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A reconstructed total precipitation framework

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Climate change is expected to alter the statistical properties of precipitation. There are two related but consequentially distinct theories for changes to precipitation that have received some consensus: (1) the time-and-space integrated global total precipitation should increase with longwave cooling as the surface warms, (2) the most intense precipitation rates should increase at a faster rate related to the increase in vapor saturation. Herein, these two expectations are combined with an analytic integration of three conceptually independent properties of the tropical hydrological cycle, the intensity, probability, and frequency of precipitation. The total precipitation in both a cloud-resolving model and tropical Global Precipitation Measurement mission data is decomposed and reconstructed with the analytic integral. By applying (1) and (2) to the precipitation characteristics from the model and observations to form a warming proxy model, it is suggested that a wide range of future distributions of precipitation intensity, probability, and frequency are possible.

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INTRODUCTION

Changes due to climate warming in the intensity and location of precipitation are some of the most potentially impactful to human and ecological systems. Climate change is projected to bring about more flooding as well as more intense droughts. But our knowledge of how the hydrological cycle may respond to climate warming remains incomplete and largely derived from highly parameterized models, although models that exhibit commendable skill in many respects and which have been thoroughly compared to available data. Such a reliance on models has limited the predictions made of hydrological changes due to warming to those that climate models are capable of simulating or, more accurately, those that climate models happen to simulate. Dozens of climate models exist and produce myriad responses in clouds and precipitation. Therefore, it is tempting to assume that the range of possible responses of precipitation to climate warming are completely suggested by the ensemble of models. The authors, however, are unaware of any evidence suggesting that all possible responses are simulated. The predicted changes are in one sense often very general and physically based,^{1,2} but in another sense, are rather limited in that they are the cumulative result of similar structural assumptions and parameterizations among the ensemble of models.

Past work may be generally imagined as a series of successful attempts to infer and physically explain the *likely* response of the hydrological cycle to warming; what we do in this study might be simply described as an attempt to infer *possible* responses. In many ways, this is a regression, but we hope to suggest that by populating a large phase space of possible responses and then building in layers of physical constraints, we may be repositioned to move forward in asking what responses are likely. Therefore, this method is not a replacement for complex, global models or contemporary theory but rather a complementary tool with its own merits and limitations.

To begin to answer our basic question “what *could* the hydrological cycle of a marginally warmer Earth look like?”, we

will invoke two physical constraints on precipitation. These postulates themselves go some way toward answering the question immediately.

1. The amount of water vapor in the atmosphere increases as the surface temperature rises following the Clausius-Clapeyron law (assuming relative humidity and convective mass fluxes do not change) which dictates ~7% increase in specific humidity per degree of surface warming (although this value ranges on Earth from 6 to 15% depending inversely on surface temperature³). Assuming the vertical structure of convection doesn't change significantly with surface warming, the magnitude of intense, convectively-generated precipitation will increase at the same rate as column moisture.^{4–7} There is some disagreement on the precise rate of the probable increase in convective precipitation intensity^{8–11} which we will discuss later, but for our purposes, the existence of an increase in intense precipitation is more important than its numerical value which we will take to be the global mean value of 7%/K.³
2. Total global precipitation must increase at approximately 2% per degree of surface warming. This is the result of an increase in the radiative flux divergence of the atmosphere at a rate of 2%/K. Total global precipitation must approximately balance the total atmospheric radiative flux divergence. So while intense precipitation grows following Clausius-Clapeyron scaling, total global precipitation grows more slowly following surface warming.^{12–15} This occurs both globally and in conditions representative of the tropics^{5,15,16} when sensible fluxes and shortwave absorption do not change significantly.

The mechanics of our new method may be illustratively compared to those of a global circulation model. In a model, global precipitation is the result of countless iterations of calculations of grid-scale physical processes. This is a bottom-up approach. In our new method described below, we largely take a

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top-down philosophy in which we start with global precipitation and use it to infer something about the possible behavior of quantities related directly to physical processes.

RESULTS

Hydrological Decomposition

To make use of these numbered postulates, we will decompose the global, long-term mean precipitation rate, I (i.e. the precipitation rate constrained by Postulate 2). There are two, commonly used independent ways to describe I . The first is through energy balance constraints. This has been done many times for the current climate in a conceptual way and with climate models. These methods conclude that, on average, $\sim 3 \text{ mm day}^{-1}$ of precipitation should fall everywhere across the globe.^{17,18} The second is by averaging all daily accumulations of precipitation across the globe over a long period of time either in nature¹⁹ or from climate models¹³ to arrive at the same result.

Next, we introduce an additional way to estimate total global precipitation. We decompose I into components that describe the conditions under which rain may be expected. For any atmospheric condition, X , the average precipitation rate in an atmospheric column with that condition, $G(X)$, is the result of the average precipitation rate when the atmosphere in X is precipitating, P , multiplied by the likelihood of precipitation, F , in X . That is

$$G(X) = F(X) * P(X). \quad (1)$$

The total global precipitation at a particular atmospheric state X , $dI(X)/dX$, is the average precipitation rate multiplied by the probability density function of a certain atmospheric condition

occurring, $J(X)$. So,

$$dI(X)/dX = G(X) * J(X) = F(X) * P(X) * J(X). \quad (2)$$

To find I , (2) must be summed over all m possible atmospheric states (each with a unique X). That is

$$I = \sum_m F(X_m) * P(X_m) * J(X_m). \quad (3)$$

In principle, (3) is perfectly generalizable to any quantifiable atmospheric state. Here we will choose to characterize the atmospheric meteorological state by the atmospheric 'column relative humidity', R . R is the vertically integrated moisture content divided by the vertically integrated saturation moisture.²⁰ As such, R can assume a value on the range $[0,1]$. R can be calculated from satellite measurements, balloon soundings, or from a model and has the appealing quality that its bounds do not change with surface temperature (i.e. climate state). R is also physically connected to precipitation since the atmospheric moisture reservoir is the preceding source of precipitation.

Rather than sum over all possible instances of each R as in (3), we develop idealized functional representations of $J(R)$, $P(R)$, and $F(R)$ and then analytically integrate over the range of R , as

$$I = \int_0^1 F(R) * P(R) * J(R) dR. \quad (4)$$

To determine the forms of $J(R)$, $P(R)$, and $F(R)$, we approximately reconstruct the results from a tropical, high-resolution, large domain, cloud-resolving, radiative convective equilibrium simulation. We will use the model to suggest fit shapes and fit parameters for components of I , but we are not attempting to reconstruct the RCE model data precisely. We need general forms for $J(R)$, $P(R)$, and $F(R)$ because, as we will show later, the utility of

Fig. 1 For each panel, the behavior of precipitation properties from RAMS (blue), fitted to the RAMS data (red), and the range of possible fit values in a +2 K warmer scenario (green shading). **a** the probability of precipitation (F) with $\lambda_F = 0.019$ and $R_F = 0.78$. **b** The conditional mean precipitation rate (P) with $\lambda_P = 0.12$ and $P_m = 97 \text{ mm h}^{-1}$. **c** The probability density of R normalized by the maximum value (J) with $\lambda_J = 0.02$ and $R_J = 0.69$. **d** The (normalized) precipitation rate ($P * J * F = dI/dR$)

the decomposition and analytic solution to (4) below is in suggesting how the ratio of parameter values between a relatively cool and warm climate state will change such that their initial values only need be approximate.

The functional form of $P(R)$ has the most precedent and has been examined from observations in the past. Ahmed and Schumacher²¹ fit $G(R)$ using a power-law in R . Bretherton et al.²⁰ used an exponential fit. We will use the latter for internal mathematical consistency, but either could be used in principle. It is important to note that $P(R)$ is conditioned on non-zero precipitation rates ($>0.1 \text{ mm h}^{-1}$) unlike in past work, but this is found not to affect the underlying shape of the function. $P(R)$ from the RCE model and the idealized fit are shown in Fig. 1b. The functional form of $P(R)$ is

$$P(R) = P_m e^{(R-1)/\lambda_P}. \quad (5)$$

In (5), P_m is the maximum precipitation rate. This occurs at $R = 1$. The exponential form of (5) implies that P , and the cascade of physical processes that lead to surface precipitation, depends nonlinearly on R . The degree of nonlinearity is implied by λ_P . In (5) and in equations below, a λ_i is a growth rate with units of humidity, and an R_i is a critical humidity or R location of an inflection in an assumed function. The curve in Fig. 1b has a slight deviation from an exponential form. This is likely due to the particulars of the model rather than something physical.

The probability of precipitation²² (or called the “fractional occurrence of precipitation” in Igel et al.²³) is the long-term likelihood of precipitation $>0.1 \text{ mm h}^{-1}$ falling when $X = R$. Igel et al.²³ suggested the steep rise in the probability of precipitation with R (see Fig. 1a below) is indicative of a system exhibiting characteristics of self-organized criticality²⁴ wherein precipitation occurrence (not intensity) responds rapidly to an increase in

moisture beyond a critical threshold to prevent runaway moistening. Given that behavior, we have chosen to approximate the RCE model probability (Fig. 1a) with a monotonically increasing sigmoid function.

$$F(R) = \frac{1}{1 + e^{-(R-R_F)/\lambda_F}}. \quad (6)$$

While we have chosen to use a sigmoid function, it is certainly possible that other similar functional types (say, a simple step function) could be appropriate for modeling $F(R)$.

The precipitating-end (i.e. high R) of the population density is approximated with a monotonically decreasing sigmoid function (Fig. 1c).

$$J(R) = \frac{1}{1 + e^{(R-R_J)/\lambda_J}}. \quad (7)$$

The fit to the RCE model data shows that the J fit is the largest leap of faith since it is only representative of the right side of the distribution. Observational data show a similar enough form at high R ^{25–27} to inspire confidence that the sigmoid is representative. The complete probability distribution of R represents the complex interplay of surface evaporation, moisture advection and diffusion, and precipitation and cloud formation, but it is easiest to imagine $J(R)$ at high R as being the result of advection moistening columns and precipitation drying them. In this framework, $J(R)$ is steep because up-gradient advection of moisture becomes more difficult as moisture content approaches its maximum R while precipitation becomes more likely (i.e. (6)) and more intense (i.e. (5)) with R .

Figure 1d shows dI/dR from the RCE model (using (3)) and from the idealized fits (using (4)). The figure shows that the idealized functional forms and (4) do an acceptable job of representing the

Fig. 2 For each panel, the behavior of precipitation properties from GPM (blue), fitted to the GPM data (red), and the range of possible fit values in a +2 K warmer scenario (green shading). **a** The probability of precipitation (F) with $R_F = 1.04$. **b** The conditional mean precipitation rate (P) with $P_m = 3.50 \text{ mm h}^{-1}$. **c** The probability density of R normalized by the maximum value (J) with $R_J = 0.8030$. **d** The (normalized) precipitation rate ($P \cdot J \cdot F = dI/dR$). λ values are shown in Fig. 3

structure of the full results using (3). Again, we are not attempting to represent the RCE model precisely but to approximate its basic characteristics with a flexible framework. So, we proceed with caveats but with confidence. It is important to note that $J(R)$ asymptotes to zero at high R much faster than $G(R)$ grows. This means dI/dR approaches zero at high R and that the bulk of accumulated precipitation falls well below $R=1$ in a regime of frequent and light precipitation.

Combined satellite observations

The forms in (5)–(7) are used to fit satellite observations of contemporary precipitation. The Global Precipitation Measurement (GPM) mission²⁸ is a multi-satellite, multi-instrument platform tasked with measuring global precipitation at high accuracy and frequently. The flagship platform includes dual downward-pointing precipitation radars (DPR) at Ka and Ku band frequencies. Their combined surface precipitation retrieval has 5 km along track resolution and can measure precipitation rates up to 300 mm h⁻¹. NASA's Atmospheric Infrared Sounder²⁹ (AIRS) retrieves atmospheric profiles of both temperature and moisture over ~45 km pixels. From these profiles, we calculate a column relative humidity.

The terms comprising I from the combined satellite analysis are shown across Fig. 2. The functional forms of $F(R)$ and $P(R)$, and $J(R)$ at high R fit well the satellite-derived data. There are two behaviors that illustrate the flexibility of the method we have developed. $F(R)$ is over-fit by the sigmoid in this case. $P(R)$ fits the exponential form well but is much lower in magnitude than it is in the cloud model; this is similar to magnitudes presented previously from observations.^{21,30} Differences in the magnitude of precipitation intensity among observing systems are common, likely due to differences in areal coverage and effective sensitivity, while the exponential pickup in precipitation is consistent.²³

Analytic solution

Substituting (5–7) into (4),

$$I = \int_0^1 \frac{1}{1 + e^{-(R-R_F)/\lambda_F}} P_m e^{(R-1)/\lambda_P} \frac{1}{1 + e^{(R-R_J)/\lambda_J}} dR, \quad (8)$$

results in an integral with no exact solution. Therefore, we are forced to approximate this integral asymptotically. The integral in (8) yields the total global precipitation as a function of the six parameters which describe the rain rate (P_m, λ_P), the probability of rain at a given relative humidity (R_F, λ_F), and the probability of observing a particular relative humidity (R_J, λ_J). We know of no exact analytic formulation of this integral: even if one exists, it would be too complicated to elucidate the essential dependence of the integral on the parameters. Instead, we note that both data and the numerical simulations imply that the centers of the two probabilities (R_F, R_J) are close to one another and far from the boundaries of the domain of integration (0,1), the widths of the sigmoids (λ_F, λ_J) are small relative to the integration domain, and the e-folding scale of the precipitation rate (λ_P) is large compared to the widths of the sigmoids. Taken together, these facts imply that the product of the two probabilities look like a hyperbolic secant squared within the domain of integration, with the exponential increase of precipitation rate skewing the integrand toward larger values of R . We provide the details of the asymptotic analysis, which yields the approximate expression in (9), in the supplementary material. We approximate (8) as

$$I = \frac{\pi}{2} \left[\overbrace{P_m e^{(\bar{R}-1)/\lambda_P}}^i \right] \left[\overbrace{\frac{1}{\lambda_F e^{-\Delta/\lambda_F}}}^{ii} \right] \left[\overbrace{e^{((\lambda_F^{-1} + \lambda_J^{-1})/\bar{\lambda} - \frac{1}{2})}}^{iii} \right] \left[\overbrace{1 - \frac{4}{\pi} e^{-(1-\bar{R})/2\bar{\lambda}}}^{iv} \right]. \quad (9)$$

This solution includes two mean quantities, $\bar{R} = (R_F + R_J)/2$ and $\bar{\lambda}^{-1} = (\lambda_F^{-1} + \lambda_J^{-1})/2$, and two difference quantities, $\Delta = (R_F - R_J)/2$ and $\sigma^{-1} = (\lambda_F^{-1} - \lambda_J^{-1})/2$. See the supplementary material for a discussion of the introduction of these quantities.

The terms in (9) are (i) the precipitation rate at $\bar{R}$, (ii) the effective width of the product of $F(R)$ and $J(R)$ or the width of the distribution of the conditional PDF of precipitating columns, (iii) a skewness, and (iv) a measure of the proximity of the peak of dI/dR to $R=1$. Terms iii and iv are often close to unity. Equation (9) represents a simple, yet meaningful distillation of the consequences of the hydrological cycle. The total global precipitation occurring in any climate state, I , depends on parameters which describe the behavior of individual components of the hydrological cycle. In principle, hydrological parameters can either be found from data (as above) or from theory.

Sensitivity to climate state

We can use (8) and postulates (1) and (2) to infer how certain parameters of the contemporary satellite-observed hydrology may change as a function of an arbitrary increase in surface temperature. For example, postulate (2) effectively states that I should increase at a rate of 2%/K. Postulate (1) effectively states that P_m should increase at 7%/K. Note that this is a very loose interpretation of Postulate (1) where with the most intense precipitation rate changes at exactly 7%/K but that less intense precipitation may change differently; the additional imposition of Postulate (2) effectively limits weaker precipitation rates (i.e. G ($0.8 < R \ll 1$)) from increasing at 7%/K. It will turn out that even with just these two limitations several conclusions can be drawn about the possible sensitivity of $J(R)$, $P(R)$, and $F(R)$ to surface conditions.

One of the benefits of using $X=R$ is the value of the initial growth of $G(R)$ does not depend strongly on climate or observing system.^{20,23} Thus, we will assume that only the “steepness” of $J(R)$, $P(R)$, and $F(R)$ change with surface temperature. That is, R_i don't change but λ_i do. As discussed in Neelin et al.,³¹ this may be an imperfect assumption but is clearly sufficient for our illustrative needs. Taken literally, R_i 's that do not depend on climate state imply changes in the vertical temperature profile with surface warming have little impact on convective development beyond their impact to moisture saturation.

Figures 1 and 2 includes a range of possible hydrological states in a 2K-warmer climate with postulates (1) and (2) imposed. These solutions to (8) with I 4% more than from the raw data are found numerically through a simple parameter sweep of parameter values that are between one-half and twice their current values. Solutions occur along a thin, curved plane in the three- λ_i phase space (not shown). This plane is approximated by (9) with fixed R_i . These solutions show that the identified physical constraints to warmer-climate precipitation (i.e. the postulates) do not preclude a variety of responses in $P(R)$ and $J(R)$ to warming. The lone exception is that P_m is 14% higher in all 2K-warmer solutions. Among the many possible numerical solutions are states with much flatter $P(R)$ (i.e. increased λ_P) and both leftward and rightward shifts in R of the peak of dI/dR . While the RAMS data tends to suggest either leftward or rightward shifts are possible, the GPM data mostly allows for rightward shifts (so called “rich-get-richer”³²). Figure 3 shows four example combinations of parameters possible in a world with 2K of warming. These are chosen so that the extreme values of the parameter phase space are represented. Figure 3a shows that the range of $F(R)$ is limited to those within a factor of 0.7 to 1.4 of λ_F . The GPM data suggest two extreme types of futures are consistent with postulates (1) and (2). The first is a slight leftward shift in dI/dR while the second is a rightward shift and change in the shape of dI/dR such that significantly more rain falls at high R . Comparing Fig. 3c, d suggests the nature of these shifts appears largely to depend on shifts in $J(R)$. We can use (9) to help interpret this suggestion. For

Fig. 3 Panels as in Fig. 1 and 2. GPM fits (red) recreated from Fig. 2. Other colored lines show numerical solutions for fit parameter value combinations within a factor of 2 of the GPM-derived values for a theoretical 2 K warmer world. Parameter values for the warmer world represent maximum and minimum parameter values that satisfy numbered postulates; they are listed in legends. ‘0’ subscripts are used to indicate values from the GPM fit. Some lines are dashed to aid visibility. The blue dotted line indicates a parameter combination assuming a twice-Clausius–Clapeyron scaling

the rightward shift cases, λ_J increases significantly. So physically, these are climates with rainfall accumulation occurring over a wider band of R but closer to $R = 1$ than in the current climate. Interestingly, these rightward shifts can occur in climates where $P(R)$ can either decrease or increase for $R < 0.9$ as the surface warms. It is possible to recast numerically our data into bins of $G([0,1])$ to try to recreate the results of Pendergrass and Hartmann³² who introduced the idea of decomposing hydrological changes into modes. Our rightward shifts correspond approximately to a combination of familiar “decrease”, “shift”, and “extreme” Pendergrass and Hartmann³² modes (not shown).

Across the panels of Fig. 3, we have included an example parameter value for which P_m is 14%/K higher (a so-called “super Clausius–Clapeyron” scaling). We have included these example lines because there is still disagreement about the probable magnitude of increases in extreme precipitation rates and to illustrate the flexibility of our method. The super Clausius–Clapeyron scaling results in a slightly narrower range of λ_F and a very slight shift in the normalized accumulation toward higher R of the most leftward-skewed dl/dR . We can use (9) to contrast parameter values between assumptions of a 7%/K or 14%/K increase in P_m under surface warming as 2%/K increase in l ; in this case, P_m increases at the expense of the effective width term.

Figure 3 suggests a wide range of parameter values are consistent with the premise of our model and the physical

constraints imposed. It is important to remember that no physics are imposed beyond consistency with components of our model and those listed. Therefore, it is possible climate change parameters may violate the laws of physics (unlike in models) while still satisfying the criteria we impose.

DISCUSSION

We have developed a way to predict possible changes to the hydrological cycle as the world warms. This framework does not rely on running a myriad of models to fill the possible hydrological phase space but rather constrains possible changes analytically by reconstructing the total global precipitation from components and invoking two physically-based, model corroborated constraints. Equation (9) might be thought of as the simplest possible hydrological climate model.

Either the analytic or numerical solutions to (8) can be used to infer the possible sensitivity of the probability of precipitation, $F(R)$, the conditional mean precipitation, $P(R)$, and the PDF of column humidity $J(R)$ to surface warming at least in the tropics. It was shown above that, in principle, with only postulates (1) and (2) invoked to constrain the climate parameter space, a physically broad set of possible changes to the hydrological sensitivity to R is consistent. But, even with just those two constraints, we can suggest that the characteristics of the probability of precipitation

will not be very different in a warmer climate than it is today if the values of critical R_i do not change with the climate state.^{20,23} Very few climate states that might be considered exotic satisfy all the criteria imposed by (8) and the two postulates. The most significant quantitative difference between hydrological components could arise in P at relatively low R if λ_p increases strongly. In this case, the weakest rain rates could strengthen significantly (Fig. 1a and 2a). The model calibrated with GPM data suggest the probability of precipitation, λ_{F_i} , will not change shape significantly due to warming.

The biggest caveat to the conclusions drawn is that our model is developed from concepts pulled from tropical hydrology. While the majority of global precipitation falls in the tropics and while tropical concepts are often used as idealizations of the climate system as a whole, we cannot state definitively whether our model can be applied to the globe. As a concept, our model should be applicable, but it may not be as a quantitative prediction tool with our choice of training data.

One of the considerable benefits of the framework developed is that as new postulates are suggested by detailed mechanistic studies, appropriate analytic or numeric solutions can be recomputed with additional physical constraints imposed and a reduced solution phase space can be found at nearly no additional cost. Such mechanistic studies are likely possible given that the components of I are related intimately to the nature of convective precipitation: F likely depends on the relative distribution of cloud types; J depends on the intensity of storms (and is not contingent on whether a storm exists in any environment); and J depends on sources, sinks, and movement of water vapor. And, there is no limitation to the number of self-consistent or types of constraints that might be imposed. For example, possible constraints may be as broad as that $F(R)$ is higher everywhere with a warmer surface or as narrow as that $F(R)$ is 10% higher at $R = 0.6$. Various imposed constraints may even be correlated.

The framework can also be used to suggest where future mechanistic studies might contribute to significant gains in narrowing the range of predictions of the future hydrological cycle. For example, better constraining how the parameters related to the probability density of R might change with surface temperature would limit strongly the range of possible solutions of (9). In contrast, better constraining the conditional mean precipitation rate would not. The method can also be used to find hidden constraints in models. It is likely that the ensemble of climate model predictions for a warmer climate occupies a narrower region of $F(R)$, $P(R)$, and $J(R)$ phase space (i.e. hydrological states) than our new method. It should be possible to break down climate models in the way we have with CRM and observations to diagnose otherwise-hard-to-detect consistencies (i.e. parameter similarities) among models by fitting climate model results to our analytic forms.

The method we have introduced is based on constraints imposed on the tropics-wide climate system. But it might be asked whether this method could be applied regionally. In principle, it could be possible to define physical constraints (along the lines of postulates (1) & (2)) that are relevant to smaller regions, an individual ocean basin, perhaps. We will not presume to know how these constraints might be obtained or what they are, but they are potentially identifiable. A regional application of this method could prove to be a powerful tool for understanding regional climate change statistics. The complicating factor is whether the relative distribution of R changes between different regions. Certainly it is possible that as the land will warm relative to the ocean that its median R may lessen relative to the ocean's.³³ Such a changes would necessitate dealing with shifts in R_i which are not addressed in our results.

METHODS

Cloud model

For full details of the model setup see Igel et al.²³ and Igel.³⁴ The Regional Atmospheric Modeling System (RAMS)³⁵ is a nonhydrostatic cloud-resolving model with a double moment microphysics scheme^{36,37} and an interactive two-stream radiation scheme.³⁸ The model was run, unperturbed, over a fixed ocean surface of 301 K for 60 days. The horizontal grid spacing was 1 km, and there were 65 vertical levels with a well resolved boundary layer and maximum vertical spacing of 650 m. The domain size was 2000 km by 400 km by 25 km. Hourly state files from the final 10 days of the simulation were used to create the precipitation and moisture statistics. For an example moisture-precipitation snapshot of the simulation, see Igel et al.²³ Convection naturally develops and aggregates into clusters.³⁹

Satellite measurements

Data from the GPM DPR and AIRS were obtained for July 2014 to December 2015. Data were limited to 20°S to 20°N to retain the conceptual simplicity of the tropics covering a large enough area to be representative. AIRS R values were calculated by vertically integrating the standard, version-6, level-2 specific and relative humidity (over liquid)⁴⁰ from the surface to the tropopause. Data with poor quality assurance flags were not used.⁴¹ Such caution likely biases $P(R)$ and $F(R)$ to be too low. Data were collocated between the satellites via a nearest neighbor interpolation with a maximum spatio-temporal spread between measurement pixel centers of 100 km and 1 h. The level-2 “near surface” rainfall rate from GPM’s dual-Ka/Ku radar retrieval was used. This product can retrieve high spatial resolution precipitation rates up to 300 mm h⁻¹ which is well more than many other standard precipitation products. Data for both land and ocean were included. While originally only discussed in the context of oceanic convection,²⁰ the precipitation statistics upon which we are basing our development have also been shown to behave similarly over land and ocean.²² Data were obtained from NASA servers in October of 2018. While GPM provides a robust measurement of precipitation, the limitations inherent in radar retrievals of precipitation intensity and difficulty in measuring rainfall in complex situations, especially over land generally and steep topography specifically,^{42–44} should be kept in mind.

DATA AVAILABILITY

The RAMS data is available at <https://doi.org/10.7910/DVN/A5FUHC>. GPM data are available via anonymous ftp at <ftp://arthurhou.pps.eosdis.nasa.gov/gpmdata>. AIRS data are available at <https://disc.gsfc.nasa.gov/datasets?page=1&source=AQUA%20AIRS&keywords=airs%20version%206>.

CODE AVAILABILITY

Analysis code is available at <https://mattigel.faculty.ucdavis.edu/code/> with no restrictions.

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AUTHOR CONTRIBUTIONS

M.R.I. conceived the work. J.A.B. articulated and solved the integral equation. M.R.I. and J.A.B. wrote the manuscript.

ADDITIONAL INFORMATION

Supplementary Information accompanies the paper on the *npj Climate and Atmospheric Science* website (<https://doi.org/10.1038/s41612-019-0090-8>).

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