Gauss transverse mode $f_{\omega,p,\ell,q}^{(0)}(x^-, x^+, x^1, x^2) = e^{i\omega_0 x^-} e^{-\frac{(\omega x^-)^2}{2}} \text{HG}_{m,n}(x^+, x^1, x^2)$ —Hermite-Gauss modes are used to extend the communication alphabet beyond bits, i.e. 0 and 1²⁷. The longitudinal and frequency distributions are assumed to be Gaussian. When σ is large, the dominant correction operator is calculated to be:

$$\mathcal{O} = -\frac{9.5ia^2}{z_R}(x^2 - y^2), \quad (10)$$

where $x = \sqrt{2}x_1/w(x^+)$ and $y = \sqrt{2}x_2/w(x^+)$ are dimensionless coordinates, $w(x^+) = w_0\sqrt{1 + \left(\frac{x^+}{z_R}\right)^2}$ is the beam radius, $z_R =$

Fig. 2 Propagation of wavepacket and associated null geodesics in the Schwarzschild spacetime geometry. **a** The null geodesics (dashed curves) for beams that propagate very close to the event horizon—we set the event horizon at 1. **b** Schematic of wavepacket propagation radially in the Schwarzschild spacetime geometry. Alice and Bob are located at r_a and r_b , respectively, while θ and φ are polar and azimuthal angles of the standard spherical coordinates.

Fig. 3 Propagation of Gaussian wavepacket along radial direction in the Schwarzschild spacetime geometry. **a** Amplitude (purple) and phase (orange) of the initial Gaussian beam at the sender (Alice), respectively, presented by $\mathcal{A}(a.u.)$ and $\chi(a.u.)$. Their units, $a.u.$, are chosen such that both are normalised to their maximum values. **b** The changes in the amplitude ($\delta\mathcal{A}$) and the phase ($\delta\chi$) of the Gaussian wavepacket around frequency ω_0 with width of σ transmitted over the radial null geodesic in the Schwarzschild geometry between r_a and $r = r_b$, the geometry is shown in Fig. 2b. The units are chosen where the Schwarzschild radius and light speed are one.

Fig. 4 Propagation of Hermite-Gaussian modes in a weak gravitational field. Hermite-Gaussian mode $HG_{m,n}(x^+, x^1, x^2)$ is not shape invariant under propagation in the Schwarzschild spacetime geometry, and both the intensity and phase profiles modify. **a** Intensity and phase distributions of the first 9 HG modes $m, n \in \{0, 1, 2\}$ prior to the free-space propagation. **b** The intensity $|\psi^{(1)}|^2$ and phase $\arg \psi^{(1)}$ distributions of the corrected term, for the first 9 HG modes $m, n \in \{0, 1, 2\}$, after the propagation in a weak gravitational field. The rows and columns are associated to $m \in \{0, 1, 2\}$ and $n \in \{0, 1, 2\}$, respectively. The dimensionless coordinates are used for these plots.

$\frac{1}{2}\omega_0 w_0^2$ is the Rayleigh range, $a = r_a/l$ is a scaling parameter, w_0 is the beam radius at Alice's position—see Supplementary Note 6 for more details. This operator contains coordinate parameters x and y , and thus, alters both the amplitude and phase of the transverse modes upon propagation. The correction for the solar system, when Alice and Bob are at the mean Earth-Sun distance from the Sun and the wavepacket passes at $l = 2R_\odot$, and for $z_R = 2.8 \text{ km} \times a^2 = 1.34 \times 10^9 \text{ m}$, remains perturbative and is given by $\varepsilon\psi^{(1)}(x^\mu) = i 0.10 (x^2 - y^2) \psi_{\text{Alice}}(x^\mu)|_{x^+ = T}$.

The amplitude and phase of the mode $\psi^{(0)}(x^\mu)$ and the correction $\varepsilon\psi^{(1)}(x^\mu)$ for a few Hermite-Gaussian modes are shown in Fig. 4. As seen, these alterations on the modes are considerable. For instance, it causes up to 12.2% crosstalk between $HG_{0,3}$ and $HG_{0,1}$ modes. The crosstalk between mode $\mathcal{M}$ and mode $\mathcal{N}$ is given by $|\langle \mathcal{N} | \mathcal{M} \rangle|^2$. Therefore, these perturbative alterations need to be accounted for when information is encrypted in the spatial modes. The action of curved spacetime geometry on the wavepacket is linear. Therefore, a target beam that does not possess the information can be used as a reference to monitor the distortion of the information carrier, and an active system can be employed for compensating the distortion in real-time—in conjunction, results in retrieving the original information. Moreover, whenever $\varepsilon \simeq 1$, the higher-order terms of correction need to be considered. For instance, for $z_R < 28 \text{ km} \times a^2$, the correction becomes larger than 1 and we need to take into account higher ε terms. Taking into account all the corrections is tantamount to knowing the Riemann tensor in whole of the spacetime geometry, a piece of knowledge which is not attainable. Thus, we tend to argue that once the perturbation breaks, in

addition to known well-studied gravitational decoherence^{28–32}, a decoherence occurs. We observe that, in addition to the known decoherence of a bipartite entangled system when each particle traverses through a different gravitational field gradient^{32,33}, a coherent beam decoheres when different segments of the spatial spread of the wave experience different tidal gravitational field gradients. The phenomenon we are reporting also occurs for geodesics passing very close to the event horizon—see Supplementary Note 7.

Finally, it is noteworthy that photon pairs $|\psi\rangle_{\text{entangled}}$, e.g. entangled in spatial, frequency or temporal modes, would be affected by the curved spacetime geometry whenever they are shared between two parties, namely Alice and Bob. The final state of the entangled photon, indeed, is given by applying the non-local operators $\mathcal{U} = (1 + \varepsilon(\mathcal{O} + \mathcal{Q}_U + \mathcal{Q}_N))$ onto the entangled states, $(\mathcal{U}_A \otimes \mathcal{U}_B)|\psi\rangle_{\text{entangled}}$ —here, $\mathcal{U}_A$ and $\mathcal{U}_B$ are associated with the correction operators at Alice and Bob's places, respectively.

Conclusion

We have presented how the curvature of the spacetime geometry affects the propagation of an arbitrary wavepacket along a general geodesic in a general curved spacetime geometry. The effect is beyond classical general relativity, residing in the same category as Hawking radiation³⁴. A set of linear operators are presented that encode the effect of the curvature. The corrections to the information carrier wavepacket are investigated in cases of de Sitter (anti de Sitter) and Schwarzschild spacetime geometries. It has been shown that the corrections accumulate overtime and distort

the wavepacket. The gravitational distortion, therefore, needs to be accounted for in quantum communication performed over long distances in a curved spacetime geometry.

Data availability

The authors declare that the data supporting the findings of this study are available within the paper and its supplementary information file.

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Author contributions

Q.E. and E.K. conceived the idea and developed the theoretical framework. E.C. checked and confirmed the computation. All authors contributed to the manuscript preparation.

Competing interests

The authors declare no competing interests.

Additional information

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