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Beyond trade statistics: how much do exports actually contribute to domestic value added?

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Koopman et al. (2014) and Los et al. (2016) decomposed gross exports into various value-added components by adopting the input–output assumption of disconnection between production and final demand. Such an assumption, however, neglects the ability of production inflows to generate income and consumption, and therefore additional impacts on production. To achieve a more complete understanding of the role played by trade, this article presents a method for quantifying the value added of exports that reflects the linkages between production and private consumption. In the tradition of Miyazawa (1968, 1976) and Sonis and Hewings (1973), the proposed model endogenously defines household consumption in the output determination, thus improving the way in which the interdependencies between income and output generation processes are revealed. The proposal is directly applicable empirically through available world trade databases.

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Introduction

In recent decades, world production has evolved towards fragmented, interconnected, and interdependent manufacturing processes so that production systems have been divided into complex trade operations, with firms carrying out specialized activities concentrated in specific territorial locations. This issue has attracted the attention of a growing literature aimed at measuring the national value-added embodied in exports.[1] Using an accounting perspective, Johnson and Noguera (2012) introduced a measurement to quantify the value-added content of bilateral trade. Their method was later applied to global automotive production by Timmer et al. (2015). Los et al. (2015) decomposed the final value of a product into the value-added contributions made by any other country in the world to analyze whether the fragmentation of value chains is mainly within or across regional blocs. More recently, product fragmentation has been conceptualized as functional specialization in trade by focusing on activities such as marketing and R&D rather than on specific products (Timmer et al., 2019).

Within this branch of literature, Koopman (2014) (hereinafter, KWW) proposed an accounting method based on the inter-sectoral and inter-country relationships of the input–output (IO) model that systematically identified several value-added components within a country's gross exports.[2] Los (2016) (hereinafter, LTV) proposed an alternative approach to KWW's that uses the input–output hypothetical extraction method (HEM) to obtain the value added in exports. Later, Los and Timmer (2018) extended the extraction procedure to provide a unified framework that includes the measures defined so far.[3]

The present paper falls within this line of IO literature but, rather than having an accounting perspective, it aims to determine the economic impact of trade. With regard to the effects of trade, a large body of the literature uses large-scale general equilibrium models with a computational focus for policy analysis. Prominent surveys of this literature have been conducted by Caliendo and Parro (2022), who reviewed the recent theoretical and empirical literature to study the impacts of trade and trade policies; and by Antràs and Chor (2022), who surveyed the literature that has analyzed the global value chains in the context of the world IO tables and IO statistics developed to date. A prominent example is Fusacchia et al. (2022), who analyzed the general equilibrium effects of trade liberalization in Africa continental, by focusing on product fragmentation and production networks and taking into account the value-added structure of international trade. Recently, Baqaee and Farhi (2023) presented a unified framework for studying output and welfare impacts in open and distorted economies via a flexible model with disaggregated and interconnected production structures.

The standard IO approaches developed by KWW and LTV are based on the assumption that final demand is exogenously defined. Accordingly, inflows to production are not channeled to (towards) final demand through subsequent income shocks and consumption increases. However, the hypothesis of invariable household consumption is contrary to the most elementary postulates of economic theory. Consumers earn income because of their endowments of factors (namely labor and capital) and, in accordance with their role as consumers, spend their income on goods and services. The sequential economic interdependencies therefore lead to increases in production to expand factor payments (i.e., employee compensations and capital revenues). Also, since consumption is directly linked to income, there is a subsequent increase in household spending, which generates a new round of output increases, and so on. Because the conventional input–output model does not allow final demand to be part of the endogenous structure that generates multiplicative effects, methods based on this framework are unable to represent the complete transmission channels transiting from the (starting) impact on production to (towards) income, continuing from income to (towards) private consumption, and returning back to production.

By considering the limited ability of the input–output model to fully reflect the circular flow of income, this paper improves the definition of the propagation mechanisms triggered by trade. The model used, which is in the tradition of Miyazawa (1968, 1976) and Sonis and Hewings (1993), reveals the interdependencies between income and output generation processes. In comparison with the traditional model, the larger economic linkages captured in this approach mean that the value added of exports is not quantitatively limited by the value of gross exports but by the value of total value added. The model's extension offers a complementary perspective of trade that, by going beyond what trade statistics strictly reflect, contributes to a better understanding of the driving forces activated by exports within an economy. From an empirical perspective, all data requirements are covered by the global input–output databases available and no further information is needed.

The rest of the paper is organized as follows. Section "Value added of trade: an extended input–output model" describes a conceptual framework that includes the linkages between production, income, and consumption, derives measurements for the value added of exports, and provides an illustrative example. Section "Empirical application" shows an empirical application, and section "Conclusions" draws conclusions.

Value added of trade: an extended input–output model

The two-country case. This section adapts the conceptual framework proposed by KWW and further developed by LTV to modify the definition of private consumption in the determination of gross output. The mutual interaction between consumption, income, and production in the IO model was first proposed by Miyazawa (1968, 1976) to capture the link between household groups and sectors and explicitly reveal income propagation impacts. Extensions of this approach can also be found in Sonis and Hewings (1993).

In what follows, the Miyazawa-Sonis and Hewings endogenization of private consumption is introduced into the model. For the sake of simplicity, assume a world with two countries or regions (s and r), each of which produces a single good that can be used either as an intermediate input or a final product.[4] In both countries, intermediate and final goods are domestically consumed or exported to each other. By focusing on country's, gross output is defined as:

$$x_s = a_{ss}x_s + a_{sr}x_r + c_{ss}m_s + c_{sr}m_r + y_{ss} + y_{sr} \qquad (1)$$

where x_s and x_r are gross output in country s and r, respectively, a_{ss} are the input–output coefficients showing the intermediate consumption of goods from s in relation to s's output that are used domestically, and a_{sr} are the input–output coefficients showing the quantities of intermediate consumption from s used in r in relation to r's output. Equation (1) also contains the final uses of the domestically produced goods, which can be consumed either at home or abroad. A component of final demand is household consumption, which responds to a constant fraction (c_{ss} and c_{sr}, respectively) of the private income in the country of destination (m_s and m_r). Final uses also include y_{ss} and y_{sr}, which contain the final demand other than private consumption (i.e., public consumption and investment) of the products from s that can be consumed in s or r, respectively.

To complete the representation of the income-generation process, private income in s (m_s) is assumed to be a fraction (γ^s)

of value added such that:

$$m_s = \gamma^s v_s x_s \tag{2}$$

where v_s is the value added to gross output ratio in country s and $0 \le \gamma^s \le 1$ is a scalar reflecting the fraction of value added that is converted into private income in s.[5] It should be recognized at this point that the rigidity of private incomes and consumption coefficients introduced into the model may overestimate the impacts since there is no possibility of adapting consumption decisions. The production impacts calculated from expression (1) may therefore be considered upper boundaries of the true effects channeled through consumption demand.

Introducing Eq. (2) and its analogy for m_r into Eq. (1), gross output can be rewritten as:

$$x_s = a_{ss}x_s + a_{sr}x_r + c_{ss}\gamma^s v_s x_s + c_{sr}\gamma^r v_r x_r + y_{ss} + y_{sr} \tag{3}$$

The input–output model characterizing the two-country system can be represented as follows:[6]

$$\begin{bmatrix} x_s \\ x_r \end{bmatrix} = \begin{bmatrix} a_{ss} + c_{ss}\gamma^s v_s & a_{sr} + c_{sr}\gamma^r v_r \\ a_{rs} + c_{rs}\gamma^s v_s & a_{rr} + c_{rr}\gamma^r v_r \end{bmatrix} \begin{bmatrix} x_s \\ x_r \end{bmatrix} + \begin{bmatrix} y_{ss} & y_{sr} \\ y_{rs} & y_{rr} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{4}$$

or alternatively:

$$\begin{bmatrix} x_s \\ x_r \end{bmatrix} = \begin{bmatrix} I - [a_{ss} + c_{ss}\gamma^s v_s] & -a_{sr} - c_{sr}\gamma^r v_r \\ -a_{rs} - c_{rs}\gamma^s v_s & I - [a_{rr} + c_{rr}\gamma^r v_r] \end{bmatrix}^{-1} \begin{bmatrix} y_{ss} & y_{sr} \\ y_{rs} & y_{rr} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{5}$$

Equation (5) includes the interdependences between production, private income, and consumption, thus showing greater economic relationships than that of the conventional Leontief system. This extension overcomes the input–output weakness of an unconnected production system from the rest of the circular flow. Note also that, since the proposed model moves private consumption from the exogenous final demand to the technical input–output matrix of coefficients and since the use of output is reconsidered but not its quantity, Eq. (5) captures larger multipliers (i.e., larger values of the inverse matrix) than in the traditional approach by KWW and LTV.

Equation (5) can also be written in a compact way:

$$x = \left(I - A^1 - A^2\right)^{-1} Yi \tag{6}$$

where the matrix of structural coefficients has been split into two matrices, one which contains the input–output coefficients (A^1) and one which contains the private consumption coefficients (A^2):

$$A^1 = \begin{bmatrix} a_{ss} & a_{sr} \\ a_{rs} & a_{rr} \end{bmatrix}; A^2 = \begin{bmatrix} c_{ss}\gamma^s v_s & c_{sr}\gamma^r v_r \\ c_{rs}\gamma^s v_s & c_{rr}\gamma^r v_r \end{bmatrix}$$

In Eq. (6), x is the column vector of sectoral production, Y is the inter-country matrix of final demand other than private consumption, and i is a column vector of compatible dimension compounded by unitary elements.

LTV and Los and Timmer (2018) proposed the 'hypothetical extraction method'[7] to quantify how much domestic value added is contained in a country's exports. The HEM simulates hypothetical situations in which some of the elements in the IO model are made equal to zero. This provides the option of comparing the real values for sectoral output (or, alternatively, value added) with those corresponding to an extreme (and unreal) case in which some selected elements did not exist. Comparing the actual and hypothetical cases is therefore a way to quantify by how much the removed element contributes to the system.

According to LTV, the difference between the gross value added (GVA) and the hypothetical value added if the economy does not export is the *domestic value added of exports* ($VAXD$).[8] Using Eq. (5), total value added in country s is obtained as:

$$GVA_s = \begin{bmatrix} v_s & 0 \end{bmatrix} \begin{bmatrix} I - [a_{ss} + c_{ss}\gamma^s v_s] & -a_{sr} - c_{sr}\gamma^r v_r \\ -a_{rs} - c_{rs}\gamma^s v_s & I - [a_{rr} + c_{rr}\gamma^r v_r] \end{bmatrix}^{-1} \begin{bmatrix} y_{ss} & y_{sr} \\ y_{rs} & y_{rr} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{7}$$

and a parallel expression could be written for country r. The hypothetical situation is modeled by canceling all exports (both intermediate and final) from s to r, so that a new (fictitious) value added (GDP_s^*) is given by:

$$GVA_s^* = \begin{bmatrix} v_s & 0 \end{bmatrix} \begin{bmatrix} I - [a_{ss} + c_{ss}\gamma^s v_s] & 0 \\ -a_{rs} - c_{rs}\gamma^s v_s & I - [a_{rr} + c_{rr}\gamma^r v_r] \end{bmatrix}^{-1} \begin{bmatrix} y_{ss} & 0 \\ y_{rs} & y_{rr} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{8}$$

Value added of exports is obtained as the difference between the actual and hypothetical value added:

$$VAXD_s = GVA_s - GVA_s^* \tag{9}$$

Similarly, the value added of exports proposed by LTV ($\overline{VAXD}$) can be obtained by adapting Eqs. (7) and (8) so that the elements in matrix A_2 are moved to the final demand matrix, as follows:

$$\overline{GVA_s} = \begin{bmatrix} v_s & 0 \end{bmatrix} \begin{bmatrix} I - a_{ss} & -a_{sr} \\ -a_{rs} & I - a_{rr} \end{bmatrix}^{-1} \begin{bmatrix} \overline{y_{ss}} & \overline{y_{sr}} \\ \overline{y_{rs}} & \overline{y_{rr}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{10}$$

$$\overline{GVA_s{}^*} = \begin{bmatrix} v_s & 0 \end{bmatrix} \begin{bmatrix} I - a_{ss} & 0 \\ -a_{rs} & I - a_{rr} \end{bmatrix}^{-1} \begin{bmatrix} \overline{y_{ss}} & 0 \\ \overline{y_{rs}} & \overline{y_{rr}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{11}$$

where the bars over variables indicate that private consumption is included in the corresponding element. Consequently, the elements in the final demand matrix are now greater than (or at least equal to) the corresponding elements in Eq. (4): $\overline{y_{ss}} \ge y_{ss}$, $\overline{y_{sr}} \ge y_{sr}$, $\overline{y_{rs}} \ge y_{rs}$, $\overline{y_{rr}} \ge y_{rr}$.[9]

The difference between the actual value added (Eq. (10)) and hypothetical value added (Eq. (11)) is the domestic value added of exports in the standard input–output model:

$$\overline{VAXD_s} = \overline{GVA_s} - \overline{GVA_s{}^*} \tag{12}$$

A quantification of the impact induced by the private consumption circuit can be done by comparing the value added of exports in the two structures. This impact, or *feedback of domestic value added of exports* (FD), is given by:

$$FD_s = VAXD_s - \overline{VAXD_s} \tag{13}$$

Johnson and Noguera (2012) introduced the *value-added consumed abroad by final users* ($VAXC$),[10] a concept that quantifies the domestic value added induced by foreign final demand or the value added effectively consumed abroad. This measurement will always be smaller than (or at least as large as) the $VAXD$ because it does not contain the value added returning to the home country embedded in the imported products. For country s, the $VAXC$ is calculated by hypothetically assuming that all final demand in country r is nil.[11] In the proposed model, the cancelation of all r's final demand implies making zero not only the components in the final demand matrix (y_{sr} and y_{rr}) but also the elements of the final consumption ($c_{sr}v_r$ and $c_{rr}v_r$). The hypothetical situation is then applied as follows:

$$GVA_s^{**} = \begin{bmatrix} v_s & 0 \end{bmatrix} \begin{bmatrix} I - [a_{ss} + c_{ss}\gamma^s v_s] & -a_{sr} \\ -a_{rs} - c_{rs}\gamma^s v_s & I - a_{rr} \end{bmatrix}^{-1} \begin{bmatrix} y_{ss} & 0 \\ y_{rs} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{14}$$

and the value added consumed abroad by final users is equal to:

$$VAXC_s = GVA_s - GVA_s^{**} \tag{15}$$

In the traditional IO framework, the hypothetical value added and value-added consumed abroad by final users ($\overline{VAXC}$) would be:

$$G\bar{VA}_s^{**} = \begin{bmatrix} v_s & 0 \end{bmatrix} \begin{bmatrix} I - a_{ss} & -a_{sr} \\ -a_{rs} & I - a_{rr} \end{bmatrix}^{-1} \begin{bmatrix} \bar{y_{ss}} & 0 \\ \bar{y_{rs}} & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{16}$$

$$\overline{VAXC_s} = \overline{GVA_s} - \overline{GVA_s}^{**} \tag{17}$$

Parallel to Eq. (13), the difference between the two approaches quantifies the value added explained by the private consumption linkages, or the *feedback of value added consumed abroad by final users* (*FC*):

$$FC_s = VAXC_s - \overline{VAXC_s} \tag{18}$$

According to Los and Timmer (2018), the hypothetical extraction enables the calculation of as many indicators as there are different components in the model's structure. This provides the option of analyzing the impact on domestic value added due to final exports or the *value added of final exports* (*VAXF*). This measurement may be useful for showing how a country's value added is influenced by factors that affect the behavior of external households, such as foreign consumers' preferences or foreign private income. All these questions may be relevant for countries that export genuinely local (final) products (e.g., agrarian goods or local handcraft) or those that occupy the final positions in the global production chain, i.e., just before the outputs are distributed for consumption (e.g., the assembly of machinery). Applied to *s*, the *VAXF* is obtained by assuming that the country's foreign final demand is zero:

$$GVA_s^{***} = \begin{bmatrix} v_s & 0 \end{bmatrix} \begin{bmatrix} I - [a_{ss} + c_{ss}\gamma^s v_s] & -a_{sr} \\ -a_{rs} - c_{rs}\gamma^s v_s & I - [a_{rr} + c_{rr}\gamma^r v_r] \end{bmatrix}^{-1} \begin{bmatrix} y_{ss} & 0 \\ y_{rs} & y_{rr} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{19}$$

$$VAXF_s = GVA_s - GVA_s^{***} \tag{20}$$

Similarly, the value added of final exports in the standard model ($\overline{VAXF}$) is calculated as follows:

$$\overline{GVA_s}^{***} = \begin{bmatrix} v_s & 0 \end{bmatrix} \begin{bmatrix} I - a_{ss} & -a_{sr} \\ -a_{rs} & I - a_{rr} \end{bmatrix}^{-1} \begin{bmatrix} \overline{y_{ss}} & 0 \\ \overline{y_{rs}} & \overline{y_{rr}} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} \tag{21}$$

$$\overline{VAXF_s} = \overline{GVA_s} - \overline{GVA_s}^{***} \tag{22}$$

Finally, the *feedback of value added of final exports* (*FF*) is the difference between the value added in the conventional and extended approaches:

$$FF_s = VAXF_s - \overline{VAXF_s}. \tag{23}$$

Figure 1 schematically compares the various measurements. The orange bars show the definitions in the extended model and the gray bars show the definitions in the conventional approach. Since the proposal is to add the feedback due to the income-consumption circuit,[12] the value added of exports (*VAXD*) is not quantitatively limited to gross exports as it is in KWW and LTV.[13] Indeed, the value added created by exports inside the economy can be greater than the strict value of exports, because the *VAXD* adds impacts circumscribed within the economy's borders that are not necessarily materialized in foreign exchanges but in domestic (production-to-consumption) inflows. This explains why Fig. 1 shows larger value-added impacts in the extended model since the inclusion of linkages beyond the production-related (input–output) connections triggers impulses beyond export statistics. These impacts mean that the upper bound for *VAXD* is determined by total value added (Fig. 1), so the contribution of export activity to domestic value added can add up, at most, to the total value added of the economy.

As reported in previous literature, the value added consumed abroad by final users ($\overline{VAXC}$) is lower than (or at least equal to) the $\overline{VAXD}$ because the former excludes the value added that, having previously crossed the border, returns to the original country embedded in goods that are ultimately consumed at home. Consequently, the two measurements could have the same value only if these back-and-forth exchanges did not exist.[14] The *VAXC* also has a larger value than the $\overline{VAXC}$ because of the higher impacts reflected in the proposed model.

Finally, the *VAXF* is the smallest of all the concepts because it is limited to quantifying the value added of final exports and therefore excludes the value added generated by intermediate exchanges. Again, the values are larger if the linkages between consumption and production are explicit in the model's structure.

Generalization to M countries and N sectors. Let us consider the extension of the model to *M* countries, each of which has *N* sectors that produce goods for both the domestic and the external markets. Production can be assigned to final demand or used as intermediate input. Each country can trade with all other countries. By adapting Eq. (5), we can write:

$$\begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_M \end{pmatrix} = \begin{pmatrix} I - A_{11}^1 - A_{11}^2 & -A_{12}^1 - A_{12}^2 & \cdots & -A_{1M}^1 - A_{1M}^2 \\ -A_{21}^1 - A_{21}^2 & I - A_{22}^1 - A_{22}^2 & \cdots & -A_{2M}^1 - A_{2M}^2 \\ \vdots & \vdots & \ddots & \vdots \\ -A_{M1}^1 - A_{M1}^2 & -A_{M2}^1 - A_{M2}^2 & \cdots & I - A_{MM}^1 - A_{MM}^2 \end{pmatrix}^{-1} \begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1M} \\ Y_{21} & Y_{22} & \cdots & Y_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{M1} & Y_{M2} & \cdots & Y_{MM} \end{pmatrix} \begin{pmatrix} I \\ I \\ \vdots \\ I \end{pmatrix}. \tag{24}$$

In this expression, the elements are blocks: x_M contain the $N \times 1$ sectoral production in each country, so the dimension of the resulting vector is $MN \times 1$; A_{MM}^1 is an $N \times N$ block of input–output inter-country coefficients and A_{MM}^2 is an $N \times N$ block of the $c_{MM}\gamma^M v_M$ coefficients, so the dimension of the inverse matrix in (24) is $MN \times MN$;[15] Y_{MM} are the $N \times 1$ elements of the consolidated final demand other than household consumption in each country, resulting in an $MN \times M$ matrix of exogenous demand. Finally, the right-hand side in Eq. (24) contains an $M \times 1$ vector, where I are unitary elements.

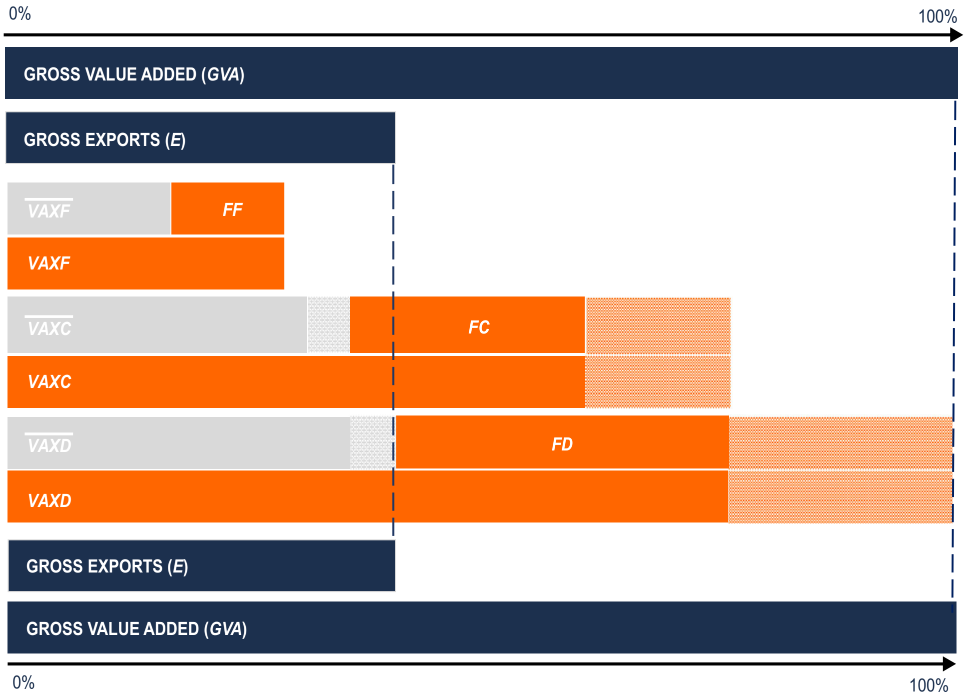

Fig. 1 Decomposition of value added in Country *s*. Author's elaboration.

			Use by country-sector							Private Consumption by country			Other Final Demand by country			Total Use
			Country 1			…	Country M			Country 1	…	Country M	Country 1	…	Country M	
			Sector 1	…	Sector N	…	Sector 1	…	Sector N							
Supply from country-sectors	Country 1	Sector 1		…		…		…			…			…		
		…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
		Sector N		…		…		…			…			…		
	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
	Country M	Sector1		…		…		…			…			…		
		…	…	…	…	…	…	…	…	…	…	…	…	…	…	…
		Sector N		…		…		…			…			…		
Value added (labor and capital)				…		…		…								
Gross Output				…		…		…								

*Adapted from Timmer et al. (2015).

Values to obtain A^1_{MM}; Values to obtain A^2_{MM}; Values of Y; Values to obtain v_M; Values to obtain A^1_{MM}, A^2_{MM} and v_M.

Fig. 2 Structure of the World Input-Output Database (WIOD)*. *Adapted from Timmer et al. (2015).

The gross value added in each country (GVA_M) is derived from:

$$\begin{pmatrix} GVA_1 \\ GVA_2 \\ \vdots \\ GVA_M \end{pmatrix} = \begin{pmatrix} \hat{v}_1 & 0 & \cdots & 0 \\ 0 & \hat{v}_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0\cdots & & \widehat{v_M} \end{pmatrix} \begin{pmatrix} I - A^1_{11} - A^2_{11} & -A^1_{12} - A^2_{12} & \cdots & -A^1_{1M} - A^2_{1M} \\ -A^1_{21} - A^2_{21} & I - A^1_{22} - A^2_{22} & \cdots & -A^1_{2M} - A^2_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ -A^1_{M1} - A^2_{M1} & -A^1_{M2} - A^2_{M2} & \cdots & I - A^1_{MM} - A^2_{MM} \end{pmatrix}^{-1} \begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1M} \\ Y_{21} & Y_{22} & \cdots & Y_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{M1} & Y_{M2} & \cdots & Y_{MM} \end{pmatrix} \begin{pmatrix} I \\ I \\ I \\ \vdots \\ I \end{pmatrix}, \quad (25)$$

<table>
<tr><td colspan="3" rowspan="3"></td><td colspan="7">Use by country-sector</td><td colspan="3">Private Consumption by country</td><td colspan="3">Other Final Demand by country</td><td rowspan="3">Total Use</td></tr>
<tr><td colspan="3">Country 1</td><td>…</td><td colspan="3">Country M</td><td rowspan="2">Country 1</td><td rowspan="2">…</td><td rowspan="2">Country M</td><td rowspan="2">Country 1</td><td rowspan="2">…</td><td rowspan="2">Country M</td></tr>
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<tr><td rowspan="7">Supply from country-sectors</td><td rowspan="3">Country 1</td><td>Sector 1</td><td colspan="3" rowspan="3">A^1_{11}</td><td rowspan="3">…</td><td colspan="3" rowspan="3">A^1_{1M}</td><td rowspan="3">A^2_{11}</td><td rowspan="3">…</td><td rowspan="3">A^2_{1M}</td><td rowspan="3">Y_{11}</td><td rowspan="3">…</td><td rowspan="3">Y_{1M}</td><td rowspan="3">x_1</td></tr>
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<tr><td>Sector N</td></tr>
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<tr><td rowspan="3">Country M</td><td>Sector1</td><td colspan="3" rowspan="3">A^1_{M1}</td><td rowspan="3">…</td><td colspan="3" rowspan="3">A^1_{MM}</td><td rowspan="3">A^2_{M1}</td><td rowspan="3">…</td><td rowspan="3">A^2_{MM}</td><td rowspan="3">Y_{M1}</td><td rowspan="3">…</td><td rowspan="3">Y_{MM}</td><td rowspan="3">x_M</td></tr>
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<tr><td>Sector N</td></tr>
<tr><td colspan="3">Value added (labor and capital)</td><td colspan="3">v_1</td><td>…</td><td colspan="3">v_M</td></tr>
<tr><td colspan="3">Gross Output</td><td colspan="3">x'_1</td><td>…</td><td colspan="3">x'_M</td></tr>
</table>

*Adapted from Timmer et al. (2015).

Fig. 3 Model's Coefficients from the WIOD structure*. *Adapted from Timmer et al. (2015).

where $\hat{v_M}$ are $N \times N$ diagonal matrices containing the ratios of sectoral value added to gross output in the main diagonal and zeros elsewhere for the M countries, thus configuring an $MN \times MN$ matrix of direct value-added coefficients.

The parameters on the right-hand side in expression (25) can be directly obtained from the available global input–output databases. For instance, using the structure of the World Input-Output Database (WIOD) (Fig. 2),[16] the model's components are calculated as follows. Final demand other than private consumption (matrix Y) is shown directly in the database (shaded blue in Fig. 2); the coefficients in matrices A^1_{MM} and A^2_{MM} are calculated by dividing the values of intermediate inter-country transactions (shaded gray in Fig. 2) and private demand (shaded pink in Fig. 2), respectively, by total output (shaded purple in Fig. 2); and the v_M coefficients are obtained by dividing value added (shaded green in Fig. 2) by total output. Specifically, Fig. 3 identifies the various models' coefficients corresponding to (i.e., obtained from) the elements in the WIOD.

In view of the model's structure in Eq. (25), the hypothetical extraction method can be used to individually cancel out any component of the model.[17] For example, when all exports in Country 1 (both intermediate and final) are set equal to zero, the aggregate domestic value added of exports ($VAXD_1$) is:

$N \times M$ elements of the final demand matrix and in the $N \times N$ coefficients of the A^1_{MM} and A^2_{MM} inter-country blocks.

Similarly, the value-added consumed abroad by final users ($VAXC$) and the value added by final exports ($VAXF$) in the general structure are calculated by adapting expressions (15) and (20) to the model defined in expression (25).

Numerical example. This section adapts the numerical example proposed by KWW to the extended methodological framework. Let us consider two countries that produce and export a unique good. Output, gross exports, and value added is identical in both countries. Specifically, the gross output in Country 1 is 200 and consists of 150 monetary units of intermediate goods (100 consumed domestically and 50 exported) and 50 monetary units of final goods (30 consumed domestically and 20 exported). Gross output in Country 2 is also 200 but consists of 50 monetary units of intermediate goods (consumed entirely at home) and 150 monetary units of final goods (80 consumed domestically and 70 exported).

The value added in Country 1 is equal to 100 (gross output$_1$ - domestic intermediate consumption$_1 = 200–100 = 100$), as is the value added in Country 2 (gross output$_2$—domestic intermediate consumption$_2$ - foreign intermediate consumption$_2 = 200–50–50 = 100$).

$$VAXD_1 = \begin{pmatrix} v_1 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} I - A^1_{11} - A^2_{11} & -A^1_{12} - A^2_{12} & \cdots & -A^1_{1M} - A^2_{1M} \\ -A^1_{21} - A^2_{21} & I - A^1_{22} - A^2_{22} & \cdots & -A^1_{2M} - A^2_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ -A^1_{M1} - A^2_{M1} & -A^1_{M2} - A^2_{M2} & \cdots & I - A^1_{MM} - A^2_{MM} \end{pmatrix}^{-1} \begin{pmatrix} Y_{11} & Y_{12} & \cdots & Y_{1M} \\ Y_{21} & Y_{22} & \cdots & Y_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{M1} & Y_{M2} & \cdots & Y_{MM} \end{pmatrix} \begin{pmatrix} I \\ I \\ \vdots \\ I \end{pmatrix} - \begin{pmatrix} v_1 & 0 & \cdots & 0 \end{pmatrix} \begin{pmatrix} I - A^1_{11} - A^2_{11} & 0 & \cdots & 0 \\ -A^1_{21} - A^2_{21} & I - A^1_{22} - A^2_{22} & \cdots & -A^1_{2M} - A^2_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ -A^1_{M1} - A^2_{M1} & -A^1_{M2} - A^2_{M2} & \cdots & I - A^1_{MM} - A^2_{MM} \end{pmatrix}^{-1} \begin{pmatrix} Y_{11} & 0 & \cdots & 0 \\ Y_{21} & Y_{22} & \cdots & Y_{2M} \\ \vdots & \vdots & \ddots & \vdots \\ Y_{M1} & Y_{M2} & \cdots & Y_{MM} \end{pmatrix} \begin{pmatrix} I \\ I \\ \vdots \\ I \end{pmatrix}. \quad (26)$$

Following a parallel procedure, the HEM could provide bilateral measures by canceling the vis-à-vis trade flows in the

In table form, the input–output relationships above have the following structure (Table 1):

According to LTV, the domestic value added of exports ($\overline{VAXD}$) is calculated as the difference between the total value added and the hypothetical value added if exports are completely removed from the model. With the information above, the components of the model are:

$$A^1 = \begin{bmatrix} a_{ss} & a_{sr} \\ a_{rs} & a_{rr} \end{bmatrix} = \begin{bmatrix} 0.5 & 0.25 \\ 0 & 0.25 \end{bmatrix}$$

$$\bar{Y} = \begin{bmatrix} \bar{y_{ss}} & \bar{y_{sr}} \\ \bar{y_{rs}} & \bar{y_{rr}} \end{bmatrix} = \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix}$$

and $v_s = v_r = 0.5$. Gross value added (GVA) is then equal to:

$$GVA_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 100$$

$$GVA_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 100$$

The $\overline{VAXD}$ is thus obtained as:

$$\overline{VAXD_1} = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 0 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 70,$$

$$\overline{VAXD_2} = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 0 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 46.67$$

In Country 1, the $\bar{VAXD_1}$ is equal to gross exports because all the intermediate goods used in production are domestic. In Country 2, the $\bar{VAXD_2}$ is lower than its exports (70) because it uses foreign intermediate goods (50), a fraction of which is ultimately embedded in its exports ($0.5 \times 0.67 \times 70 = 23.3$).

The value added consumed abroad by final users ($\overline{VAXC}$) quantifies the value added attributed to foreign final demand. This is calculated by canceling all the elements in the final demand of the importing country, as follows:

$$\overline{VAXC}_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 0 \\ 70 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 46.67$$

$$\overline{VAXC}_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 0 & 20 \\ 0 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 46.67$$

Unlike the $\overline{VAXD}$, the $\overline{VAXC}$ does not consider the value added initially exported as intermediate goods and finally returning home embedded in the final imports. For Country 1, this exclusion places the $\overline{VAXC}_1$ at a lower value (46.67) than the corresponding $\overline{VAXD}_1$ (70) because the fraction $0.5 \times 0.67 \times 70 = 23.3$ of its intermediate exports that returns through its final imports is not included. In contrast, Country 2 does not export intermediate goods, which is why the two measurements are identical ($\overline{VAXC}_2 = \overline{VAXD}_2$).

The value added of final exports ($\overline{VAXF}$) is given by:

$$\overline{VAXF}_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 0 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 20$$

$$\overline{VAXF}_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 70 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 0.67 \\ 0 & 1.33 \end{bmatrix} \begin{bmatrix} 30 & 20 \\ 0 & 80 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 46.67$$

The $\overline{VAXF}_1$ is equal to Country 1's final exports (20) because no foreign value added (i.e., foreign intermediate goods) is used to obtain these exported goods. Unsurprisingly, the $\overline{VAXF}_2$ is equal to the value added of exports ($\overline{VAXD}_2$), since exports in Country 2 are limited to final goods.

Note that although the two countries have identical values for production, domestic value added, and gross exports, a different trade structure leads to a nonidentical value-added contribution to exports. Specifically, if we assume that Country 1 does not use foreign intermediate goods while Country 2 does, the value added of exports in Country 1 is larger than it is in Country 2.

Now let us consider that private demand is an element of the structural coefficients of the model. This assumption requires additional information, namely, the amount of final demand that corresponds to household consumption. If we assume that half of the final demand values are private consumption in both countries, and that value added is completely converted into household income in both countries (i.e., $\gamma^s = \gamma^r = 1$), the system of accounts can be represented as (Table 2):

Table 1 Input–output (open) system.

	Intermediate demand		Final demand		Output
	Country 1	Country 2	Country 1	Country 2	
Country 1	100	50	30	20	200
Country 2	0	50	70	80	200
Value added	100	100			
Output	200	200			

Table 2 Input–output closed system ($\gamma^s = \gamma^r = 1$).

		Intermediate demand		Household consumption		Final demand (without household consumption)		Output
		Country 1	Country 2	Country 1	Country 2	Country 1	Country 2	
Intermediate demand	Country 1	100	50	15	10	15	10	200
	Country 2	0	50	35	40	35	40	200
Household income	Country 1	100	0	0	0	0	0	100
	Country 2	0	100	0	0	0	0	100
	Savings	0	0	50	50			
	Output	200	200	100	100			

The matrix of private consumption coefficients (A^2) and the final demand matrix (Y) are equal to:

$$A^2 = \begin{bmatrix} c_{ss}\gamma^s v_s & c_{sr}\gamma^r v_r \\ c_{rs}\gamma^s v_s & c_{rr}\gamma^r v_r \end{bmatrix} = \begin{bmatrix} \frac{15}{100}\times 1\times \frac{100}{200} & \frac{10}{100}\times 1\times \frac{100}{200} \\ \frac{35}{100}\times 1\times \frac{100}{200} & \frac{40}{100}\times 1\times \frac{100}{200} \end{bmatrix} = \begin{bmatrix} 0.075 & 0.05 \\ 0.175 & 0.2 \end{bmatrix}$$

$$Y = \begin{bmatrix} y_{ss} & y_{sr} \\ y_{rs} & y_{rr} \end{bmatrix} = \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix}$$

Note that $v_s = \frac{m_s}{\gamma^s x_s} = 0.5$ and $v_r = \frac{m_r}{\gamma^r x_r} = 0.5$, and therefore the (initial) gross value added is maintained:

$$GVA_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 3.03 & 1.66 \\ 0.97 & 2.34 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 100$$

$$GVA_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 3.03 & 1.66 \\ 0.97 & 2.34 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 100$$

On the other hand, the new structure modifies the value added of exports as follows:

$$VAXD_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 3.03 & 1.66 \\ 0.97 & 2.34 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2.35 & 0 \\ 0.75 & 1.82 \end{bmatrix} \begin{bmatrix} 15 & 0 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 82.35$$

$$VAXD_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 3.03 & 1.66 \\ 0.97 & 2.34 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2.35 & 1.28 \\ 0 & 1.82 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 0 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 63.64$$

The feedback value added of exports (FD) is obtained from the differences between the standard and the extended model:

$$FD_1 = VAXD_1 - \overline{VAXD_1} = 82.35 - 70 = 12.35$$

$$FD_2 = VAXD_2 - \overline{VAXD_2} = 63.64 - 46.67 = 16.97$$

These are measurements of how much value added can be attributed to exports when the interdependencies between production and household consumption are explicitly defined in the model. Interestingly, the feedback measurements show a higher extra stimulus in Country 2 even though that country has a lower (absolute) value added to exports. This is because, for an identical structure of income distribution to households (i.e., an equal fraction of value added transformed into income multiplied by the value added coefficients), the propensity to consume products from Country 2 ($c_{21}\gamma^1 v_1 + c_{22}\gamma^2 v_2 = 0.375$) is higher than it is to consume products from Country 1 ($c_{11}\gamma^1 v_1 + c_{12}\gamma^2 v_2 = 0.125$).

Similarly, the value-added consumed abroad by final users ($VAXC$) in the extended model is obtained as:

$$VAXC_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 3.03 & 1.66 \\ 0.97 & 2.34 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2.73 & 0.91 \\ 0.64 & 1.55 \end{bmatrix} \begin{bmatrix} 15 & 0 \\ 35 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 63.64$$

$$VAXC_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 3.03 & 1.66 \\ 0.97 & 2.34 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 1.09 \\ 0 & 1.82 \end{bmatrix} \begin{bmatrix} 0 & 10 \\ 0 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 63.64$$

And the feedback impacts are now equal to:

$$FC_1 = VAXC_1 - \overline{VAXC_1} = 63.64 - 46.67 = 16.97$$

$$FC_2 = VAXC_2 - \overline{VAXC_2} = 63.64 - 46.67 = 16.97$$

The $VAXC$ are identical in both countries for two reasons. First, since this concept is limited to considering final demand, it excludes Country 1's exported value added that returns home embedded in imports. Second, since all foreign final demand is removed from the model and both countries have equal private consumption propensities ($c_{11}\gamma^1 v_1 + c_{21}\gamma^1 v_1 = c_{12}\gamma^2 v_2 + c_{22}\gamma^2 v_2 = 0.25$), the value added attributed to foreign final consumption coincides in both countries.

The value added of final exports ($VAXF$) is obtained as:

$$VAXF_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 3.03 & 1.66 \\ 0.97 & 2.34 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2.89 & 1.32 \\ 0.92 & 2.24 \end{bmatrix} \begin{bmatrix} 15 & 0 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 28.95$$

$$VAXF_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 3.03 & 1.66 \\ 0.97 & 2.34 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2.35 & 1.28 \\ 0 & 1.82 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 0 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 63.64$$

and the corresponding feedback is:

$$FF_1 = VAXF_1 - \overline{VAXF_1} = 28.95 - 20 = 8.95$$

$$FF_2 = VAXF_2 - \overline{VAXF_2} = 63.64 - 46.67 = 16.97$$

which again shows larger impacts in Country 2 because of the higher propensity to consume products from that country. This higher propensity determines a stronger consumption circuit than in Country 1.

At this point, we should mention that the model's ability to generate production increases is determined by the proportion of household demand in relation to total uses. For instance, let us assume that private consumption is 99% of final demand. In this situation, the matrices of consumption coefficients (A^2) and final demand matrix (Y) are equal to:

$$A^2 = \begin{bmatrix} c_{ss}\gamma^s v_s & c_{sr}\gamma^r v_r \\ c_{rs}\gamma^s v_s & c_{rr}\gamma^r v_r \end{bmatrix} = \begin{bmatrix} 0.297\times 1\times 0.5 & 0.198\times 1\times 0.5 \\ 0.693\times 1\times 0.5 & 0.792\times 1\times 0.5 \end{bmatrix} = \begin{bmatrix} 0.1485 & 0.099 \\ 0.3465 & 0.396 \end{bmatrix}$$

$$Y = \begin{bmatrix} y_{ss} & y_{sr} \\ y_{rs} & y_{rr} \end{bmatrix} = \begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \end{bmatrix}$$

This modifies the value added of exports to:

$$VAXD_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 101.07 & 99.64 \\ 98.93 & 100.36 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} - \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 2.85 & 0 \\ 2.78 & 2.82 \end{bmatrix} \begin{bmatrix} 0.3 & 0 \\ 0.7 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 99.57$$

$$VAXD_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 101.07 & 99.64 \\ 98.93 & 100.36 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$- \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2.85 & 2.80 \\ 0 & 2.82 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 98.87$$

Note that as private consumption becomes larger, the $VAXD$ approximates to total value added (100) and the induced impacts are also higher:

$$FD_1 = VAXD_1 - \overline{VAXD_1} = 99.57 - 70 = 29.57$$

$$FD_2 = VAXD_2 - \overline{VAXD_2} = 98.87 - 46.67 = 52.20$$

In parallel with the previous calculations, the $VAXC$ and $VAXF$ are obtained from:

$$VAXC_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 101.07 & 99.64 \\ 98.93 & 100.36 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$- \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 4.24 & 1.41 \\ 1.96 & 1.99 \end{bmatrix} \begin{bmatrix} 0.3 & 0 \\ 0.7 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 98.87$$

$$VAXC_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 101.07 & 99.64 \\ 98.93 & 100.36 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$- \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2 & 1.97 \\ 0 & 2.82 \end{bmatrix} \begin{bmatrix} 0 & 0.2 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 98.87$$

$$VAXF_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 101.07 & 99.64 \\ 98.93 & 100.36 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$- \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 9.36 & 6.61 \\ 9.17 & 9.30 \end{bmatrix} \begin{bmatrix} 0.3 & 0 \\ 0.7 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 93.64$$

$$VAXF_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 101.07 & 99.64 \\ 98.93 & 100.36 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 \\ 0.7 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$
$$- \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 2.84 & 2.80 \\ 0 & 2.82 \end{bmatrix} \begin{bmatrix} 0.3 & 0.2 \\ 0 & 0.8 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 98.87$$

where the feedback values are:

$$FC_1 = VAXC_1 - \overline{VAXC_1} = 98.87 - 46.67 = 52.20$$

$$FC_2 = VAXC_2 - \overline{VAXC_2} = 98.87 - 46.67 = 52.20$$

$$FF_1 = VAXF_1 - \overline{VAXF_1} = 93.64 - 20 = 73.64$$

$$FF_2 = VAXF_2 - \overline{VAXF_2} = 98.87 - 46.67 = 52.20$$

Another element of the model that can be modified is the fraction of value added that is converted into household income. By assuming, for instance, that half of the value added is attributed to households in both countries, we have $\gamma^s = \gamma^r = 0.5$. In table format, the closed input–output system would then be as follows (Table 3):

The matrix of private consumption coefficients (A^2) and the final demand matrix (Y) are now equal to:

$$A^2 = \begin{bmatrix} c_{ss}\gamma^s v_s & c_{sr}\gamma^r v_r \\ c_{rs}\gamma^s v_s & c_{rr}\gamma^r v_r \end{bmatrix} = \begin{bmatrix} \frac{15}{50} \times 0.5 \times \frac{100}{200} & \frac{10}{50} \times 0.5 \times \frac{100}{200} \\ \frac{35}{50} \times 0.5 \times \frac{100}{200} & \frac{40}{50} \times 0.5 \times \frac{100}{200} \end{bmatrix}$$
$$= \begin{bmatrix} 0.075 & 0.05 \\ 0.175 & 0.2 \end{bmatrix}$$

$$Y = \begin{bmatrix} y_{ss} & y_{sr} \\ y_{rs} & y_{rr} \end{bmatrix} = \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix}$$

The value-added coefficients have not changed: $v_s = \frac{m_s}{\gamma^s x_s} = 0.5$ and $v_r = \frac{m_r}{\gamma^r x_r} = 0.5$. Also, the (initial) gross value added is maintained:

$$GVA_1 = \begin{bmatrix} 0.5 & 0 \end{bmatrix} \begin{bmatrix} 3.03 & 1.66 \\ 0.97 & 2.34 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 100$$

$$GVA_2 = \begin{bmatrix} 0 & 0.5 \end{bmatrix} \begin{bmatrix} 3.03 & 1.66 \\ 0.97 & 2.34 \end{bmatrix} \begin{bmatrix} 15 & 10 \\ 35 & 40 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 100$$

Note also that, for the two countries, the $VAXD$, $VAXC$, and $VAXF$ and the FD, FC, and FF would be identical to the preceding ones, despite the different assumption in relation to the determination of household income.

This simple example illustrates quantitative differences between the two approaches due to the greater ability of exports to create domestic value added in the extended model. Another finding is that the nature of the interdependencies added to the traditional model, especially the strength of the income distribution mechanism and the weight of private demand, determines how large the additional impulses due to exports are.

Empirical application

This empirical application uses the latest available version of the World Input-Output Database (WIOD), which is for 2014. In its original form, the WIOD covers 56 sectors from 43 countries plus a residual Rest of the World (ROW). The original database has been aggregated to show six selected countries/regions (China, the United States, the European Union -EU27-,[18] Japan, Brazil, and Australia) and the ROW. The resulting database provided all the data (i.e., inter-country input–output transactions, sectoral value added, and final demand) required in the empirical analysis.

For an overview of world trade characteristics, Table 4 shows various aggregate statistics directly obtained from the WIOD. Two main results can be seen in this table. First, intermediate exports predominate, especially in Australia, Brazil, and the ROW, which show the highest contributions from intermediate goods to gross exports (84.4, 74.2, and 71.8%, respectively). To be more precise, however, China's intermediate and final exports are roughly equal (49.9 versus 50.1%). Second, in all countries without exception, intermediate imports are higher than final imports. Particular mention should again be

Table 3 Input-output closed system ($\gamma^s = \gamma^r = 0.5$).

		Intermediate demand		Household consumption		Final demand (without household consumption)		Output
		Country 1	Country 2	Country 1	Country 2	Country 1	Country 2	
Intermediate demand	Country 1	100	50	15	10	15	10	200
	Country 2	0	50	35	40	35	40	200
Household income	Country 1	50	0	0	0	0	0	50
	Country 2	0	50	0	0	0	0	50
	Savings	50	50	0	0			
	Output	200	200	50	50			

Table 4 Aggregate trade data, 2014.

	Exports (million USD)			Imports (million USD)			E/GVA (%)	M/GVA (%)
	Intermediate goods	Final goods	Gross exports (E)	Intermediate goods	Final goods	Gross imports (M)		
EU27	1,865,900 (56.5%)	1,437,087 (43.5%)	3,302,987	1,753,846 (69.4%)	773,598 (30.6%)	2,527,444	23.6%	17.9%
China	1,212,162 (49.9%)	1,213,302 (50.1%)	2,425,464	1,374,500 (74.6%)	468,337 (25.4%)	1,842,837	11.1%	13.9%
United States	1,260,788 (65.4%)	666,304 (34.6%)	1,927,091	1,390,078 (57.7%)	1,017,590 (42.3%)	2,407,668	23.7%	18.1%
Japan	491,126 (60.1%)	326,388 (39.9%)	817,514	614,084 (69.0%)	275,269 (31.0%)	889,353	18.4%	20.0%
Australia	242,448 (84.4%)	44,713 (15.6%)	287,162	162,257 (56.1%)	126,733 (43.2%)	288,990	13.0%	15.3%
Brazil	200,562 (74.2%)	69,701 (25.8%)	270,263	217,259 (68.5%)	100,084 (31.5%)	317,343	21.2%	21.3%
ROW	3,821,212 (71.8%)	1,503,581 (28.2%)	5,324,793	3,582,174 (58.9%)	2,499,465 (41.1%)	6,081,639	21.8%	24.9%
Total	9,094,198 (63.4%)	5,261,077 (36.6%)	14,355,275	9,094,198 (63.4%)	5,261,077 (36.6%)	14,355,275	19.4%	19.4%

Source: Author's calculations based on the WIOD.

made of China, where intermediate imports show the highest relevance (almost three quarters (74.6%) of total imports).

The figures on the right-hand side of Table 4 reveal interesting differences in the relative importance of trade in the various countries. China shows the lowest ratios for both gross exports (E) and gross imports (M) in relation to value added (specifically 11.1% for E/GVA and 13.9% for M/GVA). Conversely, the United States shows the highest percentage of exports in relation to value added (23.7%) followed by the EU27 (23.6%). With regard to import ratios, Table 4 shows that the ROW, Brazil and Japan display the highest values (24.9, 21.3, and 20.0%, respectively). Although this information is extremely aggregated and hides detailed specificities, such as the categories of goods traded or the countries involved in transactions, it illustrates a notorious heterogeneity in the role played by trade in the various countries.

At the global level, the final row in Table 4 illustrates the predominance of intermediate exchanges (63.4%) compared to final exchanges (36.6%). This aggregate measurement is a simple way to grasp the complexity of production processes and the product-fragmented nature of international trade, with large movements of inputs and semi-elaborated products (i.e., intermediate transactions) moving across the world.

Table 5 summarizes the measurements described in the section "Value added of trade: an extended input–output model". These results are obtained by assuming that all value added is converted into household income, thereby assuming that the fraction γ^M in all countries is equal to 1.[19]

The ratio of the value added of exports to gross exports ($\overline{VAXD}/E$) in the input–output model provides parallel results to previous contributions that used an identical framework and/or an identical database. In Table 5, individual values fall within a narrow interval between the highest (86.5%) for the United States and the lowest (77.1%) for Brazil.

The value added consumed abroad by final users ($\overline{VAXC}$) shows lower ratios than the $\overline{VAXD}$ although, as KWW and Los and Timmer (2018) showed, the values of both indicators are very similar. Indeed, any differences are limited to small quantitative figures. Also, compared to $\overline{VAXD}$, the values fluctuate within a small interval while the ranking of countries is identical.

The value added of final exports ($\overline{VAXF}$) is led by China (27.4% of gross exports and 6.5% of value added), which confirms the important role played by foreign final demand in that country due to China's specialization in occupying end-stages within the global production chains (see Table 4). On the other hand, the lowest value added of final exports in Australia and Brazil confirms their specialization in exporting intermediate products that are ultimately transformed outside their economies.

Table 5 also shows the results of the extended model. Unlike the information provided by the IO framework, which is interpreted as the amount of domestic value added contained in a country's exports, the $VAXD$ should be interpreted as the effective (total) contribution of exports to create value added within the economy. Unsurprisingly, the percentages in Table 5 exceed gross exports in all countries because, as was described in section "Introduction", the model quantifies further interactions other than the input–output connections. Owing to the model's structure, exports are the activation mechanism that triggers a more complete (beyond-production) set of impacts on value added. From Table 5, we see that Brazil and the United States lead the $VAXD$ ratios, with percentages of 231.9 and 214.4% of gross exports, respectively (30.2 and 23.8% of value added). The figure for the United States, for example, should be interpreted as follows: American exports are generating an amount of domestic value added that is 2.14 times the figure for gross exports. Interestingly, China has the lowest percentage of all countries (115.6% of Chinese gross exports). It is also interesting that the range of values for the $VAXD/E$ is extended in comparison with the previous (conventional) values. These findings illustrate huge disparities in individual trade impacts on domestic production and offer a complementary view to the standard model.

The differences between the two models synthesize the impacts on value added due to income-consumption linkages. The bottom of Table 5 shows that household consumption channels in the USA and Brazil generate important production inflows (127.9% of gross exports in the United States and 154.8% in Brazil). Also of note is the weak feedback for the value added of exports in China which, at 33.9% of gross exports, is the lowest in Table 5. Finally, an intermediate situation is obtained by the other countries: Japan (83.5%), Australia (81.4%), the EU27 (78.7%), and the ROW (77.6%).

Table 5 Value added of exports, 2014.

	EU27	China	United States	Japan	Australia	Brazil	ROW
Input–output model							
$\overline{VAXD}/E$	78.8%	81.7%	86.5%	74.4%	83.7%	77.1%	81.0%
$\overline{VAXD}/GVA$	18.7%	19.3%	9.6%	13.7%	17.7%	10.1%	17.7%
$\overline{VAXC}/E$	76.5%	79.6%	83.0%	73.6%	83.2%	76.7%	71.4%
$\overline{VAXC}/GVA$	18.1%	18.8%	9.2%	13.6%	17.6%	10.0%	15.6%
$\overline{VAXF}/E$	21.7%	27.4%	19.9%	18.3%	10.9%	14.3%	15.0%
$\overline{VAXF}/GVA$	5.1%	6.5%	2.2%	3.4%	2.3%	1.9%	3.3%
Extended model							
$VAXD/E$	157.5%	115.6%	214.4%	157.9%	165.2%	231.9%	158.6%
$VAXD/GVA$	37.3%	27.3%	23.8%	29.1%	34.9%	30.2%	34.6%
$VAXC/E$	154.6%	112.5%	210.3%	157.9%	164.5%	231.2%	146.3%
$VAXC/GVA$	36.7%	26.5%	23.4%	29.1%	34.8%	30.2%	32.0%
$VAXF/E$	53.1%	42.8%	56.0%	41.0%	30.4%	115.5%	44.7%
$VAXF/GVA$	12.6%	10.1%	6.2%	7.6%	6.4%	15.1%	9.8%
Feedback impacts							
FD/E	78.7%	33.9%	127.9%	83.5%	81.4%	154.8%	77.6%
FD/GVA	18.7%	8.0%	14.2%	15.4%	17.2%	20.2%	17.0%
FC/E	78.1%	32.9%	127.3%	84.3%	81.3%	154.5%	74.9%
FD/GVA	18.5%	7.8%	14.1%	15.5%	17.2%	20.2%	16.4%
FF/E	31.5%	15.4%	36.1%	22.7%	19.5%	101.2%	29.6%
FF/GVA	7.5%	3.6%	4.0%	4.2%	4.1%	13.2%	6.5%
C/D	53.9%	37.1%	66.5%	56.5%	55.0%	57.7%	55.8%
C/GVA	53.3%	35.4%	68.6%	58.1%	56.1%	64.3%	58.9%

Source: Author's calculations based on the WIOD.

To understand the reasons for these asymmetrical feedback effects, the bottom rows in Table 5 show the ratio between private consumption (C) and final demand (D) on the one hand and the ratio between private consumption and value added on the other. The (relatively) low values for China (37.1% of final demand and 35.4% of value added, respectively) compared to the other countries and especially the United States (66.5% of final demand and 68.6% of value added) clearly reflect the different nature of private consumption and the different ability to expand production.

The extension of the model suggests that a high (low) accounting value added of a country's gross exports per se is not necessarily aligned with the strong (weak) economic potential of exports to cause an impact on domestic production. A broader analysis of trade statistics that captures propagation mechanisms other than production-related links clarifies the role actually played by trade in individual economies.

Conclusions

Numerous papers have proposed a framework for disentangling the value added embodied in gross exports by adopting an accounting perspective of trade statistics. These contributions have considered the production system and the links between inputs and outputs across sectors and countries. However, inter-industry linkages are just one of the economic interdependences that play a role in open economies. Households receive income for their endowments of primary inputs, particularly labor, which activates a chain of impacts throughout the link between production and the institutional sphere of the economy. Production shocks are therefore not limited to causing effect within the production system since they are spread out among the other economic agents, thus increasing income and consumption and returning to production in the form of larger (multiplied) impacts. The final impacts are therefore greater than the initial inflows because the within-production multiplier values are completed with the income-consumption (outside-production) circular multiplier impacts.

This paper provides a novel framework for calculating the value added of exports by considering economic interdependencies other than inter-industry linkages. Specifically, the proposal allows the feedback between production, income, and consumption to be captured in the model.

Some interesting aspects deserve special attention. First, the proposed method overcomes the traditional input–output weakness of an exogenous (invariable) consumption when the model is used for trade analysis. Second, the outcomes in this paper offer new insights into the importance of trade in domestic economies, showing quantitative increases in the value added that must effectively be attributed to gross exports. Third, this approach also offers qualitative outcomes that, by changing the relative importance of national economies in the global trade portrait, demonstrate a different ability of the income-consumption channel to magnify the economic impact of exports. Fourth, the method can be directly applied at the empirical level by using available global inter-country input–output statistics. Finally, since the approach offers a new way of interpreting trade statistics, it may improve decision-making in areas such as trade policy, industrial policy, and economic planning. In particular, the outcomes in the paper suggest that foreign shocks may have been under-measured in the literature since the ability of trade to increase domestic income is greater than what export data truly measure. They may also indicate that trade liberalization may have a greater potential to create income than that which is generally predicted when the economic impact of trade includes the role attributed to household consumption.

To increase understanding of the complexity of trade flows and their impact at the national level, it is crucial to have knowledge of the underlying forces through which trade causes impacts within the economy. Precise and comprehensive frameworks should be available to authorities and trade analysts to determine how and, especially by how much, trade activity affects individual countries. With a tractable method, this article reconciles the study of the economic impact of trade and the global databases available to provide results beyond what trade statistics strictly reflect.

Despite the advantages of this approach, it should also be acknowledged that the rigidity of the relationship between income and consumption places the model's impacts at the upper limit of the possible effects. Equally, the input–output structure does not enable substitution between foreign and domestic final consumption or between foreign and domestic intermediate goods to be reflected. By combining these aspects with the informative richness and treatability of the input–output structure, a promising avenue for future research can be explored.

It should also be noted that the outcomes in this paper are limited to production indicators (i.e., value added) and do not provide other important consequences of trade, such as welfare impacts. Bearing this limitation in mind, extending the methods in this paper to welfare analysis will also merit future attention and future research efforts.

Data availability

The dataset used during this study is publicly available at: https://www.rug.nl/ggdc/valuechain/wiod/?lang=en.

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Notes

1 Among these contributions, Feenstra (1998) evaluated the implications of globalization on wages and employment for various industrial countries. Hummels et al. (2001) studied product fragmentation for major trading countries both conceptually and empirically. Yi (2003) introduced product fragmentation in a dynamic Ricardian model to reconcile the empirical evidence with the model's conclusions in relation to the link between tariff barriers and trade volume.

2 The input–output model was first proposed by Leontief (1936). See Miller and Blair (2022) for a complete description of the model and its subsequent extensions since Leontief's pioneering contribution.

3 Other recent papers evaluating the value-added content of trade through input–output relations are those by Borin and Mancini (2017), Wang et al. (2018), Arto et al. (2019), and Miroudot and Ye (2020, 2021). Alternatively, Kee and Tang (2016) adopted a micro-level perspective to include firm heterogeneity in the calculation of the value added in China's exports.

4 An extension of the model to include more countries and sectors is presented in the Appendix.

5 Note that a unitary value for γ^s (i.e., $\gamma^s = 1$) completely allocates value added to households, therefore assuming that all factors of production are owned by private consumers. Although this is an extreme situation, it should be noted that the fraction of value added that corresponds to private income does not affect the results. By defining h_{ss} and h_{sr} as total household consumption, it follows that $c_{ss}\gamma^s v_s = \frac{h_{ss}}{\gamma^s v_s x_s}\gamma^s v_s = \frac{h_{ss}}{x_s}$ and $c_{sr}\gamma^r v_r = \frac{h_{sr}}{\gamma^r v_r x_r}\gamma^r v_r = \frac{h_{sr}}{x_r}$. In other words, the product of consumption coefficients and value-added coefficients is equivalent to the quotient between sectoral household consumption and sectoral output, so the fraction of value added that makes up private income does not alter the results.

6 Considering households in the endogenous part of the IO model (i.e., moving the household sector from final demand to the technical coefficients matrix) is known as 'closing the model with respect to households' (Miller and Blair (2022), page 35). It involves adding a new row and a new column for the new (household) sector. For the sake of simplicity, the model in Eq. (4) does not include an additional equation for consumption and is therefore limited to showing the output relationships of the Miyazawa model.

7 This method was originally proposed by Paelink et al. (1965), Strassert (1968) and Schultz (1977).

8 LTV used the terms 'value added in exports' to define how much (accounting) value added is actually embedded by a country in gross exports. Since this proposal calculates how much value added is generated within an economy due to its export activity, the name is modified to 'value added of exports'.

9 Since the elements in Y are equal to the difference between the total final demand minus private consumption, a (significant) reduction in total final demand could lead to negative values in the (modified) elements of Y. Although this situation is certainly possible in the model, it is unlikely to occur at the empirical level since it would require a higher reduction in the corresponding total final demand than the reference (pre-change) value in Y. Moreover, as the model is designed to cancel out exports, and no other reduction will be applied, the empirical analysis will never involve a higher reduction than the existing demand other than private consumption.

10 Johnson and Noguera (2012, 2017) defined this concept as 'value-added exports'. To avoid possible confusions with the other measurements, LTV defined the $VAXC$ as ' value-added absorbed abroad' while Los and Timmer (2018) defined it as 'value-added consumed abroad'. In this paper, I further clarify this concept to specify that it shows the 'value added consumed abroad by final users'.

11 See LTV.

12 Note that, from Eq. (6), the two models would be identical if all elements in matrix A^2 were null and the elements in the matrix of final demand were equal in both approaches (i.e., $y_{ss} = \overline{y_{ss}}$, $y_{sr} = \overline{y_{sr}}$, $y_{rs} = \overline{y_{rs}}$, and $y_{rr} = \overline{y_{rr}}$). In practice, this would correspond to an unreal situation of null private demand.

13 From an accounting perspective, this is a fundamental difference in relation to the previous approaches.

14 At the empirical level, KWW and LTV showed that the differences between $\overline{VAXD}$ and $\overline{VAXC}$ are small.

15 A typical element $c_{NN}\gamma^N v_N$, within the $M \times M$ blocks in A^2_{MM}, describes the fraction of income in sector N used for final consumption in that sector multiplied by the fraction of sectoral value added with respect to the output of this sector. As described earlier, this product of coefficients is equivalent to the quotient between sectoral household consumption and output in each sector N.

16 See Timmer et al. (2015) for details on the construction and structure of this database.

17 See Los and Timmer (2018) for an integrated analysis of both bilateral and aggregate measurements in the standard input–output framework.

18 The European Union comprises the 27 countries that are currently part of the Union.

19 Although value-added includes incomes belonging to other agents (not only to households), the fraction of value added converted into income does not affect the model's results (see footnote 4 and the empirical example in the Appendix).

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