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Topological state engineering via supersymmetric transformations

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The quest to explore new techniques for the manipulation of topological states simultaneously promotes a deeper understanding of topological physics and is essential in identifying new ways to harness their unique features. Here, we examine the potential of supersymmetric transformations to systematically address, alter and reconfigure the topological properties of a system. To this end, we theoretically and experimentally study the changes that topologically protected states in photonic lattices undergo as supersymmetric transformations are applied to their host system. In particular, we show how supersymmetry-induced phase transitions can selectively suspend and re-establish the topological protection of specific states. Furthermore, we reveal how understanding the interplay between internal symmetries and the symmetry constraints of supersymmetric transformations provides a roadmap to directly access the desirable topological properties of a system. Our findings pave the way for establishing supersymmetry-inspired techniques as a powerful and versatile tool for topological state engineering.

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Physical laws are intrinsically connected to symmetries, which can be classified in spacetime and internal symmetries. Unlike any other symmetry, supersymmetry (SUSY), originally developed as an extension of the Poincaré Group¹, offers a loophole to the Coleman–Mandula theorem², allowing the interplay of spacetime and internal symmetries in a nontrivial way³. Despite the lack of direct experimental evidence of SUSY in high-energy physics, where it establishes a relation between bosons and fermions¹, some of its fundamental concepts have been successfully adapted to numerous fields such as condensed matter⁴, statistical mechanics⁵, nonrelativistic quantum mechanics⁶, optics^{7,8}, and cosmology⁹. In particular, SUSY provides an effective theory to describe quantum phase transitions occurring at the boundary of topological superconductors¹⁰, where topological states characterized by topological invariants emerge^{11,12}. In recent years, the field of photonics has shed light on a plethora of phenomena stemming from topological phases (as shown by Lu et al.¹³ and Ozawa et al.¹⁴), and photonic lattices have been established as a versatile experimental platform^{15–18}. In a similar vein, SUSY concepts have been introduced to photonics⁸ to tackle the long-standing challenge of systematically shaping the modal content of highly multi-mode structures^{19–26}, controlling scattering characteristics^{27–30}, designing laser arrays^{31,32}, and creating band gaps in extremely disordered potentials³³ and topologically protected mid-gap states starting from trivial configurations³⁴.

In this work, SUSY transformations are applied to manipulate topological properties, which are deeply connected to internal symmetries of the systems. Specifically, we present a method for topological state engineering, e.g., to selectively suspend and re-establish the topological protection of a targeted state, which can be applied to systems described by a tight-binding Hamiltonian such as optical waveguides³⁵, coupled cavities³⁶, ultra-cold atoms³⁷, or acoustic and mechanical systems³⁸. Furthermore, it is shown how closely this behavior is linked to symmetry constraints of SUSY transformations³⁹, enabling these symmetries to be fully or partially preserved, or cancelled in their entirety. As SUSY transformations are tailored to their specific purpose, they imprint their characteristic signature on the topological invariants, as well as the related topological protection. Here, to explore the fruitful interplay between SUSY and topology, we employ femtosecond laser written photonic lattices³⁵. Specifically, to elucidate how SUSY enables the manipulation of topological properties, we apply discrete SUSY (DSUSY) transformations to photonic lattices embodying the simplest system with nontrivial topological properties, the Su–Schrieffer–Heeger (SSH) model⁴⁰. Along these lines, we show that SUSY allows for the systematic breaking and recovery of symmetries of the system and thereby constitutes a powerful tool to tailor topological transitions and to manipulate the topological properties of a system.

Results

Theory. In its general quantum-mechanical formulation, unbroken SUSY connects two superpartner Hamiltonians $\mathcal{H}^{(1)}$ and $\mathcal{H}^{(2)}$, sharing a common set of eigenvalues except for the eigenvalue of the ground state of $\mathcal{H}^{(1)}$, which is removed from the spectrum of $\mathcal{H}^{(2)}$. A step forward toward a more general Hamiltonian spectrum manipulation, allowing the removal of different eigenvalues, can be achieved by applying DSUSY transformations⁸. Considering a one-dimensional lattice composed of N evanescently coupled single-mode waveguides, the system is characterized by a discrete Hamiltonian $\mathcal{H}$ given by an $N \times N$ tridiagonal matrix, with the propagation constants occupying the diagonal elements and the coupling strengths the off-diagonal elements. For the waveguide lattices employed here,

light propagation along the z -direction can be described using coupled-mode equations⁴¹:

$$-i \frac{d}{dz} \Psi = \mathcal{H} \Psi, \quad (1)$$

where $\Psi = (\psi_1, \dots, \psi_N)^T$, with ψ_j describing the modal field amplitude in waveguide j . From the eigenvalue equation

$$\mathcal{H} \Psi_s = \lambda_s \Psi_s, \quad (2,)$$

which relates the eigenfunction Ψ_s and eigenvalues λ_s of the state s , superpartner Hamiltonians

$$\mathcal{H}_m^{(1)} = \mathcal{H} - \lambda_m I = QR \text{ and } \mathcal{H}_m^{(2)} = RQ, \quad (3)$$

can be obtained using the QR factorization⁴², where Q is an orthogonal matrix ($Q^T Q = I$), R an upper triangular matrix, and I the identity matrix. Note that, in general, the QR factorization is not unique and could lead to different solutions sharing the same eigenvalue spectrum (see Supplementary Note 1).

The superpartner Hamiltonian $\mathcal{H}_m^{(2)}$ shares a common set of eigenvalues with $\mathcal{H}_m^{(1)}$, except for λ_m that has been removed from the spectrum of $\mathcal{H}_m^{(2)}$. In addition, the standard SUSY transformation annihilating the fundamental state can still be carried out with this method, as displayed in Fig. 1a. The corresponding eigenvalue λ_m is removed because its eigenstate Ψ_m is completely localized in the fully decoupled N th waveguide and, as such, does not have any influence on the dynamics of the remaining system of $N - 1$ waveguides (see Supplementary Fig. 1). By applying these transformations in an iterative way, superpartner structures with desired eigenvalue spectra can be engineered by removing the desired number of eigenvalues, reducing the overall system size. A question that naturally arises, and to this date remains unexplored, is the impact of targeting a state with nontrivial topological properties. Does its removal irrevocably change the topological properties of the system?

The SSH model, one of the most prominent systems for illustrating topological physics, can be implemented using a one-dimensional lattice of evanescently coupled waveguides with two alternating couplings c_1 and c_2 ($c_1 < c_2$). While an infinite lattice is invariant under the exchange of couplings, the presence of edges in a finite SSH chain introduces two distinct types of edge terminations that, in turn, give rise to topological states that can be described by the bulk-edge correspondence and topological invariants. In particular, topological edge states, which can be quantified by a winding number $\mathcal{W} = \mathcal{Z}/\pi$, appear at the end of a region with nonzero Zak phase $\mathcal{Z}$, where $\mathcal{Z} = 0$ or π depending on the edge termination^{43,44}. If the lattice terminates with the weak coupling c_1 , the winding number is 1 and the lattice supports one topological edge state. On the contrary, if the lattice terminates with the strong coupling c_2 , the winding number is 0 and the structure does not support an edge state, as displayed in Fig. 1b. The topological protection of these states is directly related with the existence of internal symmetries in the system. Specifically, the chiral symmetry given by

$$\Gamma \mathcal{H} \Gamma^\dagger = -\mathcal{H}, \quad (4)$$

where Γ is an unitary and Hermitian operator satisfying $\{\mathcal{H}, \Gamma\} = 0$, is responsible of the protection of the zero-energy states. By applying the chiral symmetry operator Γ to the eigenvalue Eq. (2), one obtains $\mathcal{H}(\Gamma \Psi_s) = -\lambda_s(\Gamma \Psi_s)$, which entails that the energy spectrum of the system is symmetric around 0. This fact, in turn, guarantees that all the states Ψ_s with positive energy λ_s have a counterpart $\Gamma \Psi_s$ with negative energy $-\lambda_s$, with the exception of the zero-energy states, which are topologically protected (see Supplementary Note 2 for more details).

Fig. 1 Supersymmetric transformations and Su-Schrieffer-Heeger model.

a Schematic representation of the eigenvalue spectrum of the Hamiltonian $\mathcal{H}$ and two sets of superpartner Hamiltonians $\{\mathcal{H}_1^{(1)}, \mathcal{H}_1^{(2)}\}$ and $\{\mathcal{H}_m^{(1)}, \mathcal{H}_m^{(2)}\}$, obtained by removing the eigenvalues λ_1 and λ_m using supersymmetric transformations, respectively. **b** Representation of a Su-Schrieffer-Heeger lattice implemented using optical waveguides, which are evanescently coupled with alternating couplings c_1 and c_2 ($c_1 < c_2$). Depending on the termination of the lattice, the structure has Winding number $W = 1$ and supports a topological edge state on that edge or $W = 0$ and does not support an edge state on that edge.

DSUSY transformations applied to the Hamiltonian can be expressed in terms of a transformation matrix V as

$$V\mathcal{H}_m^{(1)}V^{-1} = VQRV^{-1} = RQ = \mathcal{H}_m^{(2)}, \quad (5)$$

where $V = Q^{-1}$. If both $\mathcal{H}_m^{(1)}$ and V possess some symmetry, e.g., chiral symmetry satisfying $\{\mathcal{H}_m^{(1)}, \Gamma\} = \{V, \Gamma\} = 0$, then this symmetry is transferred to $\mathcal{H}_m^{(2)}$:

$$\mathcal{H}_m^{(2)} = V\mathcal{H}_m^{(1)}V^{-1} = -V\Gamma\mathcal{H}_m^{(1)}\Gamma^\dagger V^{-1} = -\Gamma\mathcal{H}_m^{(2)}\Gamma^\dagger. \quad (6)$$

On the other hand, if the transformation matrix V does not obey this symmetry, it will not be reproduced in the superpartner Hamiltonian $\mathcal{H}_m^{(2)}$ either. Exploiting this connection between symmetry constraints of DSUSY transformations and symmetries of the system, superpartner Hamiltonians with modified topological properties can be engineered, and thus establishing a new method for topological state engineering by combining SUSY isospectrality and breaking of symmetries of the system. As a proof of concept, three distinct structures are investigated: (i) the superpartner SP_1 , obtained by removing the eigenvalue λ_1 corresponding to a bulk state (see Fig. 2a), (ii) the SSH supporting two topologically protected edge states (see Fig. 2b), and (iii) the superpartner $SP_{N/2}$, obtained by removing the eigenvalue $\lambda_{N/2}$ corresponding to a topological edge state (see Fig. 2c), whose lattice configurations are schematically illustrated in Fig. 2d, e and f, respectively. Note that, due to the symmetry of the eigenvalue spectrum, equivalent results to the SP_1 and $SP_{N/2}$ structures would be obtained by removing λ_N and $\lambda_{N/2+1}$, respectively. Subsequently, the degree of protection of the superpartner topological states is probed analytically with respect to their

symmetries, as well as numerically by gauging their robustness against chiral disorder⁴⁵.

Supersymmetric topological photonic structures. Figure 2c shows the eigenvalue spectrum of the $SP_{N/2}$ lattice obtained by removing the eigenvalue $\lambda_{N/2}$ corresponding to an edge state of the SSH structure. Since $\lambda_{N/2}$ is a zero-energy eigenvalue, the diagonal elements of the superpartner Hamiltonians $\mathcal{H}_{N/2}^{(1)}$ and $\mathcal{H}_{N/2}^{(2)}$ remain 0. Thus, the superpartner lattice is composed of waveguides with zero detuning (see Supplementary Note 1 for an extended discussion). Here, the transformation matrix possesses chiral symmetry, which is transferred to the superpartner Hamiltonian $\mathcal{H}_{N/2}^{(2)}$ that satisfies $\Gamma\mathcal{H}_{N/2}^{(2)}\Gamma^\dagger = -\mathcal{H}_{N/2}^{(2)}$. Therefore, the symmetries of the system are preserved and the topological properties of the remaining zero-energy eigenstate remain intact. By applying DSUSY transformations, two different superpartner lattices supporting one topological state $\Psi_{N/2+1}$ can be obtained due to the nonuniqueness of the QR factorization process (see Supplementary Note 1). One supporting an interface state, as displayed in Fig. 2f, and the other supporting an edge state, mostly maintaining the form of Fig. 2e with interchanged couplings and the last waveguide removed. For the interface state solution, the $SP_{N/2}$ structure resembles two SSH chains with different termination at the interface and strong coupling at the outer edges, as it is illustrated in Supplementary Fig. 3. The topologically protected interface state, whose position in the lattice can be controlled by changing the dimerization $|c_1 - c_2|$, is located between the two SSH lattices and decays exponentially into the bulk. The existence of this interface state is experimentally verified, as discussed in detail in the next section, and its robustness against chiral disorder maintaining the underlying symmetry of the lattice is numerically proved. In particular, by introducing chiral disorder, the deviation of the eigenvalue $\lambda_{N/2+1}$ is proved to be 0, while the eigenstate shape is slightly modified, although it remains localized at the interface. For the edge state solution, the $SP_{N/2}$ structure resembles the SSH model with interchanged couplings and $N - 1$ waveguides, except for a localized deviation in the couplings with respect to c_1 and c_2 near the leading edge. Here, the transformation constitutes a topological phase transition in the sense that the couplings are interchanged and a waveguide removed, thus one of the edge states is annihilated. As earlier, the remaining edge state is topologically protected and robust against chiral disorder. Note that, by applying another DSUSY transformation removing the remaining zero-energy eigenvalue, the system becomes topologically trivial. Besides, the protocol could be implemented the other way around by applying inverse SUSY transformations⁴⁶, i.e., starting with the topologically trivial SSH model and adding a zero-energy eigenvalue to the spectrum. To sum up, by annihilating zero-energy eigenvalues, DSUSY transformations introduce topological phase transitions, leading to the displacement and destruction of topological states.

Let us now consider the SP_1 lattice, obtained by removing the eigenvalue λ_1 corresponding to a bulk state of the SSH structure, as it is displayed in Fig. 2a. Considering that the removal of any bulk state of the system per definition breaks the inversion symmetry of the eigenvalue spectrum, one would expect that the topological protection of the edge states is necessarily destroyed. Nevertheless, the chiral symmetry of the system is partially respected by the DSUSY transformation, preserving the topological protection of one edge state. This can be explained by separating the Hamiltonian $\mathcal{H}_1^{(2)}$ into $\mathcal{H}_{1L}^{(2)}$ and $\mathcal{H}_{1R}^{(2)}$, corresponding to the left and right parts of the lattice, respectively. The chiral symmetry of $\mathcal{H}_{1R}^{(2)}$ is preserved, satisfying $\Gamma\mathcal{H}_{1R}^{(2)}\Gamma^\dagger = -\mathcal{H}_{1R}^{(2)}$ and, thus, the topological protection of

Fig. 2 Supersymmetric topological photonic structures. Eigenvalue spectrum of (a) the superpartner SP_1 lattice, obtained by removing a bulk state ($m=1$), (b) the Su-Schrieffer-Heeger (SSH) lattice, and (c) the superpartner $SP_{N/2}$ lattice obtained by eliminating an edge state ($m=N/2$). The energy gap is $2|c_1 - c_2|$ and $c_T = c_1 + c_2$. Sketch of the (d) SP_1 , (e) SSH, and (f) $SP_{N/2}$ lattices. Detuning (coupling) is indicated by the size (spacing) of (between) the circles. The intensity of the color inside each waveguide is proportional to the amplitude of the state. Experimentally observed light evolution along the propagation direction and output intensities for the (g) nontopological edge state, (h) topological edge state, and (i) topological interface state. The total length of the sample is $L = 10$ cm and the wavelength used to excite the waveguides is $\lambda = 633$ nm. The SSH, SP_1 , and $SP_{N/2}$ lattices are composed of $N = 50$, $N = 49$, and $N = 109$ waveguides, respectively. The original SSH lattice used to synthesize the SP_1 ($SP_{N/2}$) lattice is composed of $N = 50$ ($N = 110$) waveguides, $c_1 = 0.5 \text{ cm}^{-1}$ and $c_2 = 1.0 \text{ cm}^{-1}$ ($c_2 = 1.8 \text{ cm}^{-1}$). The different number of waveguides and coupling coefficients used to create the superpartner structures are due to experimental restrictions. In particular, the SP_1 lattice has weaker couplings to adjust the detunings below 2 cm^{-1} in order to guarantee that the couplings stay constant when changing the detuning.

the right edge state is maintained. On the contrary, the chiral symmetry of $\mathcal{H}_{1L}^{(2)}$ is destroyed by the appearance of nonzero diagonal elements, which take away the symmetry protection of the left edge state. However, the state remains localized at the edge due to the high detuning between waveguides. Moreover, although the left edge state loses its topological protection, its zero-energy eigenvalue is always pinned to 0 due to SUSY isospectrality. The SP_1 lattice exhibits an exponentially decaying detuning on the left side of the lattice, while still resembling the SSH model toward the right part of the lattice (see Supplementary Fig. 3 for more details). The existence of both edge states and their different origins is experimentally verified, as discussed in the next section. Also, the stability of the edge states eigenvalues in the spectrum is numerically checked by introducing chiral disorder. Specifically, for the right edge state the deviation of the eigenvalue $\lambda_{N/2+1}$ tends to 0 as N increases, whereas for the left edge state, the deviation of the eigenvalue $\lambda_{N/2}$ is not affected by the size of the system and increases linearly with the amount of disorder (see Supplementary Note 2 and Supplementary Fig. 4 for more details). Note that by applying another DSUSY transformation removing λ_N , the inversion symmetry of the system is reestablished, and the topological protection of the left edge state can be restored. Furthermore, by removing high-order bulk states only from one side of the spectrum ($1 < m < N/2$), the detuned region can be extended across the lattice to facilitate an enhanced coupling with the right edge state, which can even lose its topological protection

due to the effect of the detuning. Finally, by applying multiple DSUSY transformations symmetrically, gaps can be carved out of the eigenvalue spectrum while preserving the topological protection of the zero-energy states. Here, in short, we have transformed a lattice supporting two topologically protected edge states to a phase-matched lattice supporting one topologically protected edge state, and one that has lost its topological protection and has become sensitive to the underlying disorder.

To summarize, the effects that DSUSY transformations induce on a system supporting two topologically protected zero-energy states are as follows: (i) the removal of a bulk state that is energetically far from the energy gap (m close to 1 or N), leads to a superpartner structure that supports one topological and one nontopological zero-energy states, (ii) the suppression of a bulk state that is energetically close to the energy gap (m close to $N/2$), destroys the topological protection of both zero-energy states of the superpartner structure, and (iii) the elimination of a zero-energy state ($m = N/2$ or $N/2 + 1$), produce a superpartner structure that supports one topologically protected zero-energy state. Finally, note that, if one removes bulk states of the system in a symmetric way, e.g., $m = 2$ and $m = N - 1$, both zero-energy states will hold their topological properties since chiral symmetry is preserved.

Experimental verification. In order to experimentally corroborate the previous theoretical findings, we employ the femtosecond

direct laser-writing technology to inscribe waveguide arrays in fused silica (see “Methods,” Supplementary Note 3 and Supplementary Fig. 3 for more details). Specifically, we exploit its ability to independently tune the coupling and detuning by changing the separation between waveguides and the inscription velocity, respectively²⁰. To this aim, four different samples are fabricated: (i) the original SSH lattice described by $\mathcal{H}$, (ii) the superpartner $\text{SP}_{N/2}$ lattice described by $\mathcal{H}_{N/2}^{(2)}$, (iii) the superpartner SP_1 lattice described by $\mathcal{H}_1^{(2)}$, and (iv) the SSH lattice weakly coupled to the SP_1 lattice. By launching single site excitations, light evolution of the different states along the different structures can be measured by means of waveguide fluorescence microscopy³⁵, and output pattern intensities can be extracted. Furthermore, by using a white light source, the wavelength of the injected light can be continuously tuned to evaluate the robustness and different origins of the edge states. Finally, by placing the SSH lattice in close proximity to the SP_1 lattice, evanescent coupling can be introduced between the topological edge state in the former, and the nontopological edge state in the latter. The contrast of the resulting sinusoidal intensity oscillations serves as direct indicator for any detuning between them, or the predicted absence thereof.

The first step to verify the previous theoretical predictions is to prove the existence of the edge and interface states. To this end, we excite the nontopological edge state, the topological edge state and the interface state and observe its evolution along the propagation direction, as displayed in Fig. 2g, h and i, respectively. First, the topological edge state is excited by injecting light into the N th waveguide, as depicted in Fig. 2h. As a single site excitation is made, and the theoretically expected edge state is exponentially localized within the waveguides N , $N - 2$, and $N - 4$, as it is illustrated in Fig. 2e, other bulk states of the system are also excited and the injected intensity slightly spreads along the propagation direction. However, one can clearly observe how the output measured intensity distribution is in accordance with the predicted mode profile, showing the expected SSH edge state. Since the SSH lattice is symmetric, a mirrored propagation image would be obtained by injecting light into the first waveguide, exciting the left topological edge state. Note that the confinement of this edge state scales with the difference between the coupling coefficients c_1 and c_2 . The next step is to demonstrate the presence of the interface state of the $\text{SP}_{N/2}$ lattice. Although the expected theoretical interface state spans

approximately five odd waveguides, as depicted in Fig. 2f, it is nevertheless populated by a single site excitation at the interface waveguide, as displayed in Fig. 2i. Moreover, as can be observed from the output intensity pattern, most of the light is localized at the interface waveguide itself. Note that, for the $\text{SP}_{N/2}$ structure supporting only one edge state, light evolution and output intensity would resemble the previously obtained for the SSH lattice. The next stage is to prove the existence of the nontopological edge state of the SP_1 lattice, which has lost its topological protection due to the breaking of chiral symmetry of one part of the system. To do that, the first waveguide of the SP_1 lattice is excited, as it is displayed in Fig. 2g. While the localization is still visible, it may be noted that the output intensity distribution for the nontopological edge state does not have the staggered profile that characterizes the topological edge states. Instead, the nontopological edge state is mainly localized due to the high detuning in the first and second waveguides of the lattice, as it is depicted in Fig. 2d. Since this edge state is solely mediated by the detuning, it is less robust against perturbations than the topological state, as we numerically verified in Supplementary Note 2 and illustrated in Supplementary Fig. 4. Furthermore, a strong indication to this reasoning can be seen when we excite both edge states tuning the wavelength continuously from 500 to 720 nm. The experimental results obtained for the propagation of the different states are in good agreement with the tight-binding simulations, as shown in Supplementary Fig. 5.

To verify the different origin of the edge states of the SP_1 lattice, we excite both edges with different wavelengths and observe the output intensities after 10 cm of propagation, as can be seen in Fig. 3a, b. Experimentally, this is achieved by using a white light source combined with a narrow wavelength filter, as discussed in detail in “Methods” section. The first observation is that, despite their different topological natures, both edge states remain localized at the corresponding edges of the SP_1 lattice, represented in Fig. 3c. However, since the nontopological edge state is supported by the detuning, its degree of localization strongly decreases toward longer wavelengths (see Fig. 3a). This occurs because at longer wavelengths, the coupling substantially increases while the detuning decreases, thus the former becomes the dominant term and the confinement of the edge state is reduced. On the contrary, it gets fully localized into a single waveguide for shorter wavelengths, where the detuning is the dominant term. The confirmation that

Fig. 3 Wavelength dependence of the edge states. Experimentally observed output intensities for different wavelengths ($500 \text{ nm} \leq \lambda \leq 720 \text{ nm}$) obtained by exciting (a) the nontopological and (b) the topological edge states of the SP_1 lattice, schematically represented in c. The red dots indicate the excited waveguides. The relation between the wavelength used and its intensity measured at the output is represented in the inset. The total length of the sample is $L = 10 \text{ cm}$ and the SP_1 lattice is composed of $N = 49$ waveguides.

Fig. 4 Coupling of topological and nontopological edge states.

Experimentally observed light evolution along the propagation direction when the waveguides supporting (a) the topological and (b) the nontopological edge states of the coupled structure, schematically displayed above, are excited. Power oscillations when the waveguide supporting (c) the topological (green line) and (d) the nontopological (blue line) edge state is excited. The y axis is the normalized power extracted from the fluorescence images in a and b. The x axis is the propagation distance in cm. The solid lines correspond to the experimental results while the dashed lines correspond to the tight-binding numerical simulations. The total length of the sample is $L = 10$ cm, the wavelength used to excite the waveguides is $\lambda = 633$ nm and the lattice is composed of $N = 99$ waveguides.

the existence of this edge state is due to detuning is a strong indication for less robustness, since it does not have a topological origin. On the other hand, as shown in Fig. 3b, the topological state strictly maintains its characteristic staggered intensity structure across the investigated spectral range. Note that the slight delocalization at short wavelengths occurs as both couplings decrease and their absolute difference $|c_1 - c_2|$, which is related with the edge state confinement, becomes too small to strongly confine the state at the edge.

So far, we have proved the existence of the different topological states, as well as the different origin of the edge states of the SP_1 lattice. The last step is to verify that the nontopological edge state indeed does possess a zero-energy eigenvalue, as expected from SUSY transformation. To this aim, we weakly couple the

Fig. 5 Wavelength dependence of the coupled edge. a Experimentally observed output intensities for different wavelengths obtained by exciting the topological edge state of the coupled edge, indicated with the red dot in b. b Schematic representation of the Su-Schrieffer-Heeger (SSH) lattice weakly coupled to the superpartner SP_1 lattice. The relation between the wavelength used and its intensity measured at the output is represented in the inset. The total length of the sample is $L = 10$ cm, the wavelength range used to excite the waveguides is $500 \text{ nm} \leq \lambda \leq 720 \text{ nm}$ and the lattice is composed of $N = 99$ waveguides.

nontopological edge state with the topological state, as displayed in Fig. 4a, b. Here, if the two states have the same energy, one would expect their coupling with a full exchange of power. On the contrary, if the two states have different energies, one would expect only a partial exchange of power. In Fig. 4a, b, we show the light evolution along the propagation direction when we excite either the waveguide supporting the topological state or the nontopological edge state, respectively. Moreover, in Fig. 4c, d, one can see how the topological edge state is coupled to the nontopological one and vice versa, transmitting around 70% of the injected power to the other edge state. Furthermore, one can observe how light is completely outcoupled from the excited waveguide, indicating that both edge states share the same propagation constant. Note that the intensity oscillations are in good agreement with the tight-binding simulations, shown by the dashed lines of Fig. 4c, d. The full oscillation pattern between edge states, both experimental and simulated, can be observed in Supplementary Fig. 5. Finally, we have checked that both edge states share the same energy independently of the wavelength by exciting them with a white light source, as it is displayed in Fig. 5. By increasing the wavelength, the coupling increases, leading to a reduced effective length scale of the chip. Looking at the output intensities, the full exchange of intensity between waveguides can be observed, confirming that both superpartner share the same eigenvalue spectrum.

Discussion

In our work, we studied the interplay between topological non-trivial systems and SUSY transformations. For this, we picked one of the most prominent models for illustrating topological physics, the SSH model, and demonstrated how topological phase transitions can be induced by DSUSY transformation. In particular, we showed that while this topological transition may suspend the topological protection of a state, it can readily be reestablished by applying another DSUSY transformation. Moreover, DSUSY

transformations can also be used to annihilate topological states as well as to displace them from the edge to the inner part of the lattice. We exemplified this by transforming a lattice supporting two topological edge states to a lattice supporting (i) one topological edge or interface state, and (ii) one topological and one nontopological edge states. We experimentally demonstrated those theoretical findings implementing the superpartner structures using femtosecond laser written waveguides.

Clearly, SUSY techniques constitute a powerful tool to design structures with desirable topological properties, which can be extended to higher dimensions and chiral edge states in future works. Moreover, iterative DSUSY transformations could serve to remove any number of states from the system and reduce its overall size while preserving the desired part of the spectrum and the system's topological properties. Furthermore, inverse SUSY transformations^{29,46} could be used to introduce zero-energy states, which will be topologically protected when these transformations are applied to systems having topology protecting symmetries. Finally, note that the method for topological state engineering presented here, based on applying DSUSY transformations, can be extended to any platform allowing independent control of the coupling and detuning of the sites, which are described by discrete Hamiltonians such as coupled cavities³⁶, ultra-cold atoms trapped in lattice potentials³⁷ or acoustic and mechanical systems³⁸.

Methods

Experimental design. Our experiments were conducted in femtosecond laser written photonic lattices, where the above-mentioned structures are fabricated and characterized as described below.

Fabrication of the structures. The waveguides were fabricated in 10 cm fused silica glass (Corning 7980) samples by using the femtosecond laser-writing method³⁵. The laser system consists of a Coherent RegA 9000 amplifier seeded with a Coherent Vitar S Titanium:Sapphire laser with an energy of 250 nJ at 800 nm, 100 kHz repetition rate, and a pulse width of ~130 fs. By moving the sample with a high-precision positioning stage (Aerotech ALS 180) at speeds between 91 and 103 mm min⁻¹, the refractive index change at the focal point was around 7×10^{-4} . The created waveguides exhibit a mode field diameter of about $10.4 \times 8 \mu\text{m}$ at 633 nm. The propagation losses and birefringence are estimated to be 0.2 dB cm⁻¹ and 1×10^{-7} , respectively.

Characterization of the structures. In order to probe the propagation, the samples were illuminated with light from a Helium-Neon laser at 633 nm (Melles-Griot). The single lattice sites were excited with a 10× microscope objective (0.25 NA). In turn, the color centers that formed during the fabrication process, enable a direct observation of the propagation dynamics by using fluorescence microscopy³⁵. The recorded images were post processed to reduce noise, distortions, and the influence of background light.

The intensities at the output facet at different wavelengths were measured by using a white light source (NKT SuperK EXTREME) combined with a narrow wavelength filter (Photon ETC LLTF-SR-VIS-HP8). The light is then coupled into a single lattice site of the sample with a 10× microscope objective (0.25NA) and the resulting light at the output facet of the sample is imaged onto a CCD camera (BASLER Aviator) with another 10× microscope objective. The recorded images were post processed to reduce noise and subsequently integrated over a strip along the direction perpendicular to the lattice orientation for each wavelength. The resulting intensity distribution for the different wavelengths are then normalized to the maximum value to increase the visibility.

Data availability

All experimental data and any related experimental background information not mentioned in the text are available from the authors on reasonable request.

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Author contributions

G.Q., M.K., and M.H. developed the theory. M.K., L.M., and G.Q. fabricated the samples and performed the measurements. M.H., J.M., V.A., and A.S. supervised the project. All authors discussed the results and co-wrote the paper.

Competing interests

The authors declare no competing interests.

Additional information

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