Quenched topological boundary modes can persist in a trivial system

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Topological boundary modes can occur at the spatial interface between a topological and gapped trivial phase and exhibit a wavefunction that exponentially decays in the gap. Here we argue that this intuition fails for a temporal boundary between a prequench topological phase that possess topological boundary eigenstates and a postquench gapped trivial phase that does not possess any eigenstates in its gap. In particular, we find that characteristics of states (e.g., probability density) prepared in a topologically non-trivial system can persist long after it is quenched into a gapped trivial phase with spatial profiles that appear frozen over long times postquench. After this near-stationary window, topological boundary mode profiles decay albeit, slowly in a power-law fashion. This behavior highlights the unusual features of nonequilibrium protocols enabling quenches to extend and control localized states of both topological and non-topological origins.

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Eigenstates dominate the long-lived excitations of a quantum system and determine its response. Recently, out-of-equilibrium protocols^{1–13} have emerged as a powerful means of controlling the states accessible in solid-state^{14–17} and cold-atomic optical lattice^{18–22} platforms. For example, intense periodic drives can warp a topologically trivial electronic band structure to sustain states with non-trivial properties such as a finite Berry flux^{2,3,15,18,19,22} and induce spatially localized topological boundary modes (TBMs) that traverse a bulk gap^{20,21,23}. Such out-of-equilibrium states are maintained only when the periodic drive is turned on; they cease to be eigenstates once the drive is switched off and effective Hamiltonian quenches to a trivial phase²⁴.

Here we argue that vestiges of spatially localized TBMs, which can be initially prepared in a gapped topologically non-trivial system, will persist long after it is quenched into a gapped trivial and uniform phase. Unlike the gapless postquench case (Fig. 1a), this persistence is characterized by a frozen (slower than power-law decay regime), followed by a melting (power-law decay) regime, see Fig. 1b. In particular, we find that even though TBMs are not eigenstates of the postquench phase and exist inside the postquench bulk gap, characteristics such as a spatially peaked TBM probability density persist and appear frozen over a long and tunable time window postquench. As we explain below, this frozen regime is a consequence of the curvature of the gapped post-quenched dispersion profile, and applies to generic localized initial states.

This near-stationary nature of the TBM postquench probability density contrasts with the gapless postquench case (Figs. 1a and 2a), and is particularly surprising given the rapid evolution of the postquench wavefunction (Figs. 1b and 2b). Indeed, this contrasts with recurrent Loschmidt-echo type responses found in many quenched systems^{25–27} wherein observables oscillate as a function of time. Further, even as translational symmetry in the postquench Hamiltonian is maintained, the postquench probability density remains spatially localized (Fig. 1b) retaining its prequench profile. It decays as a power law at very long times.

As we explain in the “Results” section, the persistence of probability density arises from an interplay of TBM spatial localization and its fast dynamical evolution postquench. This will be shown to be the underlying mechanism for its frozen and melting behavioral regimes. It is a striking example of how quantum systems pushed far out-of-equilibrium can possess very unusual properties with no analog in equilibrium^{10,28–30}. We anticipate TBM persistence can be readily accessed in quenched systems that are directly sensitive to the probability density distribution that include in ultracold atoms setups^{19,20,22,31–33}, as well as in photonic³⁴ or circuit realizations^{35–38}.

Results and discussion

Probability density and quench protocol. The persistence of TBMs can be most easily illustrated by a time-dependent Dirac Hamiltonian (two-band) that undergoes a quench at time $t = 0$ (Fig. 1):

$$\hat{H}(\mathbf{p}, \mathbf{r}, t) = \hbar v \mathbf{p} \cdot \boldsymbol{\sigma} + \mathcal{M}(x, t) \sigma_3 = \mathbf{d}(\mathbf{p}, t) \cdot \boldsymbol{\sigma}, \quad (1)$$

where v is the Dirac velocity, $\mathbf{r} = (x, y)$, $\mathbf{p} = (p_x, p_y)$, σ_i , $i = 1, 2, 3$ are Pauli matrices, and time-varying mass

$$\mathcal{M}(x, t) = \begin{cases} M(x) & t < 0 \\ m & t \geq 0 \end{cases} \quad (2)$$

In the following, we shall refer to the pre- (post-) quench Hamiltonian as $\hat{H}_i$ ($\hat{H}_f$). For $t < 0$ before the quench, we have $M(x)$ satisfying $M(x < 0) < 0$ and $M(x > 0) > 0$, such that it describes a mass domain wall along the y -axis ($x = 0$). Due to the jump in

$\text{sgn } M(x)$, the domain wall carries a unit topological charge, and thus supports gapless TBM $|\psi_{p_y}\rangle$ that linearly disperse as $\hbar v p_y$. Their spatial wavefunction is well localized at the domain wall $x = 0$:

$$\langle \mathbf{r} | \psi_{p_y} \rangle = \mathcal{N} e^{ip_y y} \Phi(x) |\Psi^{(0)}\rangle, \quad |\Psi^{(0)}\rangle = (1, i)/\sqrt{2}, \quad (3)$$

where $\mathcal{N}$ is a normalization constant, and $\Phi(x)$ decays exponentially on both sides. The decay length, α , defines the spatial extent of $\Phi(x)$.

Here we will consider a TBM state $|\psi_{p_y}\rangle$ prepared in a prequench bulk gap with $|p_y| < \alpha^{-1}$. At $t = 0$, the system is quenched via Eq. (2) and the mass parameter becomes uniform in space. As a result, $|\psi_{p_y}\rangle$ no longer exist as eigenstates of the postquench Hamiltonian and evolve as $\exp[-i\hat{H}_f t/\hbar] |\psi_{p_y}\rangle$. TBM $|\psi_{p_y}\rangle$ postquench characteristics at $t > 0$ are most saliently captured by its probability density

$$\langle \rho \rangle_{t,x} = \langle \psi_{p_y} | e^{i\hat{H}_f t/\hbar} \hat{\mathbf{p}}_r e^{-i\hat{H}_f t/\hbar} | \psi_{p_y} \rangle, \quad \hat{\mathbf{p}}_r = |\mathbf{r}\rangle \langle \mathbf{r}|, \quad (4)$$

with explicit p_y dependence suppressed hereafter for brevity. Postquench, the initial boundary-localized TBM state is no longer an eigenstate of the new spatial-translation-invariant Hamiltonian $\hat{H}_f$; instead its temporal evolution can be understood from the Larmor precession of its projected components in the postquench eigenbasis of $\hat{H}_f$. Due to the tightly localized (in x) profile of the initial TBM, many postquench p_x eigenstates are accessed. As a result, their destructive interference and multi-frequency Larmor precession can lead to dephasing²⁹ and decay of the initial TBM. Indeed, for $m\alpha/(\hbar v) \lesssim 1$, we find through direct numerical integration of Eq. (4), the probability density generically decays exponentially in time, Fig. 2a.

However, when $m\alpha/(\hbar v) \gg 1$, we find that the probability density no longer exhibits fast decay. Direct numerical integration of Eq. (4) yields an unusual regime wherein the initial TBM state freezes maintaining its original localized (in x) profile (Figs. 1b and 2b) at short times. As we explain below, this frozen TBM stays locked over times $0 \leq t \ll \tau m\alpha/\hbar v$ controlled by the size of the postquench gap m ; here the characteristic time $\tau = \alpha/2v$. At longer times, the TBM melts, displaying a much slower power-law decay. As we now explain, this arises from a near-lock-step interference process between p_x components enforced in the $m\alpha/(\hbar v) \gg 1$ regime.

To more concretely understand the freezing and subsequent melting of the TBM mode, we analyze the solution of $\langle \rho \rangle_{t,x}$ obtained through direct evaluation of Eq. (4):

$$\langle \rho \rangle_{t,x} = |C|^2 + |S|^2 - 2 \text{Im}[S^* C] \cdot \hat{\mathbf{s}} - 2(\text{Re}[S] \times \text{Im}[S]) \cdot \hat{\mathbf{s}}, \quad (5)$$

where $\mathbf{S}(x, t) = S_x \hat{\mathbf{x}} + S_y \hat{\mathbf{y}} + S_z \hat{\mathbf{z}}$ and $\hat{\mathbf{s}} = \langle \Psi^{(0)} | \boldsymbol{\sigma} | \Psi^{(0)} \rangle$. The quantities $C(x, t)$ and $\mathbf{S}(x, t)$ represent the even and odd temporal components of the postquench dynamical evolution of the probability density and can be written as $C(x, t) = [(F_0(x, t) + F_0(x, -t)]/2$ and $S_{x,y,z} = [F_{x,y,z}(x, t) - F_{x,y,z}(x, -t)]/2i$ with

$$F_j(x, t) = \frac{\mathcal{N}}{2\pi} \int \Phi(p_x) e^{ip_x x} e^{i\varepsilon_p t/\hbar} f_j(p_x) dp_x, \quad (6)$$

where $j = 0, x, y, z$, and the j -dependent weights as $f_0 = 1$ and $(f_x, f_y, f_z) = \mathbf{d}(\mathbf{p})/|\mathbf{d}(\mathbf{p})|$. Here $\varepsilon_p = \sqrt{\hbar^2 v^2 |\mathbf{p}|^2 + m^2}$ is the (post-quench) Dirac eigenenergy and $\Phi(p_x)$ is the Fourier transform of TBM profile $\Phi(x)$. In obtaining Eq. (5), we expanded $\exp[-i\hat{H}_f t/\hbar]$ and $\hat{\mathbf{p}}_r$ in Eq. (4) as superpositions of the postquench energy eigenstates, and applied the Pauli matrix identity $\sigma_i \sigma_j = \delta_{ij} + i\epsilon_{ijk} \sigma_k$ repeatedly, see full details in the “Methods”.

Fig. 1 Emergence of frozen and melting regimes after a boundary mode is quenched. **a** Postquench evolution of the probability density of a topological boundary mode (TBM) initially localized at the mass domain wall at position $x = 0$; in this case where $m = 0$ after the quench at time $t = 0$, wavetrains move away from the domain wall for $t > 0$. **b** Slow decay of the same TBM probability density for large postquench uniform dimensionless mass $ma/\hbar v = 20$, α is the domain wall width, $\hbar$ the reduced Planck constant, and v the Fermi velocity. It displays a frozen probability density regime as well as a melting regime exhibiting a power-law decay of the TBM probability density. The prequench mass distribution is the same as in **a**, with a domain wall at $x = 0$, but the postquench mass $ma/\hbar v = 20$ is large (red). In all panels probability density snapshots are plotted using Eq. (4) taken at logarithmic time spacings. We used the initial TBM profile $\Phi(x) = \text{sech}(x/\alpha)$ from Eq. (3).

Initially at $t = 0$, $S(x, t = 0) = 0$ and $C(x, t = 0)$ reduces to the spatial profile $\Phi(x)$ of the topological boundary state yielding Eq. (5) that follows the profile of the TBM mode. For $t > 0$, however, both the arguments of $S(x, t)$ and $C(x, t)$ oscillate with time capturing the Larmor precession of the TBM projected components in the postquench eigenbasis. Indeed, given the wide range of p_x eigenstates that the TBM state projects to, the temporal evolution of probability density in Eq. (5) involves (momentum-space) nonlocal interference between multiple distinct p_x momentum modes, and can lead to a complex spatio-temporal evolution of the probability density.

When $ma/(\hbar v) \lesssim 1$, the dynamical phase factor $\exp[i\varepsilon_p t/\hbar]$ in Eq. (6) rapidly oscillates as a function of p_x . As a result, it suppresses the large momentum contributions to $C(x, t)$ (and $S(x, t)$). Since the tight localization of the initial TBM depends on large p_x momentum contributions, the initial state is rapidly eroded. On a physical level, this can be understood as an intrinsic dephasing phenomena where the multiple frequency oscillation get out of phase destroying the coherence of the initial state. In real space, this manifests as two wavetrains moving almost uniformly in opposite directions with a group velocity $v_g = \hbar p_x / \sqrt{\Delta^2 + \hbar^2 v^2 p_x^2} \approx \pm v$ for small $\Delta^2 = m^2 + v^2 \hbar^2 p_y^2$ (see Fig. 1a)

Fig. 2 Frozen regime and subsequent power-law decay in the melting regime. **a** Log-linear plot of dynamical evolution of postquench probability density $\langle \rho \rangle_{t,x}$ for small dimensionless mass $ma/(\hbar v) = 0.1$. This exhibits a fast exponential decay as displayed by the linear slope, with probability density attenuating rapidly. **b** Log-Log plot of postquench probability density $\langle \rho \rangle_{t,x}$ for large $ma/(\hbar v) = 20$. In contrast to **a**, this exhibits a very slow decay and has a evolution characterized by two distinct regimes: (blue shaded) a slow frozen regime where probability density hardly evolves, and (orange shaded) a power law melting regime where the probability density decays temporally as t^{-1} (indicated by dashed line). For both panels, curves from top to bottom (red to green) indicate various positions away from the domain wall $x/\alpha = 0 \rightarrow 1.2$ in steps of 0.15. The probability density was obtained from direct numerical integration of Eq. (4) using the initial boundary mode profile $\Phi(x) = \text{sech}(x/\alpha)$.

Frozen TBMs and Wick-rotated diffusion. However, when $ma/(\hbar v) \gg 1$, the Larmor precession of the postquench state for various p_x components oscillate in near lock-step, see below. Indeed, in this limit, the oscillation frequency $\varepsilon_p = \sqrt{\Delta^2 + \hbar^2 v^2 p_x^2} \approx \Delta + (\hbar v p_x)^2 / (2\Delta)$ is dominated by a large $\Delta = (m^2 + \hbar^2 v^2 p_y^2)^{1/2} \approx m$. In the latter, we focus on small $p_y < \alpha^{-1} \ll m$. As the tight localization of the TBM state arises from large $p_x \sim \alpha^{-1}$ components, when $ma/(\hbar v) \gg 1$, the differences in the frequencies are severely suppressed. This near-lock-step oscillation dramatically slows the erosion of the TBM state and as we will now discuss, at short times can even halt it. To see this explicitly, we take $\varepsilon_p \approx \Delta + (\hbar v p_x)^2 / (2\Delta)$ in Eq. (6) and directly integrate:

$$F_{0,z}(x, t) = \mathcal{N} \sqrt{\frac{i\Delta}{\hbar v^2 t}} e^{i\Delta t/\hbar} \Psi_t(x), \quad (7)$$

with the postquench profile

$$\Psi_t(x) = \int dx' \Phi(x+x') G_t(x'), \quad G_t(x) = e^{-i\Delta x^2 / (2\hbar v^2 t)} \quad (8)$$

where $G_t(x)$ is an imaginary Gaussian kernel that results from the gapped dispersion in the large $ma/(\hbar v) \gg 1$ limit, see “Methods”. This kernel $G_t(x)$ is the mathematical embodiment of the random interference processes between different momentum sectors, with its imaginary Gaussian nature paralleling Brownian motion scattering in a diffusive medium; it is fundamentally distinct from the dynamical behavior expected when the system is quenched into a gapless linearly dispersing Hamiltonian, e.g., $m=0$ above. We note that $F_{x,y}$ contributions are small (suppressed in the $ma/(\hbar v) \gg 1$ limit) and do not contribute substantially to $\langle \rho \rangle_{t,x}$ profile.

For short times $0 < t \ll ma^2/(2\hbar v^2) = \tau ma/(\hbar v)$, the imaginary gaussian $G_t(x)$ in Eqs. (7) and (8) exhibit spatial oscillations far more rapid than $\Phi(x+x')$ varies in x . Here $\tau = \alpha/2v$ is an intrinsic timescale. As a result, at these short times $F_{0,z} \approx \mathcal{N} \Phi(x) e^{i\Delta t/\hbar}$, so that $C(x, t) \approx \mathcal{N} \Phi(x) \cos(\Delta t/\hbar)$ and $S_z \approx \mathcal{N} \Phi(x) \sin(\Delta t/\hbar)$. Substituting into Eq. (5) we obtain the probability density:

$$\langle \rho \rangle_{t,x} \approx \mathcal{N}^2 |\Phi(x)|^2 + \mathcal{O}\left(\frac{\hbar v}{m\alpha}\right), \quad 0 < t < \frac{\tau m\alpha}{\hbar v}. \quad (9)$$

We note the probability density in this regime is flat to infinite order in vt/α (Fig. 2b) (blue shaded); any temporal decay in this frozen regime is non-analytic and slower than any power law. As a result, we term this a region of frozen probability density. This is particularly surprising since the prequench TBM state is not an eigenstate of the postquench Hamiltonian $\hat{H}_f$. Perhaps even more striking is the fact that the period over which the TBM’s probability density is frozen can be controlled by the postquench gap size m ; increasing m gives a wider frozen window for TBM probability density, Eq. (9). This can be understood physically by noting that the time window over which the probability density remains frozen can be recast in a more physical intuitive way as the ratio of the spatial confinement of the prequench domain wall, α , and the typical group velocity of wavepackets postquench; larger m translates to slower postquench group velocities and hence longer frozen time window.

At long times $t \gg ma^2/(2\hbar v^2)$, the situation is dramatically different with the imaginary gaussian $G_t(x)$ in Eqs. (7) and (8) slowly varying over large ranges of x . As a result, the frozen states “melts” with its spatial profile spreading slowly out according to the convolution $\Psi_t(x)$ and an overall probability density amplitude decaying as t^{-1} :

$$\langle \rho \rangle_{t,x} \rightarrow \frac{m}{\hbar v^2} \mathcal{N}^2 |\Psi_t(x)|^2, \quad t \gg \frac{\tau m\alpha}{\hbar v}. \quad (10)$$

This t^{-1} power-law melting decay (dashed line) conforms with that found from direct numerical integration in Fig. 2b (orange shaded), and even exhibits a data collapse for probability density taken at different values of x . In this regime, the spatial extent/width of $|\Psi_t(x)|^2$ grows as t thereby conserving the total probability density over all space. We note that the persistence of TBM postquench is consistent with the edge currents found in quantum quenches of Chern insulators¹⁰.

To understand this long-time behavior more intuitively, we note that $G_t(x)$ can be interpreted as a 1D heat kernel corresponding to a Wick-rotated “diffusion” process (i.e. evolution by Schrödinger’s equation involving the id/dt instead of d/dt operator) with diffusion constant $-i\hbar/2\Delta$. While scattering processes in ordinary diffusion processes lead to a “Gaussian-blurred” distribution characterized by a (real) Gaussian decaying kernel, the imaginary Gaussian kernel $G_t(x)$ represents the rapid

interference effects from multiple non-coherent contributions in Schrödinger evolution. A large Δ results in slow spreading of the initial state, which by Eq. (8) is asymptotically governed by power-law decay, instead of Gaussian decay as in the more familiar real-time diffusion scattering processes. Physically, the slow melting TBM behavior can be likened to that of slow diffusion found in classical systems with large inertial masses; the large $m\alpha/(h\nu) \gg 1$ limit corresponds to the regime where postquench modes are energetically inaccessible.

While we have concentrated on a continuum description of the frozen and melting regimes in the main text, both frozen and melting behavior persist even on a lattice model (see detailed numerics of quench dynamics on a lattice in Supplementary Note 2) as well as at hard-wall boundaries. This agreement arises because the frozen and melting regimes are most sensitive to the long-wavelength evolution of the TBM and the mismatch between the TBM confinement, α , and the postquench Compton length scale $h\nu/m$. Indeed, this mismatch and the ensuing quench dynamics we describe here can manifest for other initially localized modes, topological or otherwise.

Pseudospin precession and creep current. The frozen-in-time probability density profile at short times described in Eq. (9) hides the fact that the TBM modes are not eigenstates of the postquench Hamiltonian $\hat{H}_f$. Are all other observables similarly frozen in time as well? To further interrogate the TBM postquench, we consider its local (spatially resolved) pseudospin expectation

$$\langle \sigma \rangle_{t,x} = \langle \psi_{p_y} | e^{i\hat{H}_f t/\hbar} \hat{\mathbf{P}}_r \sigma \hat{\mathbf{P}}_r e^{-i\hat{H}_f t/\hbar} | \psi_{p_y} \rangle, \quad (11)$$

with explicit p_y dependence suppressed hereafter for notational brevity. $\langle \sigma \rangle_{t,x}$ encodes the spinor-wavefunction information of the TBM. Prequench, $\langle \sigma \rangle$ is aligned along $\hat{\mathbf{s}}$ in the $\hat{\mathbf{e}}_2$ direction; here $\mathbf{e}_{1,2,3}$ denote the $\mathbf{x}, \mathbf{y}, \mathbf{z}$ directions in a Bloch sphere. However, postquench, the eigenstates of $\hat{H}_f$ generically possess spinor components in all three directions. As a result, the dynamical evolution of $\langle \sigma \rangle_{t,x}$ postquench involves a complex intertwining of precession and interference between wave components composing the spatially localized profile of the TBM state. Indeed, $\langle \sigma \rangle_{t,x}$ possesses dynamics that generically departs from that of the Bloch equation.

To see this, we directly evaluate Eq. (11) in the same fashion as Eq. (5) above producing the closed form

$$\begin{aligned} \langle \sigma \rangle_{t,x} = & \hat{\mathbf{s}}(|C|^2 - |S|^2) + 2 \operatorname{Re}[S^*C] \times \hat{\mathbf{s}} - 2 \operatorname{Im}[S^*C] \\ & + 2 \operatorname{Re}[(S^* \cdot \hat{\mathbf{s}})S] + 2 \operatorname{Re}[S] \times \operatorname{Im}[S]. \end{aligned} \quad (12)$$

The first term tracks the persistence of the initial pseudospin direction $\hat{\mathbf{s}}$, while the second term represents a “pure” precession contribution; the other terms correspond to additional dynamical contributions from S , which arises physically from projection onto postquench eigenstates. Eq. (12) in fact describes the dynamical solution of any two-component state with an initial spatially inhomogeneous profile. In our case, Eq. (12) contains information on how the localization of the TBM interplays with precession effects. This generically yields a complex spatio-temporal and spin-dependent evolution, see explicit example in Supplementary Note 1.

However, when $m\alpha/h\nu \gg 1$, the pseudospin expectation $\langle \sigma \rangle_{t,x}$ of the TBM mode can act like a localized block spin with TBM pseudospin expectation at each x precessing in unison. Similar to the probability density described above, this also leads to a frozen regime, where the magnitude of the $|\langle \sigma \rangle_{t,x}|$ pseudospin is preserved for a sizeable period of time (Fig. 3a). Intuitively, in this limit, the $\mathbf{d} = (\nu h p_x, 0, m)$ precession axis points strongly

towards in the $\hat{\mathbf{e}}_3$ direction, causing the initial pseudospin $\hat{\mathbf{s}}$ to precess largely in the $\hat{\mathbf{e}}_1$ - $\hat{\mathbf{e}}_2$ plane. Since the precession axis does not shift much across different p_x and p_y sectors due to large m , destructive interference is minimized and the quenched TBM states periodically return to their initial configurations: $|\langle \sigma \rangle_{t,x}|$ is preserved. Indeed, in this regime, various positions in the TBM have $\langle \sigma_{x,y} \rangle_{x,t}$ that oscillate in phase (Fig. 3b). At longer times $t \gg \tau m\alpha/h\nu$, $|\langle \sigma \rangle_{t,x}|$ melts, and decays as a power-law t^{-1} (Fig. 3a).

Since $\hbar^{-1} \partial \hat{H}_f / \partial \mathbf{p} = v(\sigma_x, \sigma_y)$, the pseudospin oscillations indicate a cyclotron-type motion of the TBM mode; in the large $m\alpha/h\nu \gg 1$, the oscillations are largely locked to a frequency $2m/h$. Interestingly, when the magnitude of the pseudospin decays, the value of $\langle \sigma_x \rangle$ or $\langle \sigma_y \rangle$ does not come back to itself after a full revolution (Fig. 3c). This decay in pseudospin leads to a creep current: an uncompensated drift current of the TBM (see estimate in Fig. 3d) that drifts along the domain wall.

The persistence of TBM characteristics can form the basis for a strategy to extend TBM features long after the topological system that supports it is gone: namely, by quenching to a large gap wherein postquench eigenstates are energetically inaccessible. Since the persistent TBM probability density arises from the fast postquench Larmor precession enforced by large m , we expect that it is robust against inelastic scattering with energies far smaller than the gap scale as well as slowly varying disorder with typical lengths longer than the TBM width. The frozen and melting quenched probability density regimes can be readily prepared and measured in ultracold atom optical lattices^{19,20,22,31,32} with its pseudospin components of the wavepacket independently extracted^{31,32}, and its real-space dynamics tracked²² or, more recently, via quantum computer digital simulations^{39,40}.

While we expect that the frozen and melting quench dynamics that we discuss here to be most easily realized in ultracold-atomic optical lattices, or in photonic³⁴ or circuit platforms^{35–38,41} wherein interactions are weak, it is interesting to consider how the quench dynamics will behave in electronic systems wherein interactions can be present. In an electronic platform, interactions can lead to dephasing that can disrupt the lock-step precession of the postquench states that underlie the Frozen and melting regime²⁹. As a result, we expect the unusual frozen and melting regimes to only manifest at times shorter than the dephasing time of the electronic system.

Perhaps most striking is the contrast in behavior of TBMs across a sharp temporal boundary (as realized in the quench at $t = 0$) against those found at spatial boundaries. In the latter, boundary states (topological or otherwise) that exist in an energy gap typically exponentially decay over space into a gapped insulating bulk. Whereas in the former, TBM probability density persists, freezing for a long window then decaying in a power-law fashion even when there are no eigenstates in trivial gap. This underscores the stark difference between sharp spatial and temporal interfaces and displays the dichotomy between equilibrium and out-of-equilibrium responses.

Methods

Postquench dynamical evolution. In this section, we review how the TBM in Eq. (3) of the main text and its characteristics (namely probability density and pseudospin expectation) evolve postquench. For generality, we do not make any assumption on the form of the initial state (unless otherwise stated), other than it can be spatially inhomogeneous in the x -direction. We shall also assume the most general postquench Hamiltonian that is translation-invariant in both directions. Hence our following results are applicable for a broader class of quench settings than those discussed in the main text.

Wavefunction evolution. The postquench ($t > 0$) system is described by the Hamiltonian $\hat{H}_f(\mathbf{p}) = \mathbf{d}(\mathbf{p}) \cdot \boldsymbol{\sigma}$ with bulk eigenstates $|\epsilon_p^\pm\rangle$ with eigenenergies $\pm \epsilon_p = \pm |\mathbf{d}(\mathbf{p})| = \pm \sqrt{\hbar^2 v^2 |\mathbf{p}|^2 + m^2}$. The wavefunction of the

Fig. 3 Evolution of pseudospin expectation and its creep after the quench. **a** Log-log plot of dynamical evolution of the local pseudospin magnitude $|\langle\sigma\rangle_{t,x}|$ for large dimensionless mass $m\alpha/(\hbar v) = 20$. It exhibits a frozen (blue) and a melting regime (orange); dashed line indicates a t^{-1} temporal power-law decay. The curves from top to bottom (red to green) indicate various positions away from the domain wall $x/\alpha = 0 \rightarrow 1.2$ in steps of 0.15. **b** $\langle\sigma_x\rangle_t$ and $\langle\sigma_y\rangle_t$ oscillation at $x = 0$ (red) and $x/\alpha = 1.2$ (brown). **c** Trajectory of $\langle\sigma\rangle_{t,x}$ for $x = 0$ for $m\alpha/(\hbar v) = \pi$ exhibiting a decay in the pseudospin density over time. **d** The decay over each cycle leads to a creep current: a non-vanishing drift current of the topological boundary mode. Creep current estimated by averaging $\langle\sigma_{x,y}\rangle_{t,x}$ over a period twice, such as to smooth out fluctuations in the period. The small kinks represent subleading second-order corrections in the power-law decay.

TBM initial state [see Eq. (3) of the main text] at $t = 0$ can be written as $\langle\mathbf{r}|\psi_{p_y}\rangle = \sum_{p_x} \mathcal{N} \Phi(p_x) e^{ip_x x} e^{ip_y y} |\Psi^{(0)}\rangle$, where $\Phi(p_x)$ is the Fourier transform of $\Phi(x)$. The time-evolved wavefunction postquench can be directly evaluated as $\psi_{p_y}(\mathbf{r}, t) = \langle\mathbf{r}|e^{-i\hat{H}_t}|\psi_{p_y}\rangle$.

Writing $\exp(-i\hat{H}_t) = \sum_{p_x} e^{\mp i\epsilon_p t} |\epsilon_p^\pm\rangle \langle\epsilon_p^\pm|$, and noting that $|\epsilon_p^\pm\rangle \langle\epsilon_p^\pm| = \frac{1}{2}(\mathbb{I} \pm \hat{\mathbf{d}}(\mathbf{p}) \cdot \boldsymbol{\sigma})|\mathbf{p}\rangle \langle\mathbf{p}|$, where $\langle\mathbf{r}|\mathbf{p}\rangle = e^{i\mathbf{p}\cdot\mathbf{r}}$ is the wavefunction of the momentum eigenstate, and $\hat{\mathbf{d}}(\mathbf{p}) = \mathbf{d}(\mathbf{p})/|\mathbf{d}(\mathbf{p})|$, we obtain

$$\psi_{p_y}(\mathbf{r}, t) = \sum_{p_x} e^{i\mathbf{p}\cdot(\mathbf{r}-\mathbf{r}')} (\cos \epsilon_p t - i\hat{\mathbf{d}}(\mathbf{p}) \cdot \boldsymbol{\sigma} \sin \epsilon_p t) \langle\mathbf{r}'|\psi_{p_y}\rangle \quad (13)$$

In obtaining Eq. (13) we summed across the $\pm$ bands as well as inserted the resolution of the identity $\sum_{\mathbf{r}'} |\mathbf{r}'\rangle \langle\mathbf{r}'|$. Summing across $\mathbf{r}'$, p'_x by writing out $\langle\mathbf{r}|\psi_{p_y}\rangle = \sum_{p_x} \mathcal{N} \Phi(p_x) e^{ip_x x} e^{ip_y y} |\Psi^{(0)}\rangle$ we obtain the compact form

$$\psi_{p_y}(\mathbf{r}, t) = e^{ip_y y} [C(x, t) - iS(x, t) \cdot \boldsymbol{\sigma}] |\Psi^{(0)}\rangle \quad (14)$$

where C and S are the same as the main text. We reproduce these here for the convenience of the reader

$$C(x, t) = \mathcal{N} \sum_{p_x} \Phi(p_x) e^{ip_x x} \cos \epsilon_p t \quad (15)$$

$$S(x, t) = \mathcal{N} \sum_{p_x} \hat{\mathbf{d}}(\mathbf{p}) \Phi(p_x) e^{ip_x x} \sin \epsilon_p t \quad (16)$$

In Eq. (14), we see that $\psi(t, \mathbf{r})$ comprises two pseudospinor contributions, one proportional to $\cos \epsilon_p t$ in the direction of the original spinor $|\Psi^{(0)}\rangle$, and another proportional to $i \sin \epsilon_p t$ for which $|\Psi^{(0)}\rangle$ has undergone a $\hat{\mathbf{d}}(\mathbf{p}) \cdot \boldsymbol{\sigma}$ spinor rotation.

Probability density dynamics. The evolution of the probability density postquench can be obtained from direct evaluation of Eq. (4) of the main text. This amounts to finding the square amplitude of the wavefunction above, $|\psi_{p_y}(\mathbf{r}, t)|^2$. This can be

evaluated as

$$\langle\rho\rangle_{t,x} = \langle\Psi^{(0)}| [C^* + i\mathbf{S}^* \cdot \boldsymbol{\sigma}] [C - i\mathbf{S} \cdot \boldsymbol{\sigma}] |\Psi^{(0)}\rangle, \quad (17)$$

where we have suppressed the explicit x, t dependence of $C, \mathbf{S}$ for brevity. This expression can be readily simplified by recalling the vector identity:

$$(\mathbf{a} \cdot \boldsymbol{\sigma})(\mathbf{b} \cdot \boldsymbol{\sigma}) = \mathbf{a} \cdot \mathbf{b} \mathbb{I} + i(\mathbf{a} \times \mathbf{b}) \cdot \boldsymbol{\sigma}. \quad (18)$$

This yields the compact expression for probability density as

$$\langle\rho\rangle_{x,t} = |C|^2 + |S|^2 + i(S^* C - C^* S) \cdot \hat{\mathbf{s}} + i(\mathbf{S}^* \times \mathbf{S}) \cdot \hat{\mathbf{s}} \quad (19)$$

where $\hat{\mathbf{s}} = \langle\Psi^{(0)}|\boldsymbol{\sigma}|\Psi^{(0)}\rangle$. Writing $\mathbf{S}^* C - C^* S = 2i\text{Im}(\mathbf{S}^* C)$ we obtain Eq. (5) in the main text.

We note, parenthetically, when $\hat{\mathbf{d}}(\mathbf{p})$ describes a Dirac model, further simplifications can be made to Eq. (19). For example, for Dirac models ϵ_p is even in p_x . This forces C to be real, and $\text{Re}[S]$ ($\text{Im}[S]$) to arise solely from the even(odd) components of $\hat{\mathbf{d}}(p_x, p_y)$. Hence $\text{Im}[S] \parallel \hat{\mathbf{e}}_1$ and $\text{Re}[S] \perp \hat{\mathbf{e}}_1$, and the time-evolved probability density reads as

$$\langle\rho\rangle_{x,t} = C^2 + (\text{Im}[S] \cdot \hat{\mathbf{e}}_1 - \text{Re}[S] \cdot \hat{\mathbf{e}}_3)^2 + (\mathbf{S} \cdot \hat{\mathbf{e}}_2)^2, \quad (20)$$

with the last term nonzero only for $p_y \neq 0$. In obtaining the above, we have used $\hat{\mathbf{s}} = \hat{\mathbf{e}}_2$ for the initial TBM.

Pseudospin dynamics. The evolution of the pseudospin expectation (density) postquench can be evaluated in the same fashion as above. Using the $C, \mathbf{S}$ notation above, the pseudospin expectation density in Eq. (11) of the main text can be rewritten compactly as

$$\langle\sigma\rangle_{t,x} = \langle\Psi^{(0)}| [C^* + i\mathbf{S}^* \cdot \boldsymbol{\sigma}] [\mathbf{C} - i\mathbf{S} \cdot \boldsymbol{\sigma}] |\Psi^{(0)}\rangle, \quad (21)$$

This expression can be readily simplified by repeated application of the identity $\sigma_a \sigma_b = \delta_{ab} \mathbb{I} + i\epsilon_{abc} \sigma_c$ and recalling the identity $\epsilon_{abc} \epsilon_{ade} = \delta_{bd} \delta_{ce} - \delta_{be} \delta_{cd}$. Here ϵ_{abc} is the Levi-Civita symbol, and a, b, c, d, e indices run over x, y, z . Focussing on

the a th component of the pseudospin expectation density, we obtain

$$[(\sigma)_{t,x}]_a = |C|^2 \hat{s}_a - 2\text{Im}(\mathbf{S}_a^* C) + \epsilon_{abc}(C^* \mathbf{S}_b \hat{s}_c + C \mathbf{S}_b^* \hat{s}_c) + \mathbf{S}_a^* \hat{s}_b \hat{s}_c + \mathbf{S}_b^* \hat{s}_c \hat{s}_a - \mathbf{S}_b^* \hat{s}_c \hat{s}_a + i\epsilon_{abc} \mathbf{S}_b \mathbf{S}_c^* \quad (22)$$

Re-writing in terms of real and imaginary parts of $\mathbf{S}$ and C , we obtain Eq. (12) in the main text. In obtaining Eq. (12) we have noted the identity $(\mathbf{u} + i\mathbf{v}) \times (\mathbf{u} - i\mathbf{v}) = -2i\mathbf{u} \times \mathbf{v}$, where $\mathbf{u}$ and $\mathbf{v}$ are vectors with real components.

Similar to that discussed above, simplifications to Eq. (22) arise when ϵ_p is even in p_x . This forces C to be real. In that case, $\text{Re}[\mathbf{S}]$ ($\text{Im}[\mathbf{S}]$) arises from the integral of the even(odd) components of $\hat{\mathbf{d}}(p_x, p_y)$. If $\hat{\mathbf{d}}(\mathbf{p})$ is furthermore purely even/odd, Eq. (22) simplifies to

$$(\sigma)_{t,x}^{\text{even}} = \hat{s}(|C|^2 - |\mathbf{S}|^2) + 2\text{Re}[\mathbf{S}^* C] \times \hat{s} + 2\text{Re}[(\mathbf{S}^* \cdot \hat{s})\mathbf{S}] \quad (23)$$

and

$$(\sigma)_{t,x}^{\text{odd}} = \hat{s}(|C|^2 - |\mathbf{S}|^2) - 2\text{Im}[\mathbf{S}^* C] + 2\text{Re}[(\mathbf{S}^* \cdot \hat{s})\mathbf{S}]. \quad (24)$$

In either case, the last term of Eq. (22) always disappears, since it requires $\mathbf{S}$ to have both real and imaginary parts.

Another useful case to study is that of the TBM modes discussed in the main text. Recalling that the TBM modes have $\hat{s} = \hat{\mathbf{e}}_z$, one can obtain the squared expected pseudospin magnitude (not to be confused with $\langle \rho \rangle_{t,x}$)

$$|(\sigma)_{t,x}|^2 = (|C|^2 + |\mathbf{S}|^2 - 2(\mathbf{S} \cdot \mathbf{e}_z)(\mathbf{S} \cdot \mathbf{e}_z))^2. \quad (25)$$

We note that Eq. (22) can also be used to deduce the expressions for general initial states with $\hat{s} \neq \hat{\mathbf{e}}_z$.

Frozen and melting regimes. In this section, we provide a detailed description of how both frozen and melting regimes arise in the limit of large $ma/hv \gg 1$. Before we proceed, we note C and $\mathbf{S}$ can be expressed as $C(x, t) = [F_0(x, t) + F_0(x, -t)]/2$ and $S_{x,y,z} = [F_{x,y,z}(x, t) - F_{x,y,z}(x, -t)]/2i$ with

$$F_j(x, t) = \mathcal{N} \int \Phi(p_x) e^{ip_x x} e^{i\epsilon_p t/h} f_j(p_x) \frac{dp_x}{2\pi}, \quad (26)$$

where $j = 0, x, y, z$, and the j -dependent weights as $f_0 = 1$ and $(f_x, f_y, f_z) = \mathbf{d}(\mathbf{p})/|\mathbf{d}(\mathbf{p})|$. Since the dynamics of $C(x, t)$ and $\mathbf{S}(x, t)$ are controlled by $F_j(x, t)$, we will analyze the temporal dynamics of $F_j(x, t)$ in the limit of $ma/hv \gg 1$.

In so doing, we first note that in the large postquench mass limit we can write

$$\epsilon_p = \sqrt{\Delta^2 + \hbar^2 v^2 p_x^2} \approx \Delta + (\hbar v p_x)^2 / (2\Delta) \quad (27)$$

where $\Delta = (m^2 + \hbar^2 v^2 p_y^2)^{1/2}$. We note parenthetically, when the postquench mass is large and by focussing on small $p_y < \alpha^{-1} \ll m/hv$, we have $\Delta \approx m$.

The tight localization of the TBM state arises from large $p_x \sim \alpha^{-1}$ components. When $ma/(hv) \gg 1$, the differences in their precession frequency are very much suppressed and can be well approximated by Eq. (27). Applying Eq. (27) into the precession frequency of Eq. (26) we obtain

$$F_j(x, t) \approx \frac{\mathcal{N} e^{i\Delta t/h}}{2\pi} \int \Phi(x') e^{ip_x(x-x')} e^{i\frac{\hbar v p_x^2}{2\Delta} t} f_j(p_x) dp_x dx', \quad (28)$$

where $(f_x, f_y, f_z) \approx (\hbar v p_x/\Delta, \hbar v p_y/\Delta, m/\Delta)$, and we have re-written

$\Phi(p_x) = \int dx' e^{-ip_x x'} \Phi(x')$. Recalling the identity

$$\int_{-\infty}^{\infty} e^{iax^2} dx = e^{i\pi/4} \sqrt{\frac{\pi}{a}}, \quad a > 0, \quad (29)$$

and integrating out p_x by completing the square in the argument of the exponential function, we obtain

$$F_{0,z}(x, t) \approx \mathcal{N} \sqrt{\frac{i\Delta}{\hbar v^2 t}} \Psi_t(x) e^{i\Delta t/h} \quad (30)$$

where

$$\Psi_t(x) = \int dx' \Phi(x+x') G_t(x'), \quad G_t(x) = e^{-i\Delta x^2/(2\hbar v^2 t)}, \quad (31)$$

where we have changed dummy variables $x - x' \rightarrow x'$. We note that in obtaining the probability density profile in both frozen and melting regimes, $F_{x,y}$ contributions are small (suppressed in the large $ma/hv \gg 1$ limit) and are negligible in $\langle \rho \rangle_{t,x}$. This is because while the integrand in F_z is proportional to $m/\Delta \rightarrow 1$, the integrand in F_x, F_y are proportional to p_x, p_y respectively, which are much smaller than the gap size m .

Frozen regime. The imaginary Gaussian kernel in Eq. (31) plays a crucial role in the dynamical evolution of the probability density. At short times, $G_t(x)$ exhibit spatial oscillations far faster than the spatial variations of $\Phi(x)$. Indeed, $\Phi(x)$ varies significantly for x varying on order of the TBM width, α . In contrast, $G_t(x)$ oscillates rapidly on length scales of order $\sqrt{2\hbar v^2 t}/\Delta$, and for short times, this length scale can be far shorter than α . As a result, for short time windows (controllable by

postquench m)

$$0 < t \ll \frac{\Delta \alpha^2}{2\hbar v^2} \approx \frac{m\alpha}{\hbar v} \tau, \quad \tau = \frac{\alpha}{2v}, \quad (32)$$

we have $\Delta \alpha^2/(2\hbar v^2) \gg 1$.

In this short time window this behavior picks out $\Phi(x)$ and we can approximate

$$\Psi_t(x) \approx \Phi(x) \int G_t(x') dx' = \Phi(x) e^{-i\pi/4} \sqrt{\frac{2\pi\hbar v^2 t}{\Delta}} \quad (33)$$

As a result, at these short times $F_{0,z}(x, t) \approx \mathcal{N} \Phi(x) e^{i\Delta t/h}$, so that $C(x, t) \approx \mathcal{N} \Phi(x) \cos(\Delta t/h)$ and $S_z \approx \mathcal{N} \Phi(x) \sin(\Delta t/h)$, where we have noted that $F_{x,y}$ are suppressed for large ma/hv . Substituting into Eq. (5) in the main text we obtain the Frozen profile

$$\langle \rho \rangle_{t,x} \approx \mathcal{N}^2 |\Phi(x)|^2, \quad 0 < t < \frac{\tau m \alpha}{\hbar v}. \quad (34)$$

This frozen (i.e. near-stationary in time) profile closely conforms to that found from a direct numerical evaluation of the probability density found in Fig. 2b of the main text.

Melting regime. In the opposite regime to Eq. (32) (maintaining $ma/hv \gg 1$), $t \gg \frac{\Delta \alpha^2}{2\hbar v^2} = \frac{m\alpha}{\hbar v} \tau$, the imaginary Gaussian kernel in Eq. (31) varies slowly in x . This will tend to spread $F_{0,z}(x, t)$, and hence the density of states, out in space. The amplitude of $F_{0,z}(x, t)$ decays as $t^{-1/2}$ as in Eq. (30). We similarly obtain $C(x, t) \approx [\Delta/(h v^2 t)]^{1/2} \mathcal{N} \cos(\Delta t/h + \pi/4) \Psi_t(x)$ and $S_z \approx [\Delta/(h v^2 t)]^{1/2} \mathcal{N} \sin(\Delta t/h + \pi/4) \Psi_t(x)$. Note that $S_x, S_y \ll S_z$ because the values of p_x, p_y that contribute to the integral giving F_x, F_y are much smaller than Δ in the large $ma/hv \gg 1$ limit. Plugging this into Eq. (5) in the main text we obtain, the power-law melting profile found in Eq. (10) of the main text.

Interestingly, in this regime the imaginary Gaussian kernel in Eq. (31) varies slowly in x , and in particular, far more slowly than $\Phi(x+x')$ varies in position. As a result, for $x \lesssim \alpha$ close to the domain wall $\Psi_t \approx \int \Phi(x+x') dx'$ goes approximately as a constant. In this limit, we find that the probability density at different $x \lesssim \alpha$ collapse onto each other and decay as

$$\langle \rho \rangle_{t,x} \propto \frac{\Delta}{\hbar v^2 t}, \quad t \gg \frac{m\alpha}{\hbar v} \tau, \quad (35)$$

displaying a universal x -independent power-law decay. Indeed, this collapse of probability density at different x positions in the long-time limit is seen in a direct numerical evaluation in Fig. 2b of the main text. Indeed, we expect that at sufficiently long times, when $\Psi_t(x)$ has spread out, larger $x > \alpha$ within the width of $\Psi_t(x)$ will also similarly exhibit a universal x -independent power-law decay.

Illustration: exactly solvable example. While we have obtained approximate expressions for $\Psi_t(x)$ [and hence, $F_j(x, t)$] above, it is instructive to illustrate the “frozen” and “melting” regime behavior through an exactly solvable model. To do so we consider the case where the initial TBM state $\Phi(x)$ takes the form of a Gaussian: $\Phi(x) = e^{-x^2/2\alpha^2}$, where α is the standard deviation. We can then obtain by direct integrating Eq. (31)

$$\Psi_t(x) = \sqrt{\frac{\hbar v^2 \alpha^2 t}{\hbar v^2 t + i\Delta \alpha^2}} [\exp(-x^2/2\alpha^2)]^{1 - \frac{\hbar v^2 t}{\hbar v^2 t + i\Delta \alpha^2}} \quad (36)$$

Evidently, when $t \ll \frac{\alpha^2 \Delta}{\hbar v^2}$ then we have

$$\Psi_t(x) \rightarrow \sqrt{\frac{\hbar v^2 t}{i\Delta}} [\exp(-x^2/2\alpha^2)], \quad t \ll \frac{\alpha^2 \Delta}{\hbar v^2} \quad (37)$$

Substituting in Eq. (30), we obtain $F_{0,z}(x, t) \approx e^{i\Delta t} \mathcal{N} \exp(-x^2/2\alpha^2) \propto \mathcal{N} \Phi(x) e^{i\Delta t}$ that leads to the frozen (near-stationary in time) probability density profile as discussed above.

In the opposite limit $t \gg \frac{\alpha^2 \Delta}{\hbar v^2}$,

$$\Psi_t(x) \rightarrow \sqrt{2\pi\alpha^2}, \quad t \gg \frac{\alpha^2 \Delta}{\hbar v^2} \quad (38)$$

Substituting into Eq. (30), we similarly obtain the power-law decay in the probability density described in Eq. (35).

Data availability

The data that support the findings of this study are available from the corresponding author upon reasonable request.

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Author contributions

C.H.L. performed the computations. J.C.W.S. initiated the project. C.H.L. and J.C.W.S. wrote the manuscript together.

Competing interests

The authors declare no competing interests.

Additional information

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