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High Chern number van der Waals magnetic topological multilayers MnBi₂Te₄/hBNMihovil Bosnar^{1,2}, Alexandra Yu. Vyazovskaya^{3,4}, Evgeniy K. Petrov^{3,4}, Evgueni V. Chulkov^{1,2,3,4} and Mikhail M. Otrokov^{5,6}

Chern insulators are two-dimensional magnetic topological materials that conduct electricity along their edges via the one-dimensional chiral modes. The number of these modes is a topological invariant called the first Chern number C that defines the quantized Hall conductance as $S_{xy} = Ce^2/h$. Increasing C is pivotal for the realization of low-power-consumption topological electronics, but there has been no clear-cut solution to this problem so far, with the majority of existing Chern insulators showing $C = 1$. Here, by using state-of-the-art theoretical methods, we propose an efficient approach for the realization of the high- C state in MnBi₂Te₄/hBN van der Waals multilayer heterostructures. We show that a stack of n MnBi₂Te₄ films with $C = 1$ intercalated by hBN monolayers gives rise to a high Chern number state with $C = n$, characterized by n chiral edge modes. This state can be achieved both under the external magnetic field and without it, both cases leading to the quantized Hall conductance $S_{xy} = Ce^2/h$. Our results, therefore, pave the way to practical high- C quantized Hall systems.

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INTRODUCTION

The Chern insulator (CI) state is a quantum phase of two-dimensional (2D) gapped materials with broken time-reversal invariance and nontrivial electronic band topology^{1–4}. It is most straightforwardly probed via Hall measurements, the hallmark being a vanishing longitudinal conductance S_{xx} along with a transversal conductance S_{xy} quantized to integer multiples of the conductance quantum, Ce^2/h ^{5,6}. Here, e is the electron charge, h is Planck's constant, and C is a dimensionless integer called the first Chern number, corresponding to the number of the 1D gapless chiral modes residing at the CI film's edge. The existence of these modes is guaranteed by the nontrivial band topology of CI.

The edge modes of a CI conduct electricity without dissipation, which could be useful for the construction of novel highly efficient chiral interconnects for low-power-consumption electronics^{7,8}. However, the contact resistance between a metal electrode and CI in the envisioned interconnect devices is a bottleneck limiting their performance. To reduce this resistance as much as possible, the number of chiral edge modes, i.e., the Chern number C , should be as large as possible^{7,8}. Therefore, it is of great interest and importance to engineer CIs with high Chern numbers.

Historically, the CI state was first observed in 2D electron gases in 1980 in a transport phenomenon that is now known as the quantum Hall effect (QHE)⁵. The QHE in this system stems from the formation of Landau levels under the external magnetic field, which drives the system into a topologically nontrivial state. However, well-defined Landau levels are only possible in systems with high carrier mobility under strong external magnetic fields, which prevents this QHE from a widely applied use.

Notwithstanding, the developments in the research field of magnetic topological insulators (TIs) in the last decade have allowed a qualitative leap toward QHE without Landau levels. In particular, the quantum anomalous Hall effect (QAHE), a special kind of QHE that occurs without the external magnetic field, has

been observed⁹. It is mainly realized in the thin films of TIs of the (Bi,Sb)₂Te₃ family doped by Cr or/and V atoms^{9–14}. In these systems, $C = 1$, and although it is theoretically possible to increase C by increasing the dopant concentration and the film thickness^{15,16}, this has not been experimentally realized to date.

Instead, complex materials engineering has been resorted to in order to achieve $C = n > 1$ state based on the magnetically doped TIs using the following idea. Rather than seeking a high- C state in a particular system, it can be realized by stacking n CI layers with $C = 1$ each. In this case, it is necessary, however, that the adjacent CI layers are efficiently decoupled from each other by a normal, i.e., topologically trivial, insulator layer. In this way, C up to 4 and 5 have been achieved in (Cr,V)_x(Bi,Sb)_{2–x}Te₃/CdSe¹⁷ and heavily Cr-doped (Bi,Sb)₂Te₃/Cr_x(Bi,Sb)_{2–x}Te₃ multilayers¹⁸, respectively. Incidentally, such a multilayer heterostructure design also allows the realization of one of the very first proposals to achieve a Weyl semimetal phase¹⁹. Although the studies^{17,18} represent a proof-of-concept of the C enhancement approach, it is well known that the potential of magnetically doped TIs for QAH-based applications is quite limited. Namely, due to a strong disorder in the pnictogen sublattice, which is randomly occupied by Bi, Sb, and magnetic dopants, both electronic^{20,21} and magnetic^{22,23} properties of such materials are strongly inhomogeneous. Therefore, the observation of the QAHE in these systems appears to be limited to about 2 K at best^{12,24}, with no further improvements achieved over the last several years¹².

Recently, new systems showing the $C = 1$ QAHE have emerged, such as the intrinsic magnetic topological insulators of the MnBi₂Te₄ (MBT) family^{25–27}, the twisted bilayer graphene²⁸, and the transition metal dichalcogenide moiré superlattices²⁹ opening new opportunities for C engineering. MBT, shown in Fig. 1a, appears as particularly promising due to its van der Waals (vdW) nature, the intrinsic combination of nontrivial band topology and long-range antiferromagnetic order ($T_{\text{Néel}} = 25$ K), as well as large

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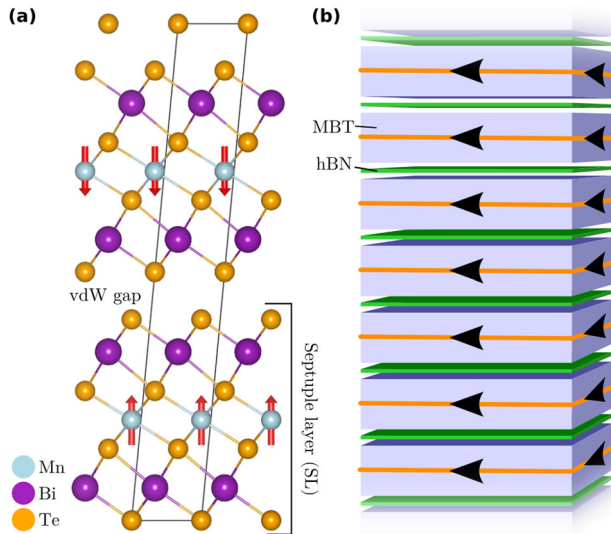


Fig. 1 Bulk MnBi_2Te_4 structure and schematic of the proposed heterostructure. **a** Side view of the bulk MnBi_2Te_4 (MBT) crystal structure. Red arrows denote Mn local moments. **b** Schematic depiction of the proposed system: MBT films are separated by hBN monolayers to make a vdW multilayer heterostructure with Chern number equal to the number of MBT films, $C = n$. Black arrows depict the direction of the edge currents.

predicted surface band gap^{25,26,30–42}. The QAHE in thin MBT flakes has been achieved up to about 1.4 K, leaving a large room for the observation temperature enhancement. Indeed, a recent study³⁶ demonstrates the actual potential of this material by registering the $C = 1$ ($C = 2$) QHE up to 30 K (13 K) in its thin flakes, ferromagnetically polarized by an external magnetic field. Remarkably, the quantization, in this case, appears without the Landau levels, in contrast to the conventional QHE observed in 2D electron gas⁵.

Here, inspired by the recent progress on the Q(A)HE in MBT, we propose a novel MBT-based high Chern number material. Namely, we design a multilayer vdW heterostructure, in which thin MBT CI films are stacked on top of each other, interlayered by hexagonal boron nitride (hBN) monolayers that decouple and insulate them from one another (Fig. 1b). As an inert wide band gap insulator, hBN is an ideal material for such a decoupling, widely used in vdW heterostructure devices as an encapsulating layer or substrate for the stacked 2D materials^{43–46}. Using the state-of-the-art density functional theory and tight-binding calculations, we show that the weak vdW bonding between MBT and hBN essentially preserves the $C = 1$ CI state in the individual MBT layers. This state can correspond to either (1) the QH insulator phase achieved in thin MBT films under the external magnetic field but without the Landau levels or (2) the QAHE insulator phase at zero field if the MBT films are made of the odd number of septuple layer blocks. In either case, stacking n MBT films with $C = 1$ interlayered by $(n - 1)$ hBN monolayers gives rise to a $C = n$ state, with n as large as allowed by the vdW heterostructures growth technology. Our results provide an excellent platform for the realization of high- C materials.

RESULTS

Crystal and electronic structure of MBT/hBN interface

MnBi_2Te_4 crystallizes in the trigonal $R\bar{3}m$ -group structure^{47–49}, made up of septuple layer (SL) blocks, in which hexagonal atomic layers are stacked in the Te-Bi-Te-Mn-Te-Bi-Te sequence, as shown in Fig. 1a. Neighboring SLs are bound by vdW forces. Below $T_{\text{Néel}} = 25$ K, MnBi_2Te_4 orders antiferromagnetically due to the

Table 1. Structural, magnetic, transport, and topological characteristics of the $\text{MBT}_{2\text{SL}}/\text{hBN}$.

	Hollow	Bridge	Top-B	Top-N
d	3.482	3.601	3.573	3.600
E_{AFM}	0.0	5.3	1.6	10.2
E_{FM}	1.2	6.4	2.9	11.3
$\Delta E_{\text{A/F}}$	-1.2	-1.1	-1.3	-1.1
S_{xy}	e^2/h	e^2/h	e^2/h	e^2/h
C	1	1	1	1

Four adsorption registries have been considered: Hollow, Bridge, Top-B, Top-N. d (in Å) is the MBT-hBN interlayer distance (see Fig. 2(a)), E_{AFM} and E_{FM} (in meV) are the energies of the respective interlayer magnetic states relative to that of the AFM-hollow case (whose energy is set to zero), $\Delta E_{\text{A/F}} = E_{\text{AFM}} - E_{\text{FM}}$ (in meV per Mn pair) is the total energy difference of the interlayer AFM and FM states, S_{xy} is the anomalous Hall conductance for the Fermi level lying within the fundamental band gap, and C is the Chern number obtained for the interlayer FM state in $\text{MBT}_{2\text{SL}}$.

antiparallel alignment between the alternating ferromagnetically ordered Mn layers^{25,34}, with the local moments pointing out-of-plane (Fig. 1a).

We start our study with the consideration of structural, magnetic, and electronic properties of the MBT/hBN bilayer made of the 2 SL thick MBT film ($\text{MBT}_{2\text{SL}}$) and hBN monolayer. This can be considered a minimal system because it contains all of the essential characteristics of MBT, such as intra- and interlayer exchange couplings as well as the nontrivial topology in the forced FM state³⁰, so it can be used to test whether they are affected by hBN.

The MBT and hBN basal planes are symmetry compatible and show a good lattice parameter matching in the MBT (1×1)/hBN ($\sqrt{3} \times \sqrt{3}$) configuration, with a mismatch of only about 0.6%. The optimal hBN adsorption geometry was then determined by the comparison of total energies of structurally optimized $\text{MBT}_{2\text{SL}}/\text{hBN}$ in four high-symmetry registries of such configuration, shown in Supplementary Fig. 3a–d. These energies were calculated for both interlayer FM and AFM spin configurations of $\text{MBT}_{2\text{SL}}$, assuming the out-of-plane magnetic moment direction, to determine a possible influence of hBN on the interlayer exchange coupling, which is in MBT significantly weaker than the intralayer one (the latter is addressed below, as well). For more details, see Supplementary Note 2.A.1.

The relevant numerical results are listed in Table 1. It can be seen that for all adsorption registries, the interlayer distance d between MBT and hBN is about 3.5–3.6 Å, and the AFM spin configuration is lower than the FM configuration by at least $\Delta E_{\text{A/F}} = E_{\text{AFM}} - E_{\text{FM}} \approx 1$ meV per Mn pair. The lowest energy is obtained for the hollow site geometry, shown in Fig. 2a and Supplementary Fig. 3a. The large d on one hand and the overall similarity of the energy differences $\Delta E_{\text{A/F}}$ in $\text{MBT}_{2\text{SL}}/\text{hBN}$ and $\text{MBT}_{2\text{SL}}$ ³⁰ on the other suggest the vdW bonding of MBT and hBN.

Having determined the MBT/hBN interface geometry, we can explore the effect of hBN on the $\text{MBT}_{2\text{SL}}$ electronic structure and topology. Since we are interested in the CI state, the FM interlayer spin alignment is considered here, for which $\text{MBT}_{2\text{SL}}$ has been predicted to have $C = 1$ ³⁰. Noteworthy, in experiments, the FM alignment in MBT is achieved by the external magnetic field application^{26,35,36,50}.

Comparison of the band structure along the $\bar{K} - \bar{\Gamma} - \bar{M}$ path for $\text{MBT}_{2\text{SL}}/\text{hBN}$ in the hollow site registry and pure $\text{MBT}_{2\text{SL}}$ is shown in Fig. 2b. Their similarity near the Fermi level is immediately obvious, and the band gaps, 63.5 and 62.7 meV, respectively, are very close. The hBN states can be found at about 1.5 eV below the Fermi level (and deeper) as well as over 3.2 eV above it. Furthermore, Fig. 2c

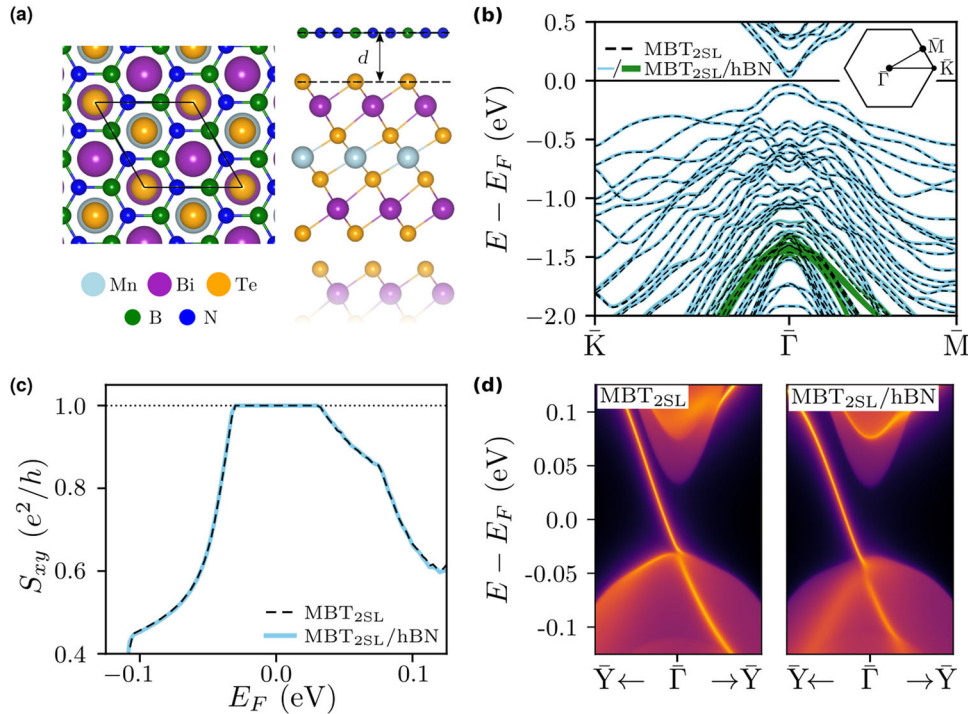


Fig. 2 Structural and electronic properties of $\text{MBT}_{2\text{SL}}/\text{hBN}$. **a** Top and side views of the hollow site MBT/hBN adsorption registry. **b** The band structures of $\text{MBT}_{2\text{SL}}$ and $\text{MBT}_{2\text{SL}}/\text{hBN}$ with hBN contribution are shown in green. **c** Fermi energy dependence of the anomalous Hall conductance $S_{xy}(E_F)$ in the units of conductance quantum e^2/h for $\text{MBT}_{2\text{SL}}$ and $\text{MBT}_{2\text{SL}}/\text{hBN}$. $E_F = 0$ corresponds to the center of the band gap. **d** The edge electronic band structures for $\text{MBT}_{2\text{SL}}$ (left) and $\text{MBT}_{2\text{SL}}/\text{hBN}$ (right). The regions with a continuous spectrum correspond to the 2D bulk states projected onto the 1D Brillouin zone. The edge crystal structure is shown in Supplementary Fig. 7. The data in (b–d) were calculated for the FM interlayer alignment in $\text{MBT}_{2\text{SL}}$ (see text).

shows that the Fermi level dependencies of the anomalous Hall conductance, $S_{xy}(E_F)$, of $\text{MBT}_{2\text{SL}}/\text{hBN}$ and $\text{MBT}_{2\text{SL}}$ are very well matched, too. In particular, in both cases, S_{xy} is constant inside the band gap, where the actual Fermi level is, and it is equal to one conductance quantum e^2/h , indicating the CI state with $C = 1$. Accordingly, the edge spectral function of $\text{MBT}_{2\text{SL}}/\text{hBN}$ in Fig. 2d features a single chiral edge mode traversing the band gap, similar to $\text{MBT}_{2\text{SL}}$. Finally, a Wilson loop method calculation for $\text{MBT}_{2\text{SL}}/\text{hBN}$ yields $|C| = 1$ as well, with the sign depending on the magnetization direction, as expected for a CI⁴.

The calculations of the band structures, $S_{xy}(E_F)$ and C for the other three high-symmetry registries show that the same results hold for all of them (see Table 1 and Supplementary Fig. 3e–k for visualization). These findings clearly confirm the vdW nature of the bond between hBN and MBT, which preserves the CI state in the forced FM phase of the $\text{MBT}_{2\text{SL}}$ film.

We note that recently it has been reported elsewhere⁵¹ that while hBN and MBT are bound by vdW interaction, the interlayer FM configuration in $\text{MBT}_{2\text{SL}}/\text{hBN}$ becomes significantly lower in energy than the AFM one (by up to 45 meV) for any adsorption registry. We have attempted to reproduce those results by retracing the steps outlined in ref. 51 (see Supplementary Note 2.A.2), but arrived at the same results that we present here. We believe that our result is correct on the physical basis that the vdW interaction, along with the insulating character of hBN, should not produce such a drastic effect on magnetism as it was found in ref. 51.

Finally, we confirm the implicit assumptions of preference for the FM intralayer spin configuration and the out-of-plane easy axis direction by total energy calculations on $\text{MBT}_{1\text{SL}}/\text{hBN}$ (see Supplementary Notes 2.A.3 and 2.A.4, respectively). The former calculation reveals that the FM configuration is 16.9 meV (per Mn pair) lower than the AFM configuration, while the latter yields a positive magnetic anisotropy energy of 0.07 meV per Mn atom (vs.

0.074 meV in pure $\text{MBT}_{1\text{SL}}$), meaning that the easy axis indeed stays out-of-plane. Thus, neither the intralayer magnetic order of MBT nor its magnetic anisotropy are changed by interfacing with hBN. The above results concerning the insensitivity of magnetic, electronic, and topological properties of MBT to hBN should hold for thicker MBT films as well because of the vdW nature of the bond. Thus, we conclude that hBN can be efficiently used to decouple MBT CI layers from each other without altering their properties.

High-C state in the forced FM phase

We can now proceed with the study of the topological properties of the MBT/hBN multilayer heterostructures. We first note that in experiments, the CI state in the forced FM phase, achieved through the application of the external magnetic field, is observed in thin MBT flakes made of both even and odd numbers of SLs^{26,35–42,52}. According to the previous density functional theory calculations³⁰, the minimal MBT film thickness required for the realization of such a $C = 1$ state is two SL blocks. Let us therefore consider $n\text{MBT}_{2\text{SL}}/\text{hBN}$, $n > 1$, heterostructures in which n films of $\text{MBT}_{2\text{SL}}$ are interlayered by $(n - 1)$ hBN monolayers, as schematically depicted in Fig. 1b. We will assume that all $\text{MBT}_{2\text{SL}}$ films are FM-polarized in the $+z$ direction by the external magnetic field. These heterostructures were constructed based on the structure of the MBT/hBN/MBT system that was determined after a series of calculations outlined in Supplementary Note 2.B.1.

Figure 3 shows the calculated low-energy band structures along the $\bar{K} - \bar{\Gamma} - \bar{M}$ path together with the respective $S_{xy}(E_F)$ for $n\text{MBT}_{2\text{SL}}/\text{hBN}$ with $n = 2, 3, 4$, and 5. The band structures basically correspond to that of a free-standing $\text{MBT}_{2\text{SL}}$ repeated n times, slightly shifted in energy due to a slight variation of the electrostatic potential across the multilayer. The bands stemming from the spatial inversion equivalent $\text{MBT}_{2\text{SL}}$ layers come in pairs,

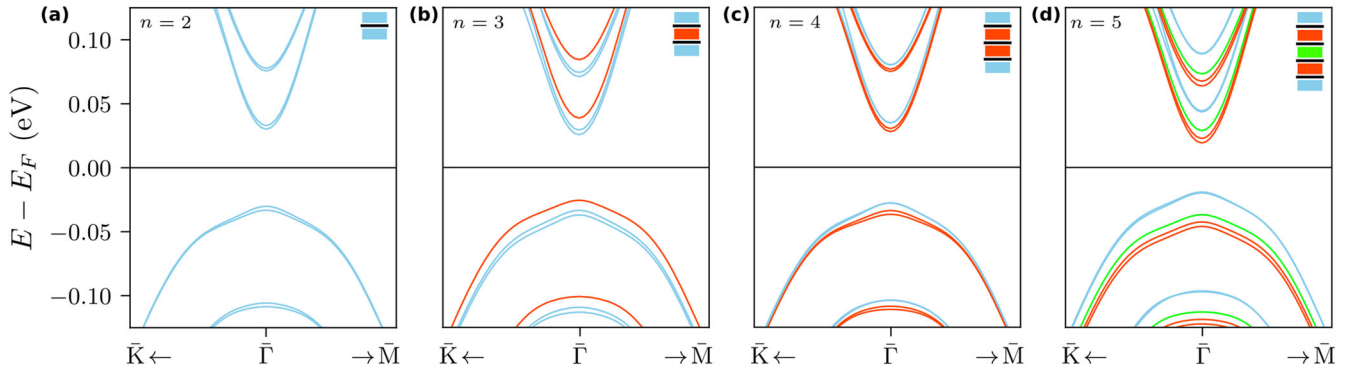


Fig. 3 Electronic band structures of the $n\text{MBT}_{2\text{SL}}/\text{hBN}$ multilayers. Calculations were performed along the $\bar{K} - \bar{\Gamma} - \bar{M}$ path in the 2D Brillouin zone for **a** $n = 2$, **b** $n = 3$, **c** $n = 4$, and **d** $n = 5$. In the insets, the colored blocks depict the equivalent $\text{MBT}_{2\text{SL}}$ layers from which the bands of the corresponding color dominantly stem. The black lines between the blocks represent the hBN layers that separate them.

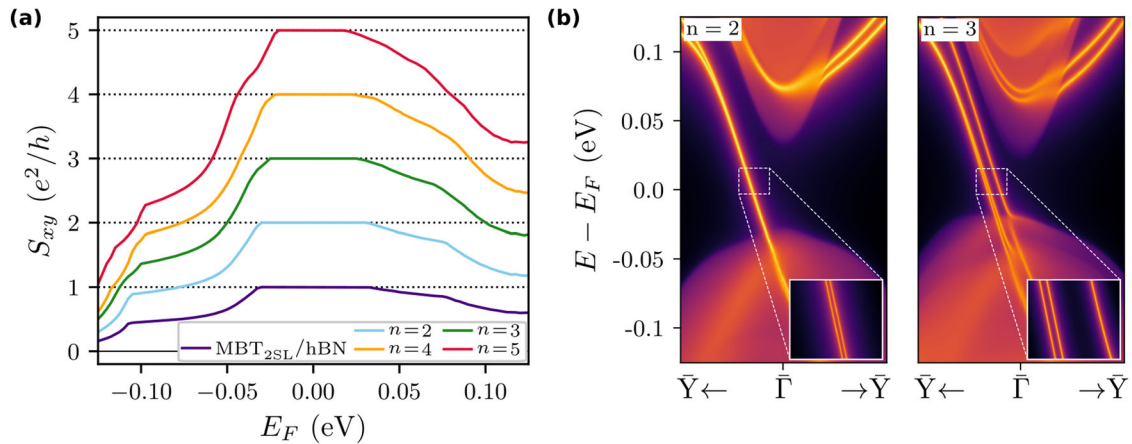


Fig. 4 Anomalous Hall conductance and edge spectra for the $n\text{MBT}_{2\text{SL}}/\text{hBN}$ multilayers. **a** The Fermi energy dependence of anomalous Hall conductance $S_{xy}(E_F)$ in the units of conductance quantum e^2/h for $\text{MBT}_{2\text{SL}}/\text{hBN}$ and $n\text{MBT}_{2\text{SL}}/\text{hBN}$, $n = 2, \dots, 5$. $E_F = 0$ corresponds to the center of the band gap in each case. **b** The edge electronic structure of $n\text{MBT}_{2\text{SL}}/\text{hBN}$ for $n = 2$ and $n = 3$.

as it is seen in the insets to Fig. 3. In the corresponding $S_{xy}(E_F)$ dependencies, shown in Fig. 4a, there are plateaus in the band gap that are equal to an integer number of conductance quanta, the integer being equal to n , suggesting $C = n$ state in the respective multilayers. Accordingly, in the plots of the calculated edge spectral functions, shown in Fig. 4b, two (three) edge modes can be seen for the $n = 2$ ($n = 3$) system.

From these results, we can conclude that $C = n$ in the examined heterostructures and infer by induction that the same will hold for heterostructures with greater n .

High-C QAH state: odd number of SLs

In the thin films made of an odd number of SLs, MBT has been predicted^{30,32} and subsequently experimentally confirmed²⁶ to show the intrinsic $C = 1$ QAHE. We therefore now turn to the topological properties of the MBT/hBN heterostructures based on the thinnest possible odd-SL MBT film, i.e., $\text{MBT}_{3\text{SL}}$ ³⁰. A comparison of $S_{xy}(E_F)$ calculated for $\text{MBT}_{3\text{SL}}/\text{hBN}$ and $\text{MBT}_{3\text{SL}}$ is shown in Fig. 5. A good matching between the two can be observed, especially for the flat portion in the band gap where $S_{xy} = e^2/h$ for both, confirming the insensitivity of the $C = 1$ QAH state of $\text{MBT}_{3\text{SL}}$ to interfacing with hBN.

As we are seeking the high-C QAH state realized at zero external field, the question arises whether it is supported by the magnetic ground state of $n\text{MBT}_{3\text{SL}}/\text{hBN}$, $n > 1$. While we have shown above that the magnetic ordering inside MBT film is not affected by hBN, the interlayer exchange coupling between the local Mn moments

through hBN should now be studied. Our total-energy calculations show that the AFM configuration is ~ 0.03 meV (per 2 Mn atoms) lower in energy than the FM one (see Supplementary Note 3). Such a small energy difference is due to the large separation between neighboring Mn planes (about 17.8 Å) and the vdW coupling between MBT and hBN. Although this number is at the limit of our calculation accuracy, assuming that its sign is correct yields a zero (non-zero) net magnetization in $n\text{MBT}_{3\text{SL}}/\text{hBN}$ with even (odd) n . As the net C is a sum of C 's of individual $\text{MBT}_{3\text{SL}}$ films, with the sign of C changing with the magnetization direction, our result predicts that an $n\text{MBT}_{3\text{SL}}/\text{hBN}$ heterostructure should have C equal to 0 (1) in its ground state.

However, the weakness of the interlayer exchange coupling through hBN makes a reliable prediction of its sign hardly possible using the density functional theory, so it should be defined in future experiments. If the FM coupling through hBN occurs experimentally, the Chern numbers of the individual MBT films will sum up to create the high-C state. However, even if the experiment would show the worst-case scenario, i.e., the AFM coupling through hBN, a metamagnetic state that is appropriate for achieving the zero-field high-C state in the $n\text{MBT}_{3\text{SL}}/\text{hBN}$ ($n > 1$) multilayers can nevertheless be “prepared” by the external magnetic field, in the manner we will describe below. In this state, the alignment of the MBT SLs across hBN would be FM, while the interlayer alignment inside MBT films stays AFM, thus guaranteeing an overall non-zero net magnetization increasing with n , in turn yielding the desired $C = n$ state. Crucial for the realization of such a state is the fact that the magnetic anisotropy

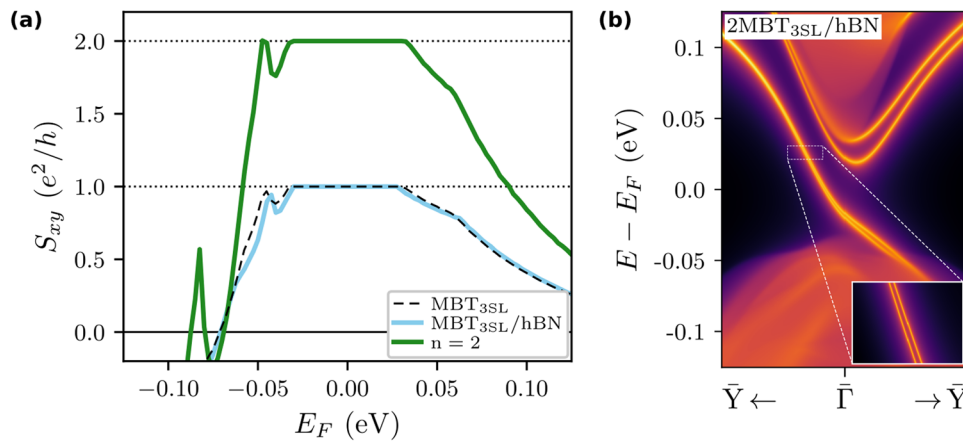


Fig. 5 Anomalous Hall conductance and edge spectra for the $n\text{MBT}_{3\text{SL}}/\text{hBN}$ multilayers. **a** The Fermi energy dependence of the anomalous Hall conductance $S_{xy}(E_F)$ in the units of conductance quantum e^2/h for $\text{MBT}_{3\text{SL}}$, $\text{MBT}_{3\text{SL}}/\text{hBN}$, and $2\text{MBT}_{3\text{SL}}/\text{hBN}$. $E_F = 0$ corresponds to the center of the band gap in each case. **b** The edge electronic structure of $2\text{MBT}_{3\text{SL}}/\text{hBN}$.

energy of $\text{MBT}_{3\text{SL}}$ is $\sim 0.22 \text{ meV}$ ³⁰, several times larger than the energy difference of AFM and FM configurations across hBN. Note that the magnetic anisotropy energy of $\text{MBT}_{3\text{SL}}$ should not change appreciably upon its interfacing with hBN, as shown in the section “Crystal and electronic structure of MBT/hBN interface”.

We, therefore, propose the following procedure to induce the above-described metamagnetic state. First, the external magnetic field of about 10 T ^{26,35,36} should be applied to overcome the AFM coupling inside individual $\text{MBT}_{3\text{SL}}$ films and polarize all SLs along the same direction. Next, the external magnetic field should be gradually reduced to zero leading to the recovery of the uncompensated interlayer AFM state with non-zero magnetization in each individual $\text{MBT}_{3\text{SL}}$ film^{26,30}. However, their net magnetizations will remain parallel to each other even if the exchange coupling across hBN is AFM because the energy barrier due to anisotropy prevents the magnetization relaxation into the only slightly more favorable AFM configuration across hBN. Indeed, a similar metamagnetic state has been recently experimentally observed in bulk MnBi_4Te_7 and $\text{MnBi}_6\text{Te}_{10}$ ^{53–57}, where the uniaxial anisotropy dominates over the AFM interlayer exchange coupling⁵⁷. The magnetic anisotropy energy of $\text{MBT}_{3\text{SL}}$ is slightly larger, while the exchange coupling across hBN is significantly weaker than those in MnBi_4Te_7 and $\text{MnBi}_6\text{Te}_{10}$ (c.f. ref. ⁵⁶), making the proposed realization of the metamagnetic state in $n\text{MBT}_{3\text{SL}}/\text{hBN}$ feasible.

We now have a firm basis to claim a possibility of the high- C QAH state realization in $n\text{MBT}_{3\text{SL}}/\text{hBN}$, $n > 1$. As in the preceding section, the high- C state can be demonstrated by the $S_{xy}(E_F)$ and the edge electronic structure calculations, the results of which are shown in Fig. 5. Analogously to the $n\text{MBT}_{2\text{SL}}/\text{hBN}$ multilayers, the $S_{xy}(E_F)$ for $n\text{MBT}_{3\text{SL}}/\text{hBN}$ with $n = 2$ shows a clear plateau within the band gap, where the conductance is equal to two conductance quanta, $2e^2/h$, i.e., $C = n = 2$. Moreover, the edge electronic structure shows two edge modes traversing the band gap.

From these results, it can be inferred that the $n\text{MBT}_{3\text{SL}}/\text{hBN}$, $n > 1$, heterostructures have $C = n$ either in the metamagnetic state or even intrinsically, depending on the actual interlayer coupling over hBN. Similar results are expected for MBT/hBN multilayers based on $\text{MBT}_{5\text{SL}}$ and $\text{MBT}_{7\text{SL}}$ films. As a side note, the weakness of the exchange interactions through hBN implies that the magnetic domain dynamics of the individual MBT subsystems should be decoupled, which is one particular difference between the here proposed high- C multilayer and non-synthetic high- C CIs^{58,59}.

DISCUSSION

Using ab initio and tight-binding calculations, we studied novel $\text{MnBi}_2\text{Te}_4/\text{hBN}$ multilayer heterostructures in which thin MnBi_2Te_4 films are interlayered by hexagonal boron nitride monolayers. The van der Waals bonding between hBN and MnBi_2Te_4 preserves the magnetic and electronic properties of the latter, in particular, the $C = 1$ Chern insulator state. Taking advantage of this, we have shown that a stack of n MnBi_2Te_4 films with $C = 1$ in the $\text{MnBi}_2\text{Te}_4/\text{hBN}$ multilayer gives rise to a high Chern number state, $C = n$, characterized by n chiral edge modes.

There are two ways to achieve this state in the proposed heterostructures. The first way is to use the external magnetic field to drive MnBi_2Te_4 films into a forced ferromagnetic state, which is nowadays widely used to observe a new kind of the quantum Hall effect that does not require the formation of Landau levels^{26,35–42,52}. In this case, both even and odd septuple layer MnBi_2Te_4 films can be used. One may well expect the high Chern number state to persist up to temperatures as high as $20\text{--}30 \text{ K}$, as previously observed for the $C = 1$ state in MnBi_2Te_4 thin flakes^{36,40}. The second way relies on the use of the odd septuple layer films as they realize the quantum anomalous Hall state intrinsically²⁶. Although a prior application of the external field might be needed to align the net magnetizations of the individual MnBi_2Te_4 slabs (as it is done in the Cr-doped $(\text{Bi,Sb})_2\text{Te}_3$ ^{9,60}), the high Chern number quantum anomalous Hall state can later be observed in remanence, i.e., at zero field. Currently, the observation temperature of the $C = 1$ quantum anomalous Hall effect in MnBi_2Te_4 is about 1.4 K . Improving the structural quality of MnBi_2Te_4 should allow the resolution of the intensely debated issue of its Dirac point gap size^{25,61–66} and push the effect observation temperature toward MnBi_2Te_4 ’s Néel point. The steps in this direction are currently underway^{38,52,67,68}.

MnBi_2Te_4 thin films currently used in the quantized transport measurements are mostly obtained by exfoliation^{26,35–42}. Using exfoliation and transfer, which is a standard technique for the construction of the van der Waals heterostructures^{45,46}, should be suitable to realize the here proposed $\text{MnBi}_2\text{Te}_4/\text{hBN}$ multilayers as well. However, this approach will not necessarily favor the formation of the interface structure with the lowest energy, but it could rather result in an arbitrary non-symmetric alignment between MnBi_2Te_4 and hBN. Fortunately, our results show that the Chern insulator state of individual MnBi_2Te_4 layers is not sensitive to the $\text{MnBi}_2\text{Te}_4/\text{hBN}$ interface registry. Therefore the exfoliation and transfer strategy should be appropriate to synthesize the here proposed $\text{MnBi}_2\text{Te}_4/\text{hBN}$ multilayers. The accumulated worldwide experience in the synthesis of two-dimensional van der Waals heterostructures is expected to greatly facilitate a prompt

realization of the high Chern number state in the $\text{MnBi}_2\text{Te}_4/\text{hBN}$ system.

In conclusion, we presented a novel concept of realization of a high Chern number magnetic topological insulator state, which relies on the use of fundamentally different but highly compatible van der Waals materials assembled in a multilayer heterostructure. While there is hardly a good alternative to hBN as an inert wide band gap monolayer insulator, other choices of the Chern insulators are in principle possible, such as transition metal dichalcogenide moiré superlattices²⁹ or compounds of the MnBi_2Te_4 family^{27,69–74}. The design proposed here allows a wide-range tuning of the Chern number, its upper limit being restricted only by the van der Waals heterostructures growth technology, making such multilayers interesting for future fundamental research and efficient interconnect technologies.

METHODS

Density functional theory calculations

The first-principles calculations were carried out on the level of density functional theory (DFT) as implemented in the Vienna Ab-initio Simulation Package (VASP)^{75–79}. All calculations shared several common parameters: the VASP's PAW datasets⁷⁹ were used, the plane wave cutoff was set to 500 eV, the spin-orbit coupling (SOC) was enabled, the exchange-correlation functional was that of Perdew, Burke, and Ernzerhof (PBE)⁸⁰, while the vdW interaction was taken into account through the Grimme D3 model with the Becke–Jonson cutoff^{81,82}. The Mn 3d-states were treated employing the GGA+ U approach⁸³ within the Dudarev scheme⁸⁴. The $U_{\text{eff}} = U - J$ value for the Mn 3d-states was chosen to be equal to 5.34 eV, as in previous works^{25,30,69,85}.

The DFT calculations were carried out with $11 \times 11 \times 1$ Monkhorst–Pack grid used to sample the Brillouin zone (BZ) for all but $n = 5$ system, where $9 \times 9 \times 1$ Monkhorst–Pack grid was used instead, and the electronic convergence threshold was 10^{-6} eV. For structural relaxations, VASP's conjugate gradient algorithm was employed until the force on each atom decreased under $0.01 \text{ eV}/\text{Å}$, and the electronic occupations were smeared by the Gaussian function of the width of 0.01 eV. For the static calculations, the tetrahedron method with Blöchl corrections was used to treat the electronic occupations instead. The noncollinear intralayer AFM configuration was calculated using hexagonal cells containing three atoms per layer [$(\sqrt{3} \times \sqrt{3})R30^\circ$ in-plane periodicity] (see Supplementary Note 2.A.3) and a $7 \times 7 \times 1$ BZ sampling. For the purpose of electronic structure visualization, the band energies for selected systems were calculated along the $\bar{K} - \bar{\Gamma} - \bar{M}$ path by a non-self-consistent DFT calculation, again with Gaussian smearing of 0.01 meV. The magnetic anisotropy energy was calculated as explained in ref. ²⁵, using the $25 \times 25 \times 1$ Monkhorst–Pack grid and electronic convergence threshold of 10^{-8} eV.

Wannier-based calculations

The obtained Kohn–Sham functions were used to construct the maximally localized Wannier functions by the Wannier90 code^{86,87}. We refer the reader to Supplementary Note 1 for further details regarding the wannierization procedure. The Wannier functions were, in turn, used to calculate anomalous Hall conductances of the proposed heterostructures through the Kubo formula as implemented in the WannierBerri code⁸⁸. In WannierBerri calculations, the broadening of 10K was used. For hBN-covered $\text{MBT}_{2\text{SL}}$ film, as well as $\text{MBT}_{2\text{SL}}/\text{hBN}/\text{MBT}_{2\text{SL}}$ heterostructure, the Chern number was also calculated by the Wilson loop method as implemented in Z2Pack^{89,90}. The edge spectral functions were calculated by the Green function method⁹¹ from the tight-binding Wannier Hamiltonian as implemented in WannierTools⁹². The number of principal layers in these

calculations was set to four, while the details of the cell used can be found in Supplementary Note 4.

DATA AVAILABILITY

Inputs and results are available from the corresponding authors upon a reasonable request.

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AUTHOR CONTRIBUTIONS

The problem was conceptualized by M.M.O. and E.V.C. Density functional theory calculations were performed by M.B., A.Y.V., E.K.P., and M.M.O. Wannier90 and surface state (WannierTools) calculations were performed by M.B. and E.K.P. Z2Pack Chern number calculations were performed by M.B. and A.Y.V. Hall conductance calculations were performed by M.B. Analysis of results was done by M.B. and M.M.O. Figures were produced by M.B. The manuscript was written by M.M.O. and M.B., with input from the other authors.

COMPETING INTERESTS

The authors declare no competing interests.

ADDITIONAL INFORMATION

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