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Platform of chiral Majorana edge modes and its quantum transport phenomena

James Jun He ^{1*}, Tian Liang¹, Yukio Tanaka² & Naoto Nagaosa^{1,3}

Majorana fermions, as electronic quasi-particle modes in solid states, have been under the focus of research due to their exotic physical properties. While the evidence of Majorana fermions as zero-dimensional bound states has been well established, the existence of one-dimensional Majorana modes is still under debate. The main reason is that the current theoretical proposals of platforms supporting such states are very challenging experimentally. Here, we propose a method to create two-dimensional topological superconductors with a heterostructure of ferromagnet, topological insulator thin film and superconductor. We show that such a system supports one-dimensional chiral Majorana edge modes in a wide range of parameters which is readily achievable in experiments. We further propose a new transport measurement to detect these modes.

¹RIKEN Center for Emergent Matter Science (CEMS), Wako, Saitama 351-0198, Japan. ²Department of Applied Physics, Nagoya University, Nagoya 464-8603, Japan. ³Department of Applied Physics, The University of Tokyo, Tokyo 113-8656, Japan. *email: jun.he@riken.jp

Majorana fermions in condensed matter systems^{1,2}, or Majorana quasiparticles, have been the subject of intense research due to their exotic properties such as fractional Josephson effect^{1,3–9}, resonant Andreev reflection^{10,11}, enhanced crossed Andreev reflection¹², spin selection^{13–18}, non-Abelian statistics^{19–23}, etc. Tremendous theoretical^{8,24–32} and experimental^{33–38} progress has been achieved to create zero-dimensional Majorana bound states in one-dimensional (1D) topological superconductors (TSCs). Promising braiding methods have been proposed based on such 1D systems³⁹ for the final proof of the non-Abelian particles, as well as for the creation of topological quantum computers⁴⁰.

On the other hand, 1D propagating Majorana modes at the edges of 2D TSCs have been realized recently in an experiment⁴¹ that combines quantum anomalous Hall insulators (QAHIs) with superconductors (SCs)⁴². In such a system, the TSC with single chiral Majorana edge mode appears only when the out-of-plane magnetization, M_z , is small compared with the superconducting gap Δ . This parameter regime may be achieved by tuning M_z with external magnetic field. In this way, narrow regions of half-quantized conductance were observed, which are considered as a signature of the chiral Majorana mode^{43–45}.

However, controversy arises based on an observation that half-quantized conductance may also appear trivially without any Majorana mode^{46,47} if the edge states of the QAH go through a long-enough path in the SC so that the SC part behaves just like a metal connecting two QAHIs in series. Due to the requirement of very small magnetization and the fact that the QAH was obtained by magnetic doping (which induces disorder), domains are likely to form and long paths for the QAH edge states can exist. This difficulty of distinguishing the TSC explanation from the trivial one stems from the theoretical model where both surfaces of the TI have the same magnetization that competes directly against the superconductivity order parameter, resulting in the requirement $M_z < \Delta$ ⁴². Thus, Majorana systems beyond this limitation are desired.

In this letter, we propose an alternative method to create 2D TSCs with a heterostructure of ferromagnet (FM), topological insulator (TI) thin film and superconductor, in which the two surfaces of the TI thin film form a two-dimensional system. One surface is superconducting due to the proximity effect and the other feels an exchange field from the FM. We show that there is a topological phase with single chiral Majorana edge mode that exists in readily achievable parameter regions and does not require magnetization to be smaller than the SC gap. An experimental setup containing a Josephson junction is proposed to uniquely determine the existence of Majorana chiral modes, in which a smoking-gun evidence is a change of conductance from $1/2 \leq \sigma_{12} \leq 1$ to $\sigma_{12} = 1/3$ (the unit of conductance is e^2/h throughout this paper) as the current is tuned up across the Josephson critical current. We also show that multiple chiral Majorana edge modes may appear when unconventional superconductors are used.

Results

Model. Assuming the FM to be insulating, the low-energy properties of the system is determined by the two surface states of the TI thin film. The two surfaces, however, experience different environments. The bottom surface is in good contact with the FM and feels a strong out-of-plane exchange field, whereas the top surface becomes superconducting due to proximity of the SC, as shown in Fig. 1. In the Nambu basis $\{c_{t\uparrow}(\mathbf{k}), c_{t\downarrow}(\mathbf{k}), c_{b\uparrow}(\mathbf{k}), c_{b\downarrow}(\mathbf{k}), c_{t\uparrow}^\dagger(-\mathbf{k}), c_{t\downarrow}^\dagger(-\mathbf{k}), c_{b\uparrow}^\dagger(-\mathbf{k}), c_{b\downarrow}^\dagger(-\mathbf{k})\}$, the effective

Fig. 1 Schematic picture of the system. A heterostructure of ferromagnet/topological-insulator/superconductor (FM/TI/SC). The two surfaces of the topological insulator thin film have Dirac dispersions. The top surface is superconducting due to the proximity effect and the bottom one feels an out-of-plane exchange field from the ferromagnet. Δ is the superconductivity order parameter and M_z denotes the magnetization. The constant t_c is the coupling between the two surfaces.

Hamiltonian of the system is

$$H(\mathbf{k}) = v(k_x s_y \tau_z - k_y s_x \tau_0) \sigma_z + M_z s_z \tau_z \frac{\sigma_z - \sigma_0}{2} + t_c s_0 \sigma_x \tau_z - s_0 (\mu \sigma_0 + \delta E \sigma_z) \tau_z + [\Delta_s(\mathbf{k}) + \mathbf{d}(\mathbf{k}) \cdot \boldsymbol{\sigma}] i s_y \tau_y \frac{\sigma_z + \sigma_0}{2}. \quad (1)$$

The Pauli matrices $s_{x,y,z}, \sigma_{x,y,z}, \tau_{x,y,z}$ act on spin, layer and particle-hole spaces, respectively. M_z is the exchange field felt by the bottom layer, $\Delta(\mathbf{k})$ and $\mathbf{d}(\mathbf{k})$ are the singlet and triplet SC order parameters on the top layer. The constant t_c is the hybridization energy between the two layers, which depends on the film thickness. μ is the chemical potential while δE denotes the energy shift between two surfaces. We assume the decay length of the exchange field to be smaller than the TI film thickness so that the top surface does not couple with the exchange field directly, although an indirect coupling can be conveyed by the hybridization t_c .

The only discrete symmetry of this system is the redundant particle-hole symmetry and thus it belongs to D class^{48,49} in which topological phases in two dimensions can be identified by Chern numbers. This model describes a currently accessible experimental system where the heterostructure can be fabricated by molecular beam epitaxy (MBE) method with controllable thickness⁵⁰. Note that it becomes a QAH if the SC on the top surface is replaced by another FM same as the bottom surface⁵¹. This property is useful when we discuss junctions of QAHIs and TSCs.

With s-wave superconductors. In the limit $t_c = 0$, the stacked TI surfaces may be unfolded and regarded as two sections of one surface aligned side by side. The edge of the stacked system then becomes the boundary between the sections. When they are gapped by superconductivity and magnetization, respectively, a chiral Majorana mode appears at the boundary^{24,52}. Similarly, such a mode is expected at the edge of the heterostructure in Fig. 1. The phase diagram obtained by calculating the total Berry curvature γ is shown in Fig. 2a. In normal states, γ corresponds to Hall conductance. If a bulk gap exists, it is quantized so that $N_c = \gamma/2\pi$ with the integer N_c being the Chern number⁵³. In superconducting states, the Chern number can be defined in the same mathematical way using the Bogoliubov-de Gennes Hamiltonian (although it is no longer related to Hall conductance). For $|\mu| < |M_z|$, the system is gapped and $N_c = \text{sign}[M_z]$, corresponding to a single chiral Majorana edge mode whose chirality is determined by the magnetization direction. If $|\mu| > |M_z|$, the bottom surface becomes gapless and the total Berry curvature is not quantized.

Fig. 2 Phase diagrams. The colors denote the summation of Berry curvature of all the occupied electronic states divided by 2π . It is an integer (Chern number N_c) when the system is gapped. The surface states Fermi velocity $v = 1$, superconductivity order parameter $\Delta_s = 0.1$ for all the calculations. **a** The phase diagram when $\delta E = 0, t_c = 0$ where δE is the difference in the chemical potential between the two surfaces and t_c is the inter-surface coupling. There are parameter regions where the system is gapless and the colors there between dark blue and yellow indicate the crossover between the two gapped regions schematically. The black lines are boundaries between the gapful and gapless phases. **b** The phase diagram when $\delta E = 0, t_c = 0.2$. **c** Schematic normal state band structures with $\delta E = 0$ and $\delta E = 0.2$. It resembles Rashba bands when δE is large. **d** The phase diagram when $\delta E = 0.2, t_c = 0.2$.

When $t_c \neq 0$, it competes with M_z trying to generate a trivial hybridization gap. Consequently, larger $|M_z|$ is required to obtain a non-zero Chern number at $\mu = 0$, as shown in Fig. 2b. If $M_z > 0$ is small and μ deviates from zero, the system first enters a non-trivial region with $N_c = \text{sign}[M_z]$ and then transitions into a trivial phase again for large $|\mu|$, as shown by the dashed line. This is easily understood by looking at the normal state band structures as shown in Fig. 2c. The bands are non-degenerate due to the inter-layer hybridization t_c and the exchange field M_z . The two subbands closest to zero energy have a trivial gap opened by t_c and thus SC has no effect if $\mu = 0$, giving $N_c = 0$. As $|\mu|$ increases, the Fermi level cuts a single band and the resulting SC is topological with single chiral Majorana edge mode. As $|\mu|$ further increases, the Fermi level cuts two bands and the system becomes a trivial SC again.

This even-odd effect of the number of Fermi surfaces resembles that of Rashba systems^{8, 28}. The similarity becomes clearer in band structure when a difference in chemical potentials of the top and bottom surfaces, $\delta E = 0.2$, is considered. As shown in Fig. 2c, the conduction (valence) band looks just like a Rashba band with positive (negative) mass and a Zeeman field which splits the degeneracy at $k = 0$. However, the splitting of the valence band is larger than that of the conduction band. This is because the states of the conduction (valence) band near $k = 0$ are from the top (bottom) surface of the TI film, and the top surface only couples with the FM indirectly through t_c while the bottom surface feels the exchange field directly. Consequently, the topological region (where the number of Fermi surfaces is odd) is wider when $\mu < 0$, as shown by the dashed line in Fig. 2d where the whole phase diagram is obtained. The effect of δE has been discussed⁵⁴ and it happens in real experiments⁵⁵. Note that no new phases emerge from this energy difference of two surfaces and its effect on the phase diagram is only quantitative.

Experimental detection. The chiral Majorana modes in the heterostructure discussed above may be detected by a QAHI-

TSC-QAHI junction as shown by previous researches^{41, 43, 44}. To avoid trivial explanations^{46, 47} and uniquely determine the existence of chiral Majorana modes, however, we propose an experimental setup including a Josephson junction^{56, 57}. As shown in Fig. 3a, the setup is achieved by adding SCs and FMs alternately on the top surface of the TI thin film while attaching a uniform FM to the bottom surface. Regions with both top and bottom surfaces coupled to FMs are QAHI⁵¹ while those with one surface coupled to FM and the other to SC are TSCs with single Majorana edge mode, as we have discussed. The two TSCs form a Josephson junction, which are connected to external electrodes 1 or 2 through another QAHI at each end. Note that the SCs are not grounded, contrary to that of reference⁵⁷. Figure 3b schematically shows how the edge states propagate in Majorana basis in which each normal edge state of the QAHI is regarded as two Majorana states. As we shall see in the following, a simple measurement of the current-dependent conductance from lead 1 to lead 2 provides smoking-gun evidence for the chiral Majorana edge modes.

Consider a current $I < I_c^{\text{bulk}}$ (I_c^{bulk} being the critical current of the bulk SCs) flowing from lead 1 to lead 2. When $I > I_c$, where I_c is the Josephson critical current, the current through the junction is normal and a voltage difference between two SCs exists. (Here we ignore the ac Josephson effect which can also contribute to the DC current when $I \sim I_c$. As will be seen later, the details near $I \sim I_c$ is not our focus and it is further discussed in Supplementary Note 1.) The current across the junction is carried by the normal edge states of the QAHI and thus it is determined by the occupation numbers of the edge states. We can define the chemical potential μ_i ($i = 1, 2, \dots, 6$) for each QAHI edge state as shown in Fig. 3b. They must satisfy $\mu_1 - \mu_6 = \mu_3 - \mu_4 = \mu_5 - \mu_2 = eI$ due to the quantized Hall conductance of the QAHI. In addition, the left half of the system is a QAHI-TSC-QAHI junction whose scattering matrix has been obtained⁴³. A simple application of the scattering coefficients to multi-terminal measurement leads to $\mu_3 = \mu_6 = (\mu_1 + \mu_4)/2$.

Fig. 3 Detection with a junction. **a** Experimental setup including a Josephson junction of two topological superconductors (TSCs). The regions without SC are quantum anomalous Hall insulators (QAHI). **b** The Majorana edge modes corresponding to the system in **(a)**. **c** Two-terminal conductance σ_{12} as functions of the applied current I . I_c is the Josephson critical current and ϕ_0 is the kinetic phase acquired by propagating across the junction. Note that all the curves coincide when $I/I_c > 1$. Dashed curves are used near $I/I_c = 1$ to indicate that the results in this range is not accurate due to the ignorance of ac Josephson effect, the finite temperature, etc. The dashed horizontal line is a guide for the eyes corresponding to the value of $1/3$.

Similarly, we get $\mu_4 = \mu_5 = (\mu_2 + \mu_3)/2$ for the right half of the system. Combination of these relations leads to

$$\sigma_{12}^> = \frac{eI}{\mu_1 - \mu_2} = \frac{1}{3}, \text{ when } I > I_c. \quad (2)$$

This result turns out to be the same as the case where the SCs are replaced by normal metals. However, it should be emphasized that the current I we consider here is smaller than the bulk critical current of the SCs and the TSCs with chiral Majorana edge states remain intact. Although Eq. (2) cannot distinguish the TSC from trivial metals, we show in the following that the behavior in the other regime $I < I_c$ is qualitatively different. Comparison of the two cases can provide us signals of Majorana edge modes.

When $I < I_c$, the current flowing through the junction (the middle QAHI region) is a supercurrent carried by Cooper pairs and the aforementioned relations of μ_i no longer hold. There is no voltage drop between the two TSCs and they can only differ by a phase of the order parameter ϕ which is related to the current by $I = I_c \sin(\phi)$ or $\phi = \arcsin(I/I_c)$. The existence of ϕ can affect σ_{12} by inducing interference between different Majorana paths and thus changing the tunneling amplitude. As a result, it has been shown that $\sigma_{12} = \frac{\cos^2(\phi/2)}{\cos^2(\phi/2) + \cos^2\phi_0}$ ⁵⁷. Note that this relation is obtained by assuming a given phase difference ϕ between the two SCs without considering any current-phase relation. To understand the origin of this interference, note that, when two Majorana modes propagate from one SC to another, a phase

difference of the SCs indicates a mismatch between the Majorana basis and a rotation of the basis must be done to compensate this mismatch. As a result of this basis rotation, the Majorana mode from point 1 of the Fig. 1b, which would propagate through point 3 to reach point 5 if $\phi = 0$, partially transforms to the other mode which propagates from point 3 to point 4. Considering such interference effect due to the phase induced by the supercurrent according to the above current-phase relation, we obtain

$$\sigma_{12}^< = \frac{1 + \sqrt{1 - I^2/I_c^2}}{1 + \sqrt{1 - I^2/I_c^2} + 2 \cos^2\phi_0} \quad (3)$$

where $\phi_0 = k_F L$ is the kinetic phase acquired by the edge states across the junction. (k_F is the Fermi wave vector and L is the length of the junction.) When $I \ll I_c$, ϕ is small and its effect on σ_{12} is negligible. As I approaches I_c , ϕ increases from zero to $\pi/2$ and σ_{12} decreases, as shown in Fig. 3c. Note that in the special case with $\phi_0 = \pi/2 + n\pi$, $\sigma_{12}^<$ is constantly unity.

In summary, as the current increases, the two-terminal conductance σ_{12} starts with a value between $1/2$ and 1 and decreases until it exceeds the Josephson critical current, above which σ_{12} is constantly $1/3$. When ϕ_0 is varied by changing the length, for example, the value of $\sigma_{12}(I \rightarrow 0)$ oscillates between $1/2$ and 1 . This is a unique consequence of the chiral Majorana edge states in the TSCs.

When the ac Josephson effect is considered, Cooper pairs also contribute to the junction current when $I \gtrsim I_c$. Thus the actually transition near $I \simeq I_c$ would look different. But the results away from the transition point are still applicable.

Multiple Majorana modes. With the setup in Fig. 1, it is interesting to consider unconventional SCs rather than s-wave⁵⁸. Particularly, previous studies show that multiple chiral Majorana edge modes may be achieved in two-dimensional Rashba system using p-wave and d-wave SCs with Zeeman field^{59–61}.

Consider a general pairing potential $\hat{\Delta}(\mathbf{k}) = (\Psi + \mathbf{d} \cdot \mathbf{s})i\sigma_y$ that acts on TI surface states with the Hamiltonian $\hat{h}(\mathbf{k}) = \mathbf{g} \cdot \mathbf{s} + V_z s_z - \mu_0$ where V_z is a perpendicular Zeeman field and $\mathbf{g} = (k_y, -k_x, 0)$ is the spin-orbit coupling vector. Let us, for clarity, assume the Fermi level to be above the band crossing point ($\mu_0 > 0$), then the superconductivity gap function in the band basis becomes $\Delta_+ = e^{i\theta}[(d_x \cos \theta + d_y \sin \theta) \sin \alpha + i(d_y \cos \theta - d_x \sin \theta - \Psi \cos \alpha)]$ with $\alpha = \arcsin(V_z/\sqrt{k^2 + V_z^2})$ and $\theta = \arg(k_x + ik_y)$.

If only singlet (s-wave and d-wave) pairings are considered, the gap function on the top surface state is simply $\Psi \cos \alpha = (\Delta_s + \Delta_d \cos 2\theta) \cos \alpha$. When $|\Delta_s| > |\Delta_d|$, it is fully gapped and the topological property is the same as the pure s-wave case discussed previously. When $|\Delta_s| < |\Delta_d|$, it becomes nodal and there may be edge states depending on the direction of the open boundaries, similar to well-known usual d-wave superconductors^{5, 62–65}. For the heterostructure in Fig. 1, the energy dispersion on the [010] and [110] edges are shown in Fig. 4 for pure d-wave. (The s-wave component only shifts the positions of the nodal points.) If $M_z = 0$, this system behaves just as usual d-wave SCs in which flat bands appear in [110] direction (Fig. 4a) but not in [010] direction (Fig. 4b). However, when M_z is turned on (Fig. 4c, d), the change of the state is quite different from that of usual d-wave SCs. Particularly, for large M_z , dispersive edge modes appear on [010] edges, as shown in Fig. 4d.

When p-wave pairing is included, we consider three typical cases, $\mathbf{d}_{||} = \Delta_p(\sin \theta, -\cos \theta, 0)$, $\mathbf{d}_{\perp} = \Delta_p(\cos \theta, \sin \theta, 0)$ and $\mathbf{d}' = \Delta_p(\sin \theta, \cos \theta, 0)$. In the first two cases, the d-vectors are either parallel or perpendicular to the spin-orbit vector $\mathbf{g}$. For $\mathbf{d}_{||}$,

Fig. 4 Energy spectra with d-wave superconductors. The energy dispersion with terminated edges in y-direction for d-wave pairing. Only the lowest 100 energy levels are shown. The y-direction is defined as the [110] direction in (a) and (c) but as the [010] direction in (b) and (d). In (a) and (b), the magnetization $M_z = 0$. In (c) and (d), $M_z = 0.4$. Other parameters are: The surface states Fermi velocity $v = 1$, inter-surface coupling $t_c = 0.2$, chemical potential $\mu = 0.3$ and the d-wave superconductivity order parameter $\Delta_d = 1.9$.

the gap function is $(\Delta_p + \Psi \cos \alpha)$ so that p-wave behaves the same as s-wave. For $\mathbf{d}_\perp$, the gap function becomes $\Delta_p \sin \alpha - i\Psi \cos \alpha$, which is always fully gapped when $\Delta_p V_z \neq 0$. In the heterostructure, it can support single chiral Majorana mode. With the third choice, $\mathbf{d}'$, the gap function is $\Delta_p \sin 2\theta \sin \alpha + i(\Delta_p - \Delta_d \cos \alpha) \cos 2\theta$. Note that, when $\Delta_p V_z \neq 0$, this is also fully gapped except at $\Delta_p = \Delta_d \cos \alpha$ where the gap changes sign. A phase diagram with this pairing potential ($\Delta_p = 0.3, \Delta_p = 0.1$) is obtained in Fig. 5, where each phase is identified by a Chern number that is equal to the number of chiral Majorana modes. A maximum of four Majorana modes can exist in this system. (More details about the phases with different $\mathbf{d}$ vectors are provided in Supplementary Note 2.)

Although p-wave and d-wave pairings are not known to exist together in nature, we may find them separately in real materials. Cuprates are well-known d-wave SCs. For p-wave pairing, some heavy fermion SCs (such as UPt_3 ⁶⁶) may be used, as well as some organic SCs such as $(\text{TMTSF})_2\text{PF}_6$ ⁶⁶ and the ruthenate superconductor Sr_2RuO_4 ⁶⁷.

Discussion

We have proposed a platform of Majorana edge channels by using superconductor/topological insulator/ferromagnet (SC/TI/FM) heterostructures. The topological phase is much easier to be realized compared with the setups studied thus far. The phase diagrams are revealed for s-wave, p-wave, and d-wave pairings for the SC. A smoking-gun experiment is also proposed to confirm the Majorana edge channels which exclude the other possibilities. The heterostructures including TI have been already realized experimentally^{50, 55}. By this technique, the quantized anomalous Hall effect is realized at higher temperature due to the suppressed inhomogeneity of the exchange gap⁵⁰. Also the different energy position of the Weyl point between the top and bottom surfaces enables the insulating phase which can support the topological magnetoelectric effect⁵⁵. With these artificial structures, one can design various Majorana edge channels to realize the circuits with dissipationless current and even quantum computation⁶⁸.

Fig. 5 Phase diagram with high Chern numbers. The phase diagram with the s-wave, p-wave and d-wave superconductivity order parameters $\Delta_s = 0, \Delta_p = 0.1$ and $\Delta_d = 0.3$. For p-wave superconductivity, the triplet order parameter is given by the vector $\mathbf{d}' = \Delta_p(\sin \theta, \cos \theta, 0)$ where θ is the polar angle of the wave vector $\mathbf{k}$. Colors denote Chern numbers. The other parameters are: the nearest neighbor hopping $t = 1$; the inter-surface coupling $t_c = 0.1$.

Methods

Berry curvature and Chern number. In two-dimensional systems, the total berry curvature of the occupied electron states determines the topological property of the system. It is obtained as⁶⁹

$$\gamma = -\text{Im} \sum_{E_n < 0} \sum_{n' \neq n} \int d^2\mathbf{k} \left[\langle n | (\partial_{k_x} H_{\mathbf{k}}) | n' \rangle \langle n' | (\partial_{k_y} H_{\mathbf{k}}) | n \rangle - \langle n | (\partial_{k_y} H_{\mathbf{k}}) | n' \rangle \langle n' | (\partial_{k_x} H_{\mathbf{k}}) | n \rangle \right] / (E_n - E_{n'})^2. \quad (4)$$

When the system is gapped, we can define the Chern number as

$$N_c = \gamma / 2\pi. \quad (5)$$

Derivation of the conductance when $I > I_c$. Consider the left half of Fig. 3b, which is a QAHI-TSC-QAHI junction. The electron–electron and electron-hole

tunneling probabilities from edge channel j to edge channel i are⁴³

$$T_{31}^{ee} = T_{31}^{he} = T_{61}^{ee} = T_{61}^{he} = 1/4, \quad (6)$$

$$T_{34}^{ee} = T_{34}^{he} = T_{64}^{ee} = T_{64}^{he} = 1/4, \quad (7)$$

$$T_{16}^{ee} = T_{43}^{ee} = 1. \quad (8)$$

Other terms vanish. The current–voltage relation (at zero temperature) is given by⁷⁰

$$I_i = \sum_j K_{ij}(\mu_j - \mu_s)/e, \quad (9)$$

with

$$K_{ij} \equiv \delta_{ij} - T_{ij}^{ee} + T_{ij}^{he}, \quad (10)$$

$$K_{ii} = 1, K_{16} = K_{43} = -1 \quad (11)$$

and other terms vanish. So we have

$$e \begin{pmatrix} I_1 \\ I_6 \\ I_3 \\ I_4 \end{pmatrix} = \begin{pmatrix} 1 & -1 \\ 0 & 1 \\ & 1 & 0 \\ & -1 & 1 \end{pmatrix} \begin{pmatrix} \mu_1 - \mu_s \\ \mu_6 - \mu_s \\ \mu_3 - \mu_s \\ \mu_4 - \mu_s \end{pmatrix}, \quad (12)$$

$$\Rightarrow \begin{pmatrix} \mu_1 - \mu_s \\ \mu_6 - \mu_s \\ \mu_3 - \mu_s \\ \mu_4 - \mu_s \end{pmatrix} = e \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ & 1 & 0 \\ & 1 & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_6 \\ I_3 \\ I_4 \end{pmatrix}, \quad (13)$$

Setting $I_1 = -I_4 = I$ and $I_3 = I_6 = 0$, we obtain

$$\mu_1 - \mu_s = -(\mu_4 - \mu_s) = eI, \quad (14)$$

$$\mu_6 - \mu_s = \mu_3 - \mu_s = 0, \quad (15)$$

so that

$$\mu_3 = \mu_6 = \mu_s = (\mu_1 + \mu_4)/2, \quad (16)$$

$$\mu_1 - \mu_6 = \mu_3 - \mu_4 = eI. \quad (17)$$

Similar analysis for the right QAHI-TSC-QAHI junction gives

$$\mu_4 = \mu_5 = \mu'_s = (\mu_2 + \mu_3)/2, \quad (18)$$

$$\mu_5 - \mu_2 = \mu_3 - \mu_4 = eI. \quad (19)$$

TI surface with general pairing. Consider a general pairing potential $\hat{\Delta}(\mathbf{k}) = (\Psi + \hat{\mathbf{d}} \cdot \mathbf{s})i\sigma_y$ that acts on a single Dirac cone $\hat{h}(\mathbf{k}) = \mathbf{g} \cdot \mathbf{s} + V_z s_z - \mu_0$. The constant V_z is a Zeeman field along z -direction and $\mathbf{g} = (k_y, -k_x, 0)$ is the spin-orbit coupling vector. In the band basis, the pairing potential $\hat{\Delta}(\mathbf{k})$ is transformed to

$$\tilde{\Delta}(\mathbf{k}) = (\tilde{\Psi} + \tilde{\mathbf{d}} \cdot \mathbf{s})i\sigma_y, \quad (20)$$

with

$$\tilde{\Psi} = -id_z e^{i\theta}, \quad (21)$$

$$\tilde{d}_x = [-\cos \alpha (d_x \cos \theta + d_y \sin \theta) + i\Psi \sin \alpha]e^{i\theta}, \quad (22)$$

$$\tilde{d}_y = (d_y \cos \theta - d_x \sin \theta)e^{i\theta}, \quad (23)$$

$$\tilde{d}_z = [\sin \alpha (d_x \cos \theta + d_y \sin \theta) + i\Psi \cos \alpha]e^{i\theta}, \quad (24)$$

where we have defined two angles α and θ as

$$\alpha = \arccos \frac{V_z}{\sqrt{k^2 + V_z^2}}, \quad \theta = \arg(k_x + ik_y). \quad (25)$$

When the Fermi level is not close to the band crossing point, the effect of inter-band pairing ($\tilde{\Psi}$ and $\tilde{d}_z$) can be ignored. The remaining intra-band pairing is

$$\Delta_{\pm} = i\tilde{d}_y \mp \tilde{d}_x \quad (26)$$

$$= e^{i\theta} \left[\pm (d_x \cos \theta + d_y \sin \theta) \cos \alpha + i(d_y \cos \theta - d_x \sin \theta \mp \Psi \sin \alpha) \right]. \quad (27)$$

Assuming the Fermi level to be above the band crossing point, then the gap function is just Δ_+ .

Data availability

All essential data are available in the paper. Additional data are given in the supplementary file. Further supporting data can be provided from the corresponding author upon request.

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Author contributions

N.N. and J.J.H. conceived the ideas. J.J.H. carried out the calculations. T.L. and Y.T. involve in the analysis of results and discussions. J.J.H. and N.N. prepared the paper with the help from the other authors.

Competing interests

The authors declare no competing interests.

Additional information

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Correspondence and requests for materials should be addressed to J.J.H.

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