Humanities & Social Sciences Communications

REVIEW ARTICLE

https://doi.org/10.1057/s41599-023-01718-7 OPEN

Check for updates

Learning to teach through noticing: a bibliometric review of teacher noticing research in mathematics education during 2006–2021

Yicheng Wei [1], Qiaoping Zhang [1✉], Jing Guo [1] & Min Chen [2,3]

Teacher noticing, as an expertize of teacher competency, has received increased attention by researchers in recent years. This bibliometric review systematically compares the current research status, research hotspots, and development trends of teacher noticing in mathematics education. It provides a reference for the future research development on mathematics teacher noticing. Articles related to mathematics teacher noticing in the Scopus online database were selected for the period 2006–2021. CiteSpace, R, and other bibliometric softwares were used to analyze the number of publications, research strengths of countries and institutions, core journals, keyword co-occurrences, and co-citation analyses. The results indicated that the number of mathematics teacher noticing publications is increasing, but as yet the total body of literature is too small and has many gaps. By country, the United States is at the forefront of research in this area and, by institutions, UC Irvine and Northwestern University are the leaders. Among the reviewed studies, framework is the main research theory in this field; the use of videos is the main research instrument; prospective teachers are the main research participants; and developing teachers' teaching skills is the main research aim. International or regional cooperation is relatively lacking. Considering the lack of richness in the current perspectives, this review has also offered suggestions for future research directions.

Introduction

Teacher noticing is a crucial component of a teacher's professional competence. To notice students' thinking is fundamental to teaching or learning to teach (González & Vargas, 2020). Recently, König, Santagata, Scheiner, Adleff, Yang and Kaiser (2022) have conducted a systematic review of the literature on teacher noticing published over the past two decades; and they reviewed the development of the concept of noticing, arguing that most of the past studies had been based on small samples and lacked generalizability. The current conclusions are not universal due to the lack of rich, diverse perspectives and cultural dimensions. From a specific perspective of research methodology, Amador, Bragelman and Superfine (2021)

[1] Department of Mathematics and Information Technology, The Education University of Hong Kong, 10 Lo Ping Road, Tai Po, New Territories, Hong Kong SAR, China. [2] Zhejiang New Ideas Institute of Education Sciences, 17 Zhitou Lane, Shangcheng District, HangZhou, China. [3] College of Teacher Education, East China Normal University, 3663 North Zhongshan Road, Shanghai, China. ✉email: zqiaoping@eduhk.hk

provided a more in-depth summary of past research on teacher noticing, suggesting that most studies have used one of the two main noticing frameworks (*Professional Noticing* or *Learning to Notice*). The use of these two frameworks has led to different conclusions, with the *Learning to Notice* framework reporting more positive results than the *Professional Noticing* framework (Amador et al., 2021). All the previous reviews have attempted to summarize and review existing research on teacher noticing with different focuses, and thus provided some suggestions for future research. However, there are still certain gaps, such as a lack of analysis of the trends in this research field, research results and connections of different countries and institutions, relevant journals, hot keywords, and important literature. As König et al. (2022) stated, most of the current research on teacher noticing is about mathematics teaching and learning. Focusing on mathematics can lead to more detailed and accurate findings, and can also, from a more comprehensive perspective, guide the future direction of teacher noticing research. There is a need for a more thorough understanding about the state-of-the-art research and development trends in mathematics teacher noticing. Therefore, in this article, we employed the approach of the bibliometric review (Hawkins, 2001; Linnenluecke et al., 2020), to analyze systematically the existing works focusing on teacher noticing in mathematics education; by sorting out the research progress, drawing the visual knowledge map of the relevant literature, and visually revealing the progress of mathematics teacher noticing research and its development trends in this filed.

Literature review

Definition of teacher noticing. Various scholars have proposed different definitions of teacher noticing. Teacher noticing has its origin in Goodwin's (1994) professional vision of coding, highlighting, and producing material representations; it consists "of socially organized ways of seeing and understanding events that are answerable to the distinctive interests of a particular social group" (p. 606). Regarding mathematics education, three definitions of teacher noticing are dominant in the literature (Amador et al., 2021). The first was from Mason's (2002) work, which defined teachers' professional noticing as "what we do when we watch someone else acting professionally and become aware of something that they do which we think we could use ourselves" (p. 30). The second definition is given by van Es and Sherin (2002, 2008). In their studies, van Es and Sherin (2002) defined teacher noticing as "(a) identifying what is important or noteworthy about a classroom situation; (b) making connections between the specifics of classroom interactions and the broader principles of teaching and learning they represent; (c) using what one knows about the context to reason about classroom interactions" (p. 573). The third definition of teacher noticing was conceptualized by Jacobs, Lamb and Philipp (2010) as "a set of three interrelated skills: attending to children's strategies, interpreting children's understanding, and deciding how to respond based on children's understandings" (p. 171). Generally, most definitions of teacher noticing include two key components: attention to what is noticed (attending), and interpretation of what is attended to (interpreting). Some have a third step of the noticing process, which relates to the observer's actions. Similarly, Kaiser and her research team (Kaiser et al., 2015) proposed that teacher noticing shall consist of perceiving typical events in an instructional setting, interpreting the perceived activities in the classroom, and decision-making—either as anticipating a response to students' activities or as proposing alternative instructional strategies. These three aspects of teacher noticing are situation-specific skills that complement the knowledge-based construct of teachers' competence. It is notable that research on teacher noticing is not only about what participating teachers notice and how they notice, but about what they will do in their instruction, which is considered essential for teachers' professional development—to be reflective practitioners.

Analytical frameworks for mathematics teachers noticing. Research on teacher noticing not only needs to investigate what teachers notice and how they notice those events during observation but also examine their responses (i.e., connecting or deciding) to the situations shown in the video clips and understand why they make such decisions. An effective analytical framework used for approaching and guiding teachers noticing is necessary (Jacobs et al., 2010; Kaiser et al., 2017; Rodgers, 2002; van Es & Sherin, 2010). In a recent review article, Amador et al. (2021) found that there were two widely used analytical frameworks in existing studies, the *Learning to Notice* and the *Professional Notice* frameworks. For example, in van Es and Sherin's (2010) research, which adopted the *Learning to Notice* framework, participants were first given the teaching video segment and transcript. After watching, the researchers asked the following types of questions to direct teachers to focus on student mathematical ideas and use evidence from the given materials to support their claims: "What method was Marisa using to solve that problem?"; "Where is that in the transcript?"; and "What does that tell us about her understanding of fractions?". Through the noticing process, Sherin and van Es (2008) found that teachers could extend what they learned from the noticing research to their classroom instruction.

Regarding the complexity of the teaching and learning process, it might be argued that the framework of *Learning to Notice* is too vague and open, making it difficult for participating teachers to understand the details of classroom practice. Hence, different aspects of noticing are adopted in existing studies. For example, using the TIMMS classroom videos, Santagata, Zannoni and Stigler (2007) have provided a *lesson-analysis framework* consisting of three parts for observing and reasoning about classroom events. This framework was used to help prospective teachers gain teaching expertize in the identification of (a) learning goals, (b) student learning concerning those goals, and (c) alternative teaching strategies to accomplish those goals. To highlight the nature of mathematics teaching, Star and Strickland (2008) restricted their noticing research to the first part (identifying what is important or noteworthy about a classroom situation) of van Es and Sherin's (2008) definition. Star and Strickland (2008) asked participants to recall classroom features and events in five observation categories: classroom environment, classroom management, mathematical content, tasks, and communication. Later, Jacobs et al. (2010) used the framework of *Professional Noticing* to focus on noticing children's mathematical thinking instead of all the events in the observed classroom teaching. Like Jacobs et al.'s (2010) framework, Leatham, Peterson, Stockero and Zoest (2015) conceptualized the *Mathematically significant pedagogical Opportunities to build on Student Thinking* (MOST) framework to describe and emphasize the intersection of three characteristics of important components in teaching mathematics, which included mathematically significant content, pedagogical opportunity, and students' mathematical thinking. It is worth noting that, besides the teacher noticing frameworks, there are other theoretical frameworks that can also contribute to the improvement of teachers' teaching competencies, such as Onto-semiotic Approach (OSA) to mathematical cognition and instruction (Breda et al., 2021). Such frameworks are not the focus of this paper, but should be considered in teachers' education research for enhancing teaching quality.

Although numerous studies have been conducted on exploring or developing teacher noticing, the analytical frameworks were mainly developed in Europe and North America (Amador et al., 2021; König et al., 2022). Few theoretical frameworks of teacher noticing are developed in a non-Western classroom context. Teacher noticing skills may differ among cultures. As Louie (2018) argued, the current cognitive orientation towards mathematics teacher noticing ignored the cultural and ideological dimensions of what and how teachers pay their attention. Lee and Choy (2017) extended the *Learning to Notice* framework to a *Three-Point Noticing* matrix, so as to compare what and how pre-service teachers in the US and in-service teachers in Singapore noticed during lesson study. The *Three-Point Noticing* framework was adopted in analyzing the Chinese lesson study (Yang & Ricks, 2012a; 2012b). Like the three goals in Santagata et al.'s (2007) the lesson-analysis framework, the *Key points* refer to the mathematical concept targeted in the lesson, just like the learning goals; and the *Difficult points* are the cognitive obstacles or stumbling blocks which students face when learning the *Key point*, which is close to students' learning concerns. This can refer to students' persistent errors or common misconceptions associated with the concepts being taught in the lesson; Lastly, the *Critical point*—and like the alternative teaching strategies—highlights the approaches that teachers use to support their students to overcome the *Difficult points* and learn the *Key points* (Yang & Ricks, 2012a, p. 43). Recently, through technology-aided interventions, Lee (2021) applied this extended *Three-Point Noticing* framework to enhance the US elementary PSTs noticing skills.

These theoretical frameworks also have an important role in the research on teachers' education programs. For example, some empirical studies have used frameworks to quantify teachers' teaching competencies. In Barnhart and van Es' study (2015), they used the framework of *Learning to Notice* to produce criteria that quantified preservice science teachers' ability to attend to, analyze, and respond to students' thinking; and their study explored the relationship between these competencies. Some empirical studies used theoretical frameworks to conduct intervention experiments to improve teachers' teaching competencies. For example, Mitchell and Marin (2015) used the framework of *Learning to Notice* and video club to intervene pre-service teachers in an elementary teacher education program. After the intervention, participants were able to explain the classroom events more completely and, as a result, they paid more attention to the characteristics of mathematics teaching.

Outside of the work conducted by the above researchers, it is unclear how teacher noticing skills and mathematics teacher education are being linked in the broader literature. In contrast to the above earlier studies, this bibliometrics study examines the empirical studies on mathematics teacher noticing over 15 years, thereby providing a more detailed picture of mathematics teacher noticing topics, development trends, cooperative partners, collaborative organizations, topic distributions of the prolific countries/regions and institutions, annual topic distributions. The study further discussed the representative research work, and research implications, as well as suggested a possible pathway for future mathematics teacher noticing research. Specifically, the following research questions (RQs) guided this review:

RQ 1: What are the trends in mathematics teacher noticing research published from 2006 to 2021? What are the prospects for the development?

RQ 2: Which countries/institutions have done the most in-depth research in this area?

RQ 3: What are the most important journals in the mathematics teacher noticing field?

RQ 4: What are the current hotspots and emerging research frontiers in this area?

Data and methods

Research tools. The research tools for this review were mainly the Bibliometrix package for R, Charticulator, and Citespace. Quantitative methods and techniques were adopted to review the literature published from 2006 to 2021. By using the above three information visualization software packages, a domain-specific collection of literature could be analyzed econometrically to explore the key paths of the evolution of the discipline area; and a series of visualizations could also be produced to develop an analytical summary of the discipline (Aria & Cuccurullo, 2017; Chen, 2016; Ren, Lee & Brehmer, 2019).

Data collection. The Webs of Science and Scopus are two major databases used in educational research. Scopus was chosen as the data source for this review because it is the largest abstract and citation database of peer-reviewed literature and quality web sources. While Web of Science also provides many exportable metadata to facilitate bibliometric analysis, its coverage of educational disciplines is less extensive than that of Scopus (Hallinger, 2020). The selection of data for the bibliometric review was done using Preferred Reporting Items for Systematic Reviews and Meta-Analyses (PRISMA) method as a strategy for retrieving articles. The PRISMA methos is the most used method for conducting systematic reviews and meta-analyses (Page et al., 2021). In this study, the extraction of data consists of the following four steps: identification, screening, eligibility, and included, as shown in Fig. 1. The review aimed to collect all English-language published literature on mathematics teacher noticing in Scopus. Regarding the timeline, to ensure the accuracy of the annual publication statistics analysis, the publications after 2021 were excluded, resulting in a final search of 139 articles.

Methods. A bibliometric review is based on statistical analysis (Linnenluecke et al., 2020; McBurney & Novak, 2002). This method is used to visualize information about annual publication statistics, national research strengths, institutional research strengths, keyword analyses, and literature co-citations (Chen et al., 2018). Analyses of these data can provide answers to the five research questions posed in the Introduction. Based on bibliometric and applied statistical methods, R, Charticulator, and CiteSpace were used to analyze the mathematics teacher noticing database of 139 articles concerning the annual number of articles published, the research strengths of nations/institutions, and the co-occurrence of keywords, cluster analyses, co-citation analyses, and burst detections. The quantitative process of analyzing these aspects of mathematics teachers noticing knowledge is shown in Fig. 2. Visual analysis was conducted as regards the development of mathematics teacher noticing research, the current hotspots, and potential problems; and the relevant evaluation indicators were selected as follows.

(1) Trends. The volume of literature is, to some extent, a proxy for the level of interest and importance attached to the field of study by the academic community. The number of publications was obtained by creating citation analysis reports in the Scopus database and de-duplicating the data through CiteSpace. These data were analyzed in a time-series distribution to infer trends in the field of mathematics teacher noticing research. A functional curve was also fitted to the cumulative number of publications. The four stages of literature growth proposed by Price, de (1961) are referred to as follows: (i) the early stage of the birth of the discipline, a period of unstable literature growth and imperfect theories that are difficult to express in mathematical formulas; (ii) the period of great development of the discipline, a period of rapid development of professional theories and a sharp increase in the absolute number of papers that strictly obey the law of

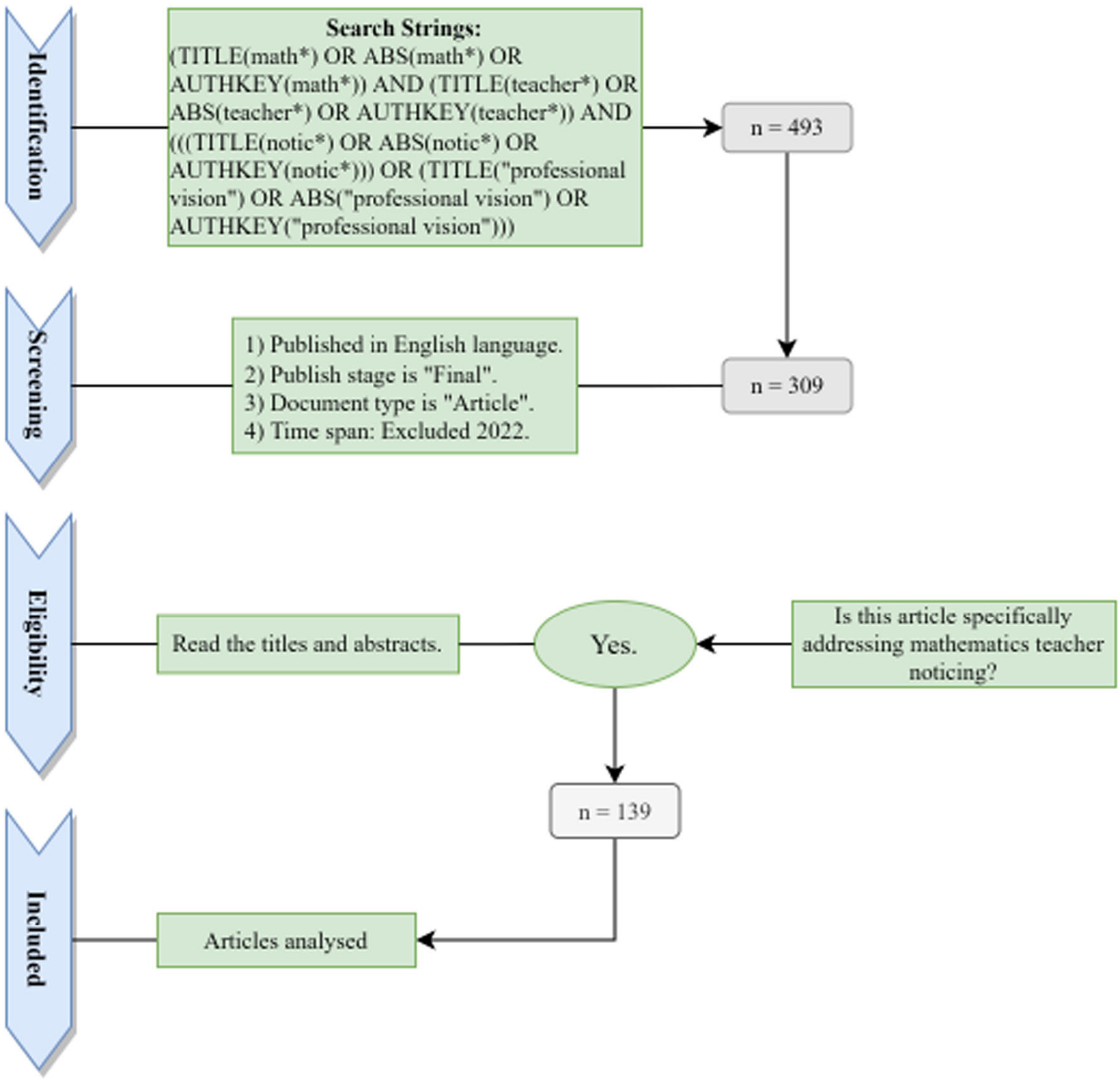

Fig. 1 Flowchart of data collection. Four steps are used for searching and screening articles and 139 avaliable publications meet the selection criteria.

exponential growth; (iii) the increasing maturity of the discipline's theories, characterized by slow growth in the number of papers that evolve into linear growth; and (iv) the completion of the disciplinary theory when the number of papers in the discipline decreases, and the curve gradually becomes parallel to the horizontal axis or fluctuates irregularly. The curve is fitted to the cumulative number of publications to determine the stage of disciplinary development of the discipline and to give practical research recommendations and projections of trends.

(2) National/institutional research strengths. VOSviewer and Gephi were used to process the source data retrieved from the Scopus database, and an appropriate number of national/institutional data was selected for analysis in Charticulator. By plotting the number of national/institutional publications and collaborations and chord diagrams, the research strengths of nations/institutions in the field of mathematics teacher noticing were obtained.

(3) Core journals. The current core journals in the mathematics teacher noticing field were obtained using the source data from the Scopus database, before being processed by using VOSviewer to compare the number of publications and citations.

(4) Research hotspots. The keywords are a high level of condensation of the research content of the literature and can be used to analyze research hotspots through keywords co-occurrence analysis in CiteSpace. In addition to analyzing keyword frequency, we also used CiteSpace to calculate keyword centrality. Centrality is a measure of the relative importance and connectivity of keyword nodes in a research area. It is used to test whether high-frequency keywords are located centrally, reflecting the position of nodes in the overall network (Landherr et al., 2010). Simply put, the greater the centrality, the more important the node (Landherr et al., 2010). CiteSpace uses an algorithm to cluster closely related keywords and calculates the important value of each keyword, with the largest value in the same cluster elected as the category's representative and becoming the label of the clustered network. Further analysis of high-frequency and high centrality keywords and clustering labels allows us to obtain the current hot topics of research.

(5) Key literature. The concept of co-citation was first introduced by the American intelligence scientist Small in 1973 as a way of measuring the degree of relationship between documents (Small, 1973). When two (or more) papers are cited by one or more subsequent papers, they are said to be co-cited; and the more frequently they are cited, the more important they are in the field (Small, 1973). The 139 relevant papers retrieved from Scopus were analyzed for co-citations, and the information visualization function of CiteSpace was used to create a co-citation map of the literature to identify the current key literature in mathematics teacher noticing. By reading the key literature, the

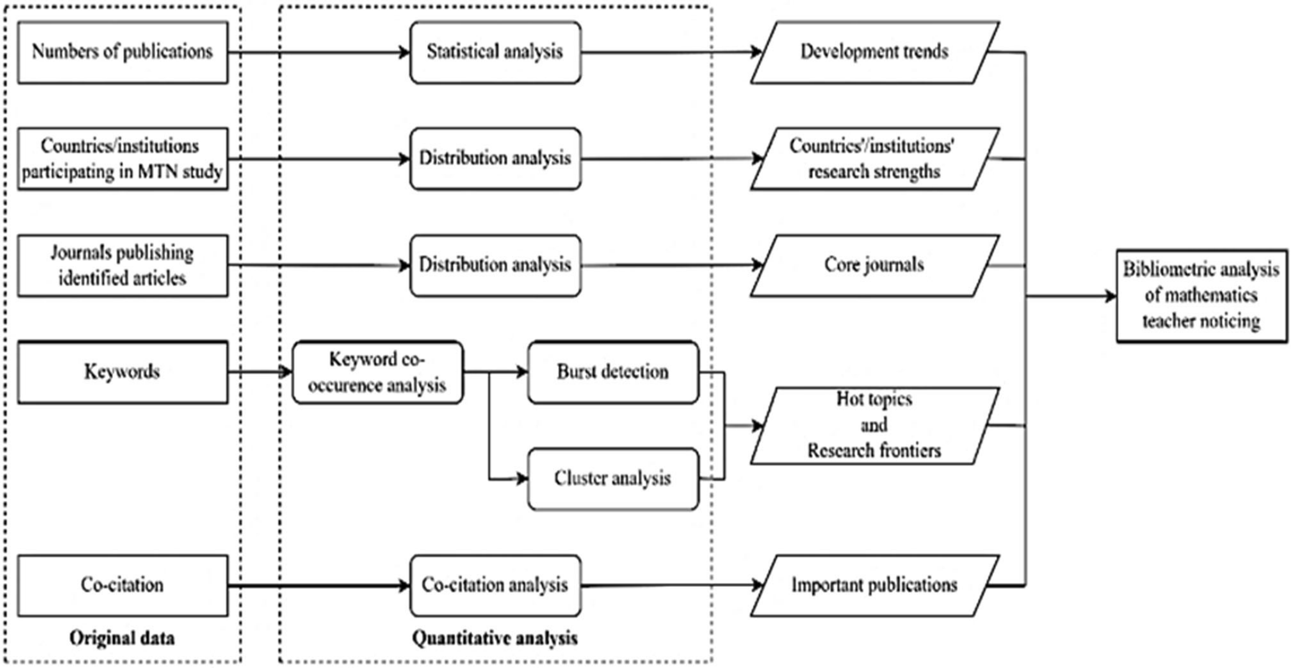

Fig. 2 Flowchart of review methods. A quantitative analysis was used in the bibliometric review in terms of the number of publications, countries of publication, institutions of publication, journals of publication, keywords, and co-citations.

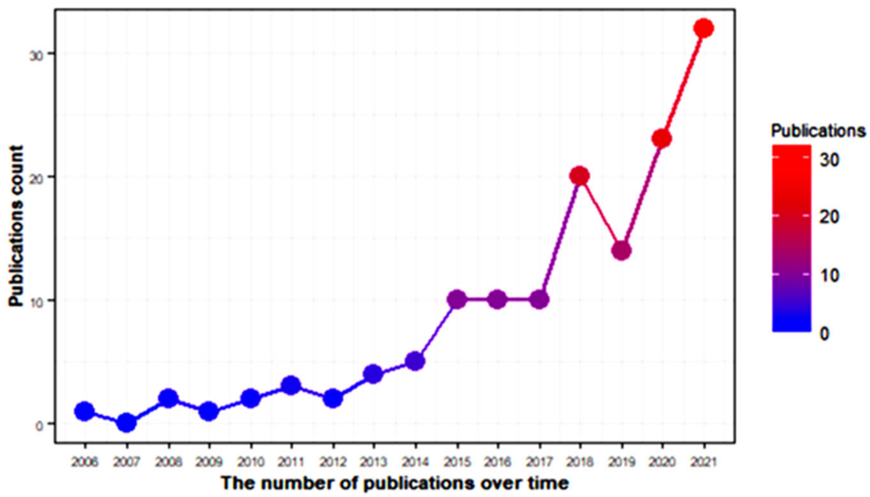

Fig. 3 Annual publications count. One hundred and thirty-nine articles pertaining to mathematics teacher noticing have been published from 2016 to 2021.

researcher was able to gain a more specific understanding of the current research findings.

(6) Research frontiers. The concept of "research frontiers" is used to reveal the dynamic evolution of a research field. "Research frontiers" are defined as a set of emergent concepts in a specific field. Using CiteSpace's word-frequency probing technology, the temporal distribution of subject terms was analyzed to select those with high rates of change in frequency and to identify frontier areas and trends based on the trend of word-frequency change.

Results and analysis

Trends based on annual publication statistics from 2006 to 2021. According to the citation analysis report in the Scopus database, the number of mathematics teacher noticing publications from 2006 to 2021 was 139, with an average of 9 publications per year, as shown in Fig. 3. Except for a relatively large drop in 2019, a general increasing trend over this period is also shown here, indicating that the research system was being constructed steadily. At the same time, a logarithmic transformation of the annual cumulative number of publications was conducted, as shown in Fig. 4. It can be seen that the linear fit curve of the logarithm, where $R^2 = 0.9953$, is very close to 1, *thus* judging it a good fit. This implies that the logarithm of the annual cumulative number of articles showed a consistent linear growth pattern since 2008, suggesting that the annual cumulative number of articles had a consistent exponential growth pattern. Based on Price, de (1961) four stages of literature growth, it could be inferred that mathematics teacher noticing research was at the beginning of the second stage of literature growth in 2021, as shown in Fig. 5; Price, de (1961) argued that research fields in the

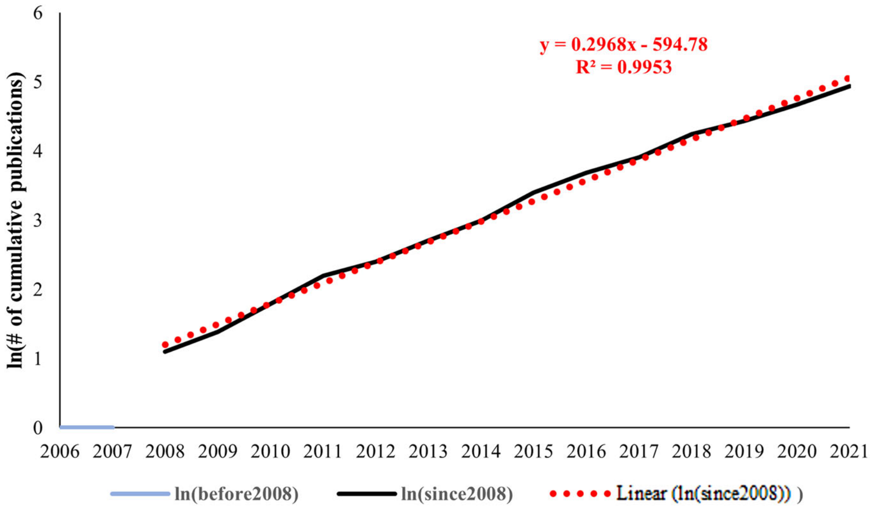

Fig. 4 Linear regression for cumulative publications. The solid black line is the true curve of ln (cumulative publications), and the dashed red line is a linear fit to ln (cumulative publications).

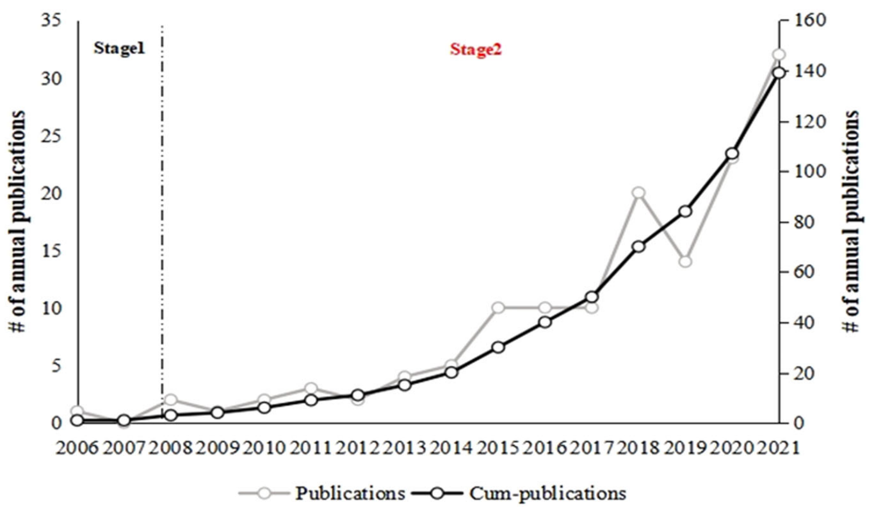

Fig. 5 Stages of literature growth. The annual cumulative number of articles had a consistent exponential growth pattern since 2008.

second stage of literature growth would see a rapid development of specialist theories and a dramatic increase in the number of papers. It is clear from the above analysis that as recently as 2021, mathematics teacher noticing research was still of great interest and had a promising future.

National/Institutional research strengths. To further understand the research status and actual contribution of the research nations/institutions in the field of mathematics teacher noticing, the numbers of publications were counted by nation/institution. The data were converted into GML files with the help of VOSviewer software, and the node data and edge data were exported by adjusting the data format with Gephi. The node data and edge data were then imported into Charticulator to create a chord diagram of the country/institutional research power, as shown in Figs. 6–9. Generally, in a chord diagram, the chord length is considered to represent the node; that is, the longer the chord length, the greater the influence of that node. The line between the two chords represents the relationship between the nodes: the thicker the line, the stronger the association between the nodes. In this analysis, the chord length was considered to represent national/institutional research power, with longer chord lengths representing stronger national/institutional research power. From Figs. 6 and 7, the US was far ahead of other countries in terms of both the number of publications and the number of citations.

In terms of publishing, the US was followed by Germany, Spain, Norway, and Turkey, but the US was clearly more dominant. In terms of citations, the distribution of other countries or regions was slightly scattered. Although the United States is still the leading country, the proportion of other countries or regions has increased. Therefore, we believe that most of the most recent mathematics teacher noticing research results were concentrated in the US. Other countries and areas have produced relatively few studies, but a few outstanding ones have received general recognition. At the same time, there were some collaborative relationships between countries, but the links were not diversified enough, and the collaborating countries were relatively fixed. As the country with the most research achievements, the US has cooperative relationships with many countries or regions, including Turkey, Australia, Hong Kong (China), Mozambique, and United States Virgin Islands. Among them, the US cooperated with Turkey the most. By contrast, Germany is working more closely with Norway. To ensure the beauty and clarity of the institutional chord diagram in Figs. 8 and 9, we only selected research institutions with (a) publications greater than 3,

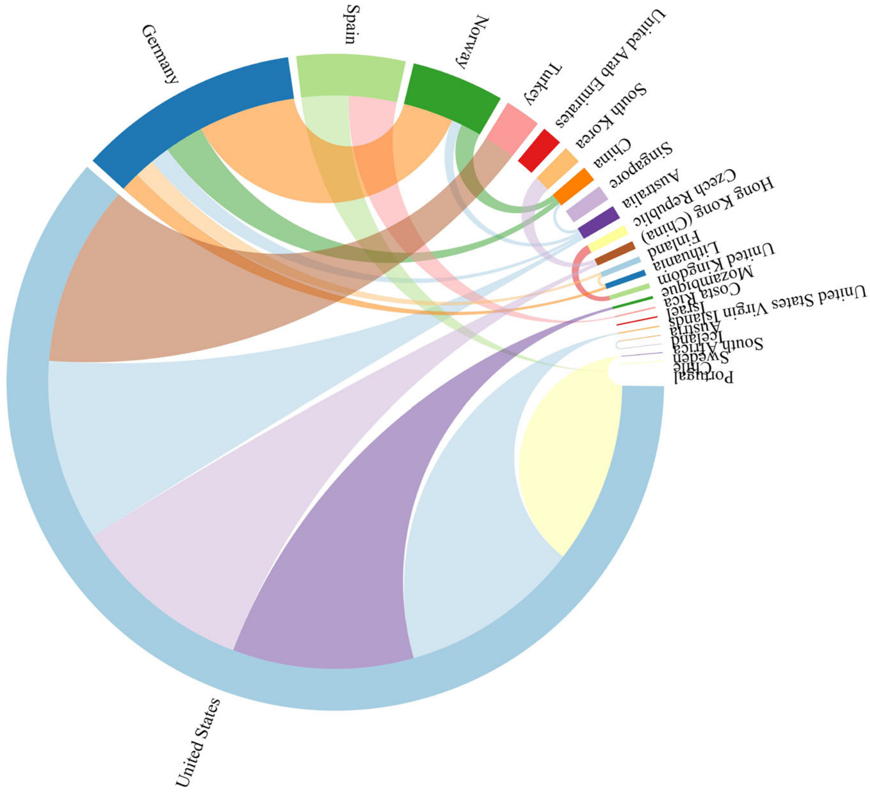

Fig. 6 Chord diagram of research countries' strengths (ranked by # of publications). The chord length is considered to represent the node and the United States, Germany and Spain have made the largest contributions to research in this area.

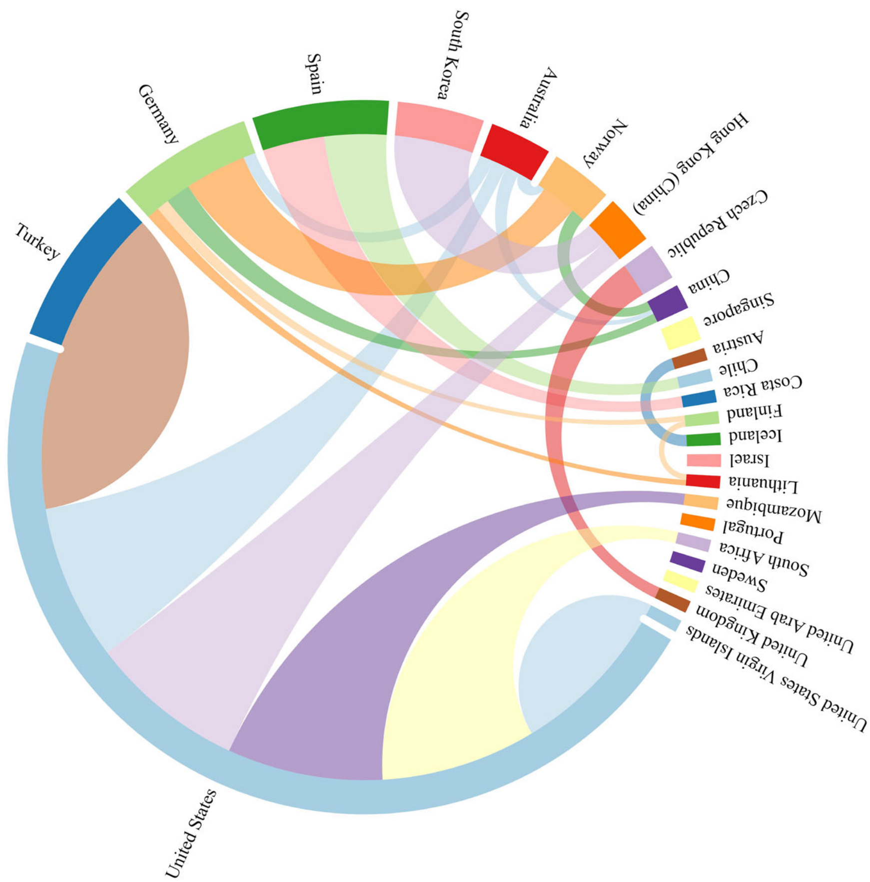

Fig. 7 Chord diagram of research countries' strengths (ranked by # of citations). The chord length is considered to represent the node and the United States, Turkey and Germany have made the largest contributions to research in this area.

Fig. 8 Chord diagram of research institutions' strengths (ranked by # of publications ≥ 3). The chord length is considered to represent the node and the University of Idaho has made the largest contributions to research in this area.

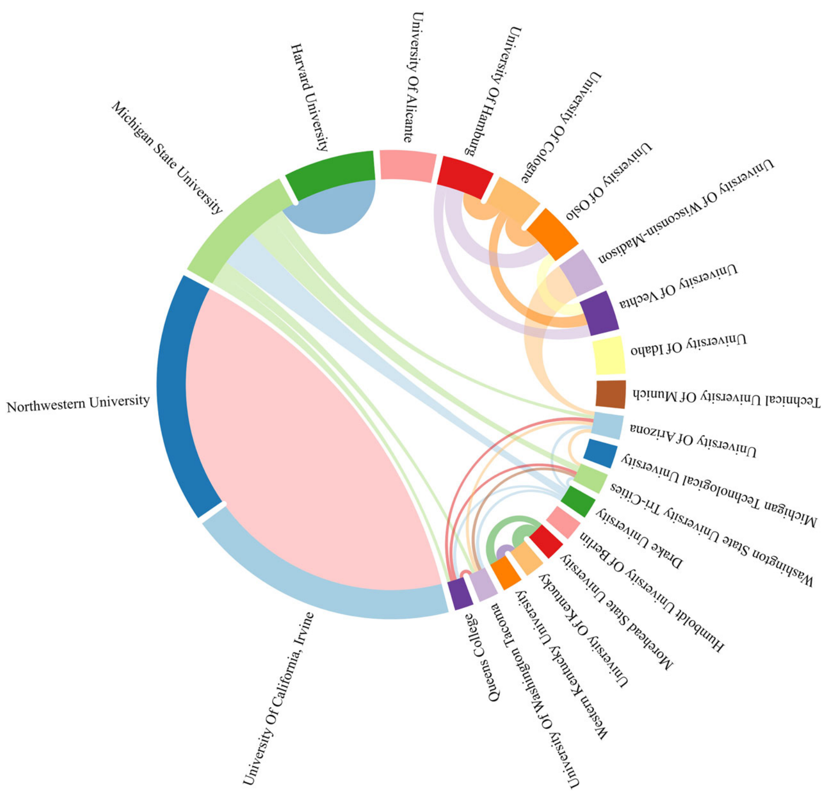

Fig. 9 Chord diagram of research institutions' strengths (ranked by # of citations ≥ 80). The chord length is considered to represent the node and the University of California, Irvine has made the largest contributions to research in this area.

Table 1 Core Journals.

(a) Ranked by # of publications ≥ 4		
Journal	Impact factor (2021–2022)	Number of publications ≥ 4
Journal of Mathematics Teacher Education	1.786	26
International Journal of Science and Mathematics Education	2.051	14
Teaching and Teacher Education	3.782	9
ZDM-Mathematics Education	2.481	7
Educational Studies in Mathematics	2.853	6
Journal of Teacher Education	4.13	6
Eurasia Journal of Mathematics, Science and Technology Education	2.588	4
(b) Ranked by # of citations ≥ 95.		
Journal	Impact factor (2021–2022)	Number of citations ≥ 95
Journal of Mathematics Teacher Education	1.786	918
Teaching and Teacher Education	3.782	722
Journal of Teacher Education	4.13	590
International Journal of Science and Mathematics Education	2.051	311
Educational Studies in Mathematics	2.853	255
ZDM-Mathematics Education	2.481	187
Journal for Research in Mathematics Education	2.278	96

and (b) citations greater than 80. From Figs. 8 and 9, the average number of institutional publications was also small due to the relatively small volume of total literature, resulting in a nonsignificant gap between research institutions (Fig. 8). However, when Fig. 9 was ranked according to the number of citations, the gap between the research institutions was more apparent, with UCI and NWU dominating by a large margin. Consequently, it was considered that UCI and NWU were the prominent contributors to research in the recent mathematics teacher noticing field. At the same time, it can be noted that there was less collaboration between research institutions.

Core Journals. To identify the core mathematics teacher noticing journals, the medium-term journal source data were extracted from the Scopus database through VOSviewer. And the journals were ranked according to the amount of included literature and the number of citations, as shown in Table 1. To ensure the effect of visualization and clarity of the table, we only select core journals with (a) publications greater than 4, and (b) citations greater than 95. There were six journals with more than five mathematics teacher noticing related articles; and, except for the *Journal of Mathematics Teacher Education* and *International Journal of Science and Mathematics Education*, there was no apparent difference in these numbers. It is worth noting that the journals with more than five literature titles were also in the top six in terms of citations and were far ahead of the other journals. In-depth analysis of the journals' impact factors, the numbers of mathematics teacher noticing related articles published, and the numbers of citations, in this review it was found that *Teaching and Teacher Education* and *Journal of Teacher Education* were the core journals for mathematics teacher noticing, although they did not have the largest number of articles. Regarding the number of citations, the mathematics teacher noticing related articles published in them had achieved greater research impact.

Research hotspots by keywords: co-occurrence analysis and cluster analysis. Citespace software was used to capture snapshots of certain areas based on time-series, to infer the direction of the mathematics teacher noticing related research. The basic information collected from 139 articles was transferred into CiteSpace for keyword analysis, setting the time interval as 2006–2021 and the time slice as 1, as shown in Fig. 10. In total, 262 nodes and 557 links were generated in this figure. Twenty-two keywords with three or more occurrences were selected; and this information was obtained using CiteSpace software, as shown in Table 2. Figure 10 and Table 2 show that, in addition to the keywords searched, "mathematics education", "professional development", "teacher education", "video", "preservice teacher education", etc. had high frequencies, indicating that they represent the hotspots of mathematics teacher noticing research. Their high node centrality indicates their importance in mathematics teacher noticing research and that they act as the "bridges" among the research hotspots.

Based on the keyword co-occurrence network mapping, for this review, the LSI method was used to do a clustering analysis of keywords, using them to name the clusters. The clustering results are shown in Fig. 11. The number of network nodes was 262, and the clustering results were evaluated by using the clustering module value (Modularity, *Q*-value) and the clustering average profile value (Silhouette, *S*-value). The higher the Modularity value, the better the clustering result. The Silhouette value is an indicator of whether a clustering network is trustworthy, and when the Silhouette value is closer to 1, it indicates that the network is more reliable. In general, when $Q > 0.3$, this means that the structure of the delineated associations is significant, and $S > 0.7$ indicates a convincing clustering (Chen, Chen, Luan, Hu & Wang, 2015). In Fig. 10, $Q = 0.8482 > 0.3$ and $S = 0.9517 > 0.7$, indicating that the clustering results were better and had high reliability. Further analysis showed that having ignored some clusters that were not shown because the Silhouette value was too low, 12 clustering labels were obtained. By sifting out the search terms and combining synonymous labels, it can be inferred that the main theory in this field was the analytical frameworks described by Amador et al. (2021) as trying to figure out what teachers notice and how they notice ("framing"). "Video" was the primary research method/tool used in this field. For example, Ulusoy and Çakıroğlu (2021) used micro-case videos to analyze students' mathematical understanding and prospective teachers' noticing skills. Copur-Gencturk and Rodrigues (2021) explored what teachers noticed in short videos of a real classroom. The main target group was "Prospective teachers" ("prospective teacher", "preservice teacher education"). The main research focus was "Developing teachers' teaching skills" ("teacher education", "preservice teacher education"). Many researchers tried to design intervention experiences to improve teachers' noticing skills; for example, Wallin and Amador (2019) used video clubs to support three rural teachers' development.

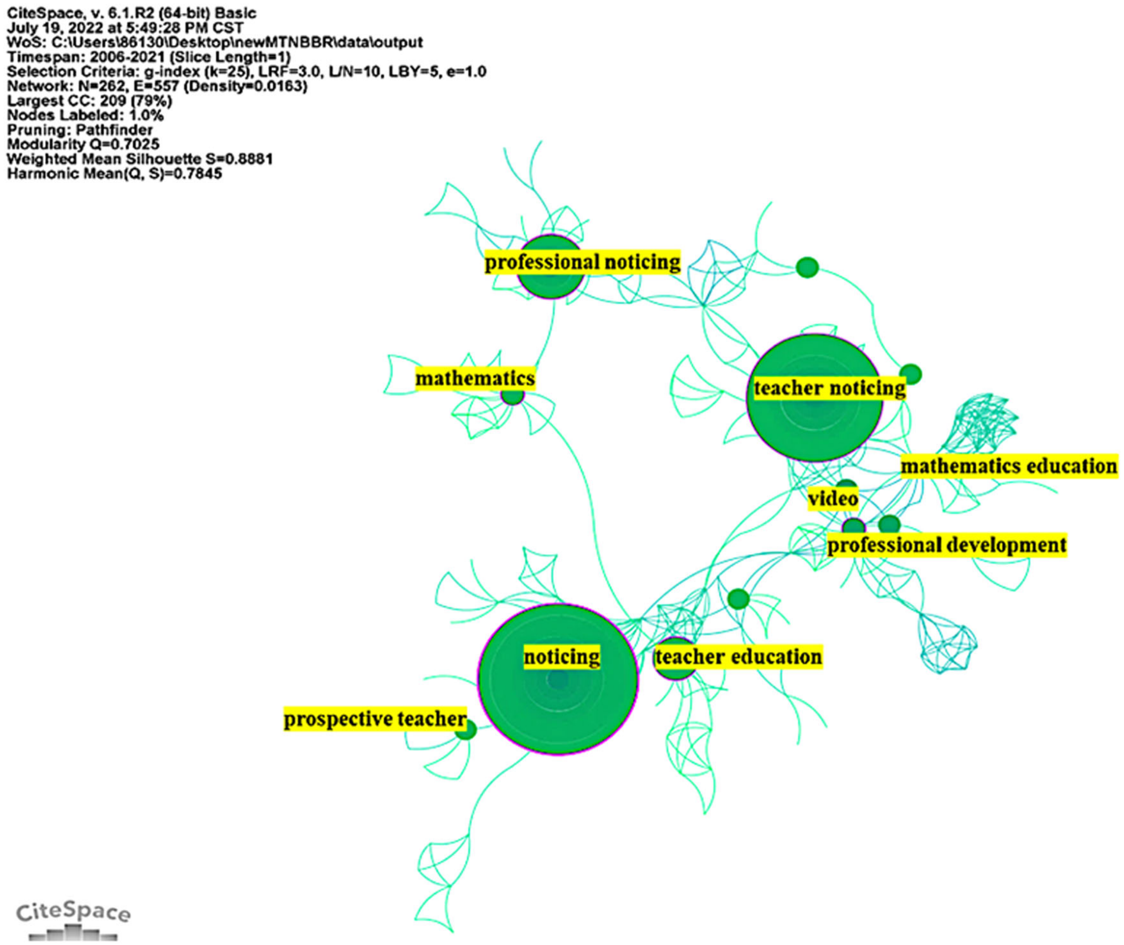

Fig. 10 Keywords co-occurrence map. The highlighted keywords represent the hotspots of mathematics teacher noticing research. "Video", "prospective teacher" and "professional development" have the largest nodes, implying their importance and relevance in the field of mathematics teacher noticing research.

Table 2 Keywords that appear three times or more.

Count	Centrality	Keywords
35	0.4	noticing
32	0.25	teacher noticing
22	0.27	professional noticing
15	0.35	mathematics education
12	0.36	professional development
9	0.28	mathematics
8	0.32	teacher education
8	0.05	prospective teacher
6	0.31	video
6	0.05	teacher learning
5	0.05	mathematics teacher education
5	0.01	preservice teacher
5	0.04	professional vision
4	0.19	equity
4	0.07	teacher cognition
4	0.22	preservice teacher education
4	0.02	lesson study
3	0.02	pedagogical content knowledge
3	0.02	in service teacher
3	0.02	clinical interview
3	0.28	teacher knowledge
3	0.02	elementary teacher education

Key literature by co-citation analysis. Co-citations appear in publications that share common research content and references with the reviewed literature. In the scientific map, co-citation literature contains a large amount of scientific knowledge. The number of citations in the literature is an essential indicator of the impact of scholarship and can be used to trace the evolution of research in a scientific field. The higher the number of co-citations, the greater the relevance of the literature. Co-citation analysis is a correlational analysis. A co-citation network map was obtained by setting the network node to Reference in CiteSpace, as shown in Fig. 12. In the co-citation map, larger nodes mean more frequently cited literature. The most frequently cited literature and the corresponding years are shown in Fig. 12. Some nodes also have a purple circle at the outermost part, representing the node's centrality; that is, the thicker purple circles represent the more important literature. From the key nodes in the graph, we can see that Barnhart and van Es (2015)'s article had the largest node and the thickest purple circle, which imply that this article was cited the most and had the greatest impact on the field of mathematics teacher noticing research. The article *Studying Teacher Noticing*, written by Barnhart and van Es (2015), quantified the effects of a video curriculum intervention through an experiment with preservice science teachers while testing the relationships among teachers' attending, analyzing, and responding. In addition, there are some sub-important literature

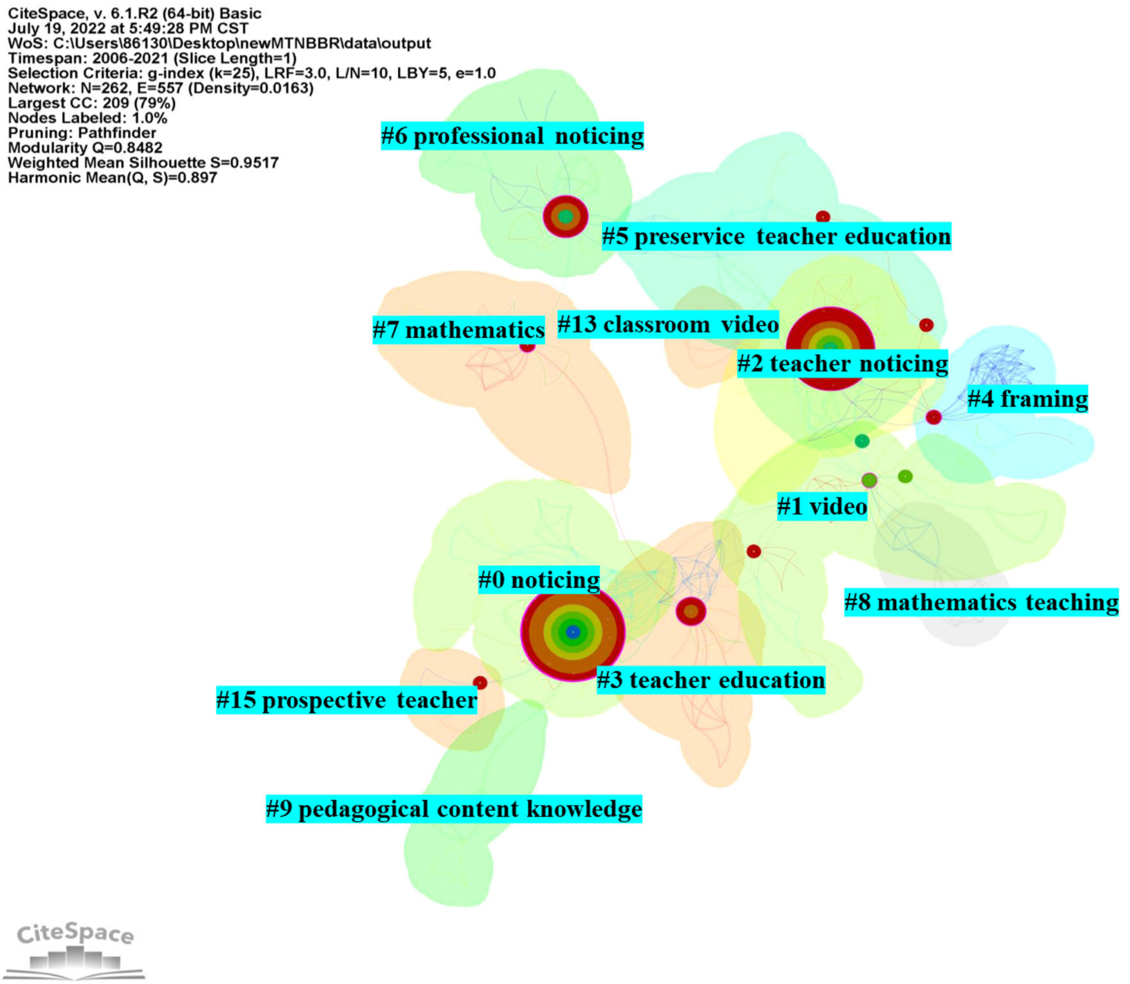

Fig. 11 Keywords clustering map. Fourteen clusters are identified by using the clustering module value and the clustering average profile value and "video" is a trend and a hot topic in the current teacher noticing research field.

nodes. The publication time of these secondary node literature is concentrated between 2015 and 2017. Together with Barnhart and van Es (2015)'s article, a relatively close research network has been formed among them. There is only one secondary node literature, which is from Mason (2011)'s work.

Research frontiers by burst detection. Using the default parameters of CiteSpace, burst detection did not detect any bursts for the keywords. It can be inferred from this that the research perspective of the mathematics teacher noticing field was not rich enough and was rather narrow at the time of this review. Adjusting the parameter $\gamma[0,1] = 0.3$ and Minimum Duration = 1, 27 keywords were detected. The top 20 of these are shown in Fig. 13. The keywords with high emergence were the emerging hotspots in the research area, and the red line corresponds to the duration of the emergence of the keyword; and the blue line represents the other period of the research period when no emergence occurred. Further analysis showed that the emergence of the terms "prospective mathematics teacher" and "learning trajectory" in 2020–2021 was likely to be a hot topic for future research in mathematics teacher noticing, i.e., a research frontier. As "teacher education" also has a strong citation, we can argue that promoting the improvement of the quality of teacher education will be the consistent goal of mathematics teacher noticing research.

Discussion

The core goals in mathematics education are enhancing the quality of mathematics teaching and ameliorating the effectiveness of mathematics teacher education. Teacher noticing is an attribute that a professional mathematics teacher should have. As well as designing classroom activities and lesson plans, they need to be able to make sense of, and attend to, classroom events (critical or non-critical); and make decisions or judgments about students' misconceptions or understanding. However, not all teachers see what they should see during classroom instruction. Even if some preservice teachers have a chance to observe and take part in their mentor's teaching or field experiences, they often fail to notice what teacher educators hope they will notice from the observation (Star et al., 2011). This shows the realistic demand for teacher noticing research. However, it is unclear as to what researchers are currently doing and focusing on. Therefore, by the bibliometric method we carried out a review on mathematics teacher noticing. According to this bibliometric review, since 2008 the academic community in mathematics education has been paying more and more attention to the field of mathematics teacher noticing. And mathematics teacher noticing research was at the beginning of the second stage of literature growth, which has now become a hot research direction. Specifically, research on mathematics teacher noticing presents in the main the following characteristics.

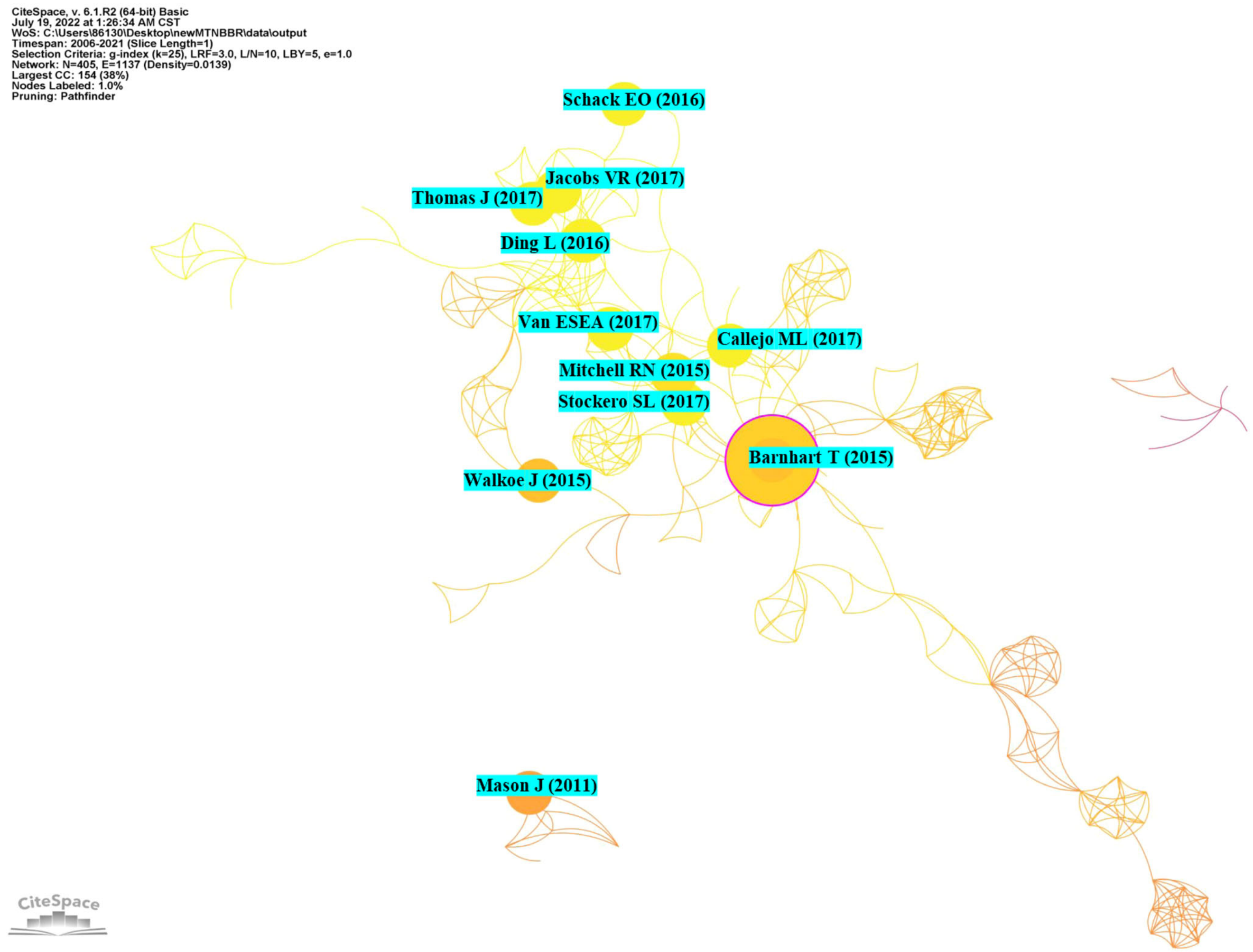

Fig. 12 Co-citation map. A larger node means a more frequently cited literature. Barnhart and van Es (2015) was co-cited by most literature, which is one of the more important and influential literature in the field.

First, the culture where the relevant studies were conducted is too concentrated, and cross-cultural research is still lacking. Most of the recent research in mathematics teacher noticing has focused on the United States (US) and US colleges and universities, with few links between countries and institutions. It is worth noting that Germany and Norway work most closely at the national level. This cooperative relationship may be due to the fact that they are homologous in education and culture. Still, they are exchanges between similar cultures, which does not reach the level of cross-cultural research in a real sense. This suggests that the cultural dimension is missing to some degree. For example, most of the studies on frameworks have discussed the US teacher noticing framework: van Es (2011)'s framework about *Learning to Notice* and Jacobs et al.'s (2010) *Professional Noticing* of children's mathematical thinking, but scarcely any research has been done on the Three-point framework (Lee & Choy, 2017), which is more familiar to Chinese teachers. As Louie (2018) argued, the current cognitive orientation towards mathematics teacher noticing has ignored the cultural and ideological dimensions of what and how teachers pay their attention. Future researchers may try to supplement the cross-cultural mathematics teacher noticing research in theory and practice and explore the differences and commonalities from the perspective of different educational cultures and teacher training systems.

Secondly, in the co-citation analysis, it is found that an article discussing science teachers noticing was the most important node (Barnhart & van Es, 2015), which gives a hint that teacher noticing can be an active interdisciplinary research topic. Among the relevant research literature on mathematics teacher noticing, Barnhart and van Es (2015)'s article about science teachers noticing was the most influential. It is believed that the value or impact of this research article is not only its interdisciplinary nature, but also its research design, which has become a crucial reference to mathematics teacher noticing research. Hence, it would be worthwhile to learn more about the current status of the whole TN research progress, which can guide the future development of specific mathematics teacher noticing and general TN research. In addition, the publication times of central node and secondary node literature on mathematics teacher noticing were concentrated in the period from 2015 to 2017; and there was a relatively close relationship network between these two nodes, indicating that the research on mathematics teacher noticing, gradually throughout this period, grew into a research field of mutual connection and mutual influence.

Thirdly, in terms of the keywords, those with high frequencies are "mathematics education", "professional development", "teacher education", "video", "framing", and "preservice teacher education". These can be seen as mathematics teacher noticing's current hotspots. The research focuses on "mathematics education", "professional development", and "teacher education"; which embodies the importance of noticing skills for teachers as well as for teacher education. It is found that current research is

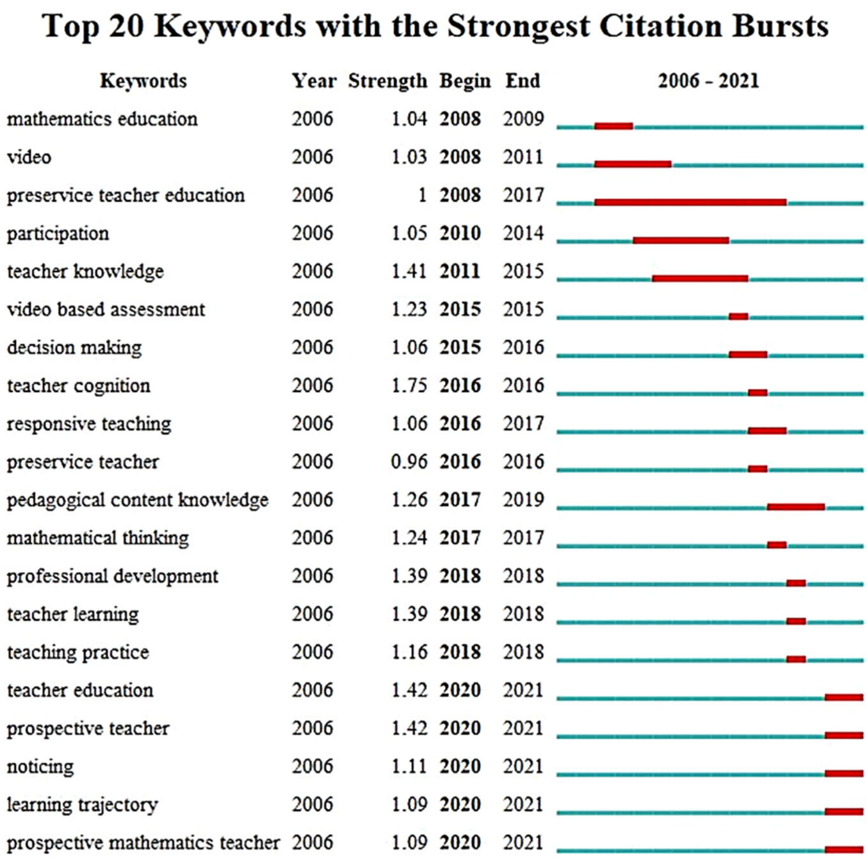

Fig. 13 Keywords with the strongest citation bursts. The red line corresponds to the duration of the emergence of the keyword and the blue line represents the other period of the research period when no emergence occurred.

limited to exploring noticing frameworks (corresponding to keyword of "framing"), and there is no general theory on other theoretical research, such as how to measure mathematics teachers' noticing skills. We also notice that video related to teaching and learning mathematics are the mainstream research medium used in mathematics teacher noticing studies, while other new media technologies such as Virtual Reality (VR) glasses, eye tracking, and animation have been sparsely used. Compared to traditional video-watching methods, more interactive technologies may improve the effectiveness of mathematics teacher noticing interventions on teachers' teaching skills, such as the stage of attending and responding during the process of noticing. Lee (2021)'s study could be a good try, in which the software *LessonSketch* was used as a technology-aided intervention tool to measure PSTs noticing. More empirical studies of teacher noticing based on innovative and interactive technologies can be developed.

In addition, the results have shown that recent research in teacher noticing mainly focused on the target of preservice teachers, with relatively little research on teacher educators, or on specialist teachers with extensive teaching experience. Expanding the research population would provide us with more perspectives of how and in which way teacher noticing may differ or change among different groups. The small number and concentration of keywords may reflect the lack of diverse research contents in the current field. Researchers can explore or develop different new research areas or mathematical topics on mathematics teacher noticing to make research breakthroughs. For example, shifting from traditional focus on students' mathematical thinking to computational thinking, from domain-specific learning activity to mathematical modeling activity, or from mathematical topics to cross-curricular context. The high degree of overlap in research keywords, along with the results of burst detection, has reminded us that existing research may be more imitative than innovative. Hence, exploring new research methods and measurement tools can be the focus in the future. Also, prospective mathematics teachers and their learning trajectories at different stages may be among the hot future research frontiers.

This bibliometric review has presented, adopting a visual approach, current research findings and research trends in mathematics teacher noticing. The results have identified research gaps that still exist in cultural perspectives, technological approaches, theoretical studies, and research targets. Considering the role of mathematical teacher noticing in teacher education, this study provided several future research directions in mathematics education. It suggested a possibility for integrating mathematical teacher noticing and other research topics, such as innovative technologies, the applications in mathematical modeling, and teacher professional development. These may ultimately contribute to the enhancement of preservice and in-service teachers' teaching competencies.

Conclusions

To grasp more clearly the future research trends of mathematics teacher noticing, we adopted in this review the method of bibliometrics to analyze 139 articles from the Scopus database. It can be found that the overall situation of mathematics teacher noticing research is good and is experiencing a steadily rising stage. Although studies on mathematics teacher noticing have appeared in many countries or regions, the core research strength is still concentrated in the United States, and that international or regional cooperation is relatively lacking. Over-concentrated noticing research in one area may result in an impeded communication between different educational cultures. It is suggested that cross-cultural studies on mathematics teacher noticing would be profitable. Our analysis also observed that current studies on mathematics teacher noticing have mainly focused on preservice or prospective groups, while video-based stimulation is a common research approach. Thus, prospective mathematics teachers and learning trajectory research may become hotspots in the next few years. All the above conclusions are from the econometric analysis of mathematics teacher noticing literature in Scopus. Other databases have not been included in the analysis scope. Also, to ensure the reliability of the subsequent analysis, we screened out some articles from other language sources, such as five articles in Spanish and Portuguese, which had less than 5 citations. Considering the small number of selected articles and citations in other languages, we believe that screening out these articles did not affect the credibility of this study. However, in future studies, researchers may consider avoiding screening that affects the conclusions. By this, a quick overview of the research context and directions can be formed. We must admit that, to a certain extent, the results of our research can only predict the general trends of research in teacher noticing, which may not accurately guide more specific research plans or questions. Therefore, more databases could be considered, with suitable statistical methods, for a deeper systematic review in the context of mathematics teacher noticing research.

Data availability

The datasets generated during and/or analyzed during the current study are available in the Scopus repository: https://www.scopus.com/home.uri.

Received: 21 October 2022; Accepted: 24 April 2023;
Published online: 09 May 2023

References

Amador JM, Bragelman J, Superfine AC (2021) Prospective teachers' noticing: a literature review of methodological approaches to support and analyze noticing. Teach Teach Educ 99:103256. https://doi.org/10.1016/j.tate.2020.103256

Aria M, Cuccurullo C (2017) Bibliometrix: an R-tool for comprehensive science mapping analysis. J Informet 11(4):959–975. https://doi.org/10.1016/j.joi.2017.08.007

Barnhart T, van Es E (2015) Studying teacher noticing: Examining the relationship among pre-service science teachers' ability to attend, analyze and respond to student thinking. Teach Teach Educ 45:83–93. https://doi.org/10.1016/j.tate.2014.09.005

Breda A, Pochulu M, Sánchez A, Font V (2021) Simulation of teacher interventions in a training course of mathematics teacher educators. Mathematics 9(24):3228. https://doi.org/10.3390/math9243228

Chen C (2016) CiteSpace: a practical guide for mapping scientific literature. Nova Science Publishers, Hauppauge NY

Chen X, Hao J, Chen J, Hua S, Hao T (2018) A bibliometric analysis of the research status of the technology enhanced language learning. In: T Hao, W Chen, H Xie, W Nadee, R Lau (eds) Emerging technologies for education, Springer International Publishing, pp. 169–179

Chen Y, Chen C, Luan C, Hu Z, Wang X (2015) The methodology function of CiteSpace mapping knowledge domains. Stud Sci Sci 33:242–253. https://doi.org/10.16192/j.cnki.1003-2053.2015.02.009

Copur-Gencturk Y, Rodrigues J (2021) Content-specific noticing: a large-scale survey of mathematics teachers' noticing. Teach Teach Educ 101:103320. https://doi.org/10.1016/j.tate.2021.103320

Goodwin C (1994) Professional vision. Am Anthropol 96(3):606–633. https://doi.org/10.1525/aa.1994.96.3.02a00100

González G, Vargas GE (2020) Teacher noticing and reasoning about student thinking in classrooms as a result of participating in a combined professional development intervention. Math Teach Educ Dev 22(1):5–32. https://mted.merga.net.au/index.php/mted/article/view/449

Hallinger P (2020) Science mapping the knowledge base on educational leadership and management from the emerging regions of Asia, Africa and Latin America, 1965–2018. Educ Manag Admi Leadersh 48(2):209–230. https://doi.org/10.1177/1741143218822772

Hawkins DT (2001) Bibliometrics of electronic journals in information science. http://informationr.net/ir/7-1/paper120.html. Accessed 18 Aug 2022

Jacobs VR, Lamb LLC, Philipp RA (2010) Professional noticing of children's mathematical thinking. J Res Math Educ 41(2):169–202. https://doi.org/10.5951/jresematheduc.41.2.0169

Kaiser G, Blömeke S, König J, Busse A, Döhrmann M, Hoth J (2017) Professional competencies of (prospective) mathematics teachers—Cognitive versus situated approaches. Educ Stud Math 94(2):161–182. https://doi.org/10.1007/s10649-016-9713-8

Kaiser G, Busse A, Hoth J, König J, Blömeke S (2015) About the complexities of video-based assessments: Theoretical and methodological approaches to overcoming shortcomings of research on teachers' competence. Int J Sci Math Educ 13(2):369–387. https://doi.org/10.1007/s10763-015-9616-7

König J, Santagata R, Scheiner T, Adleff A-K, Yang X, Kaiser G (2022) Teacher noticing: A systematic literature review of conceptualizations, research designs, and findings on learning to notice. Educ Res Rev 36:100453. https://doi.org/10.1016/j.edurev.2022.100453

Landherr A, Friedl B, Heidemann J (2010) A critical review of centrality measures in social networks. Bus Inform Syst Eng 2(6):371–385. https://doi.org/10.1007/s12599-010-0127-3

Leatham KR, Peterson BE, Stockero SL, Zoest LRV (2015) Conceptualizing mathematically significant pedagogical opportunities to build on student thinking. J Res Math Educ 46(1):88–124. https://doi.org/10.5951/jresematheduc.46.1.0088

Lee MY (2021) Improving preservice teachers' noticing skills through technology-aided interventions in mathematics pedagogy courses. Teach Teach Educ 101:103301. https://doi.org/10.1016/j.tate.2021.103301

Lee MY, Choy BH (2017) Mathematical teacher noticing: The key to learning from lesson study. In: EO Schack, MH Fisher, JA Wilhelm (eds) Teacher noticing: Bridging and broadening perspectives, contexts, and frameworks, Springer International Publishing, pp. 121–140

Linnenluecke MK, Marrone M, Singh AK (2020) Conducting systematic literature reviews and bibliometric analyses. Aust J Manage 45(2):175–194. https://doi.org/10.1177/0312896219877678

Louie NL (2018) Culture and ideology in mathematics teacher noticing. Educ Stud Math 97(1):55–69. https://doi.org/10.1007/s10649-017-9775-2

Mason J (2002) Researching your own practice: the discipline of noticing. Routledge

Mason J (2011) Noticing: Roots and branches. In: Sherin MG, Jacobs VR, Philipp RA (eds) Mathematics teacher noticing: Seeing through teachers' eyes, New York, pp. 35–50

McBurney MK, Novak PL (2002) What is bibliometrics and why should you care? Proceedings. IEEE International Professional Communication Conference. pp. 108–114

Mitchell RN, Marin KA (2015) Examining the use of a structured analysis framework to support prospective teacher noticing. J Math Teach Educ 18(6):551–575. https://doi.org/10.1007/s10857-014-9294-3

Page MJ, McKenzie JE, Bossuyt PM, Boutron I, Hoffmann TC, Mulrow CD, Shamseer L, Tetzlaff JM, Akl EA, Brennan SE, Chou R, Glanville J, Grimshaw JM, Hróbjartsson A, Lalu MM, Li T, Loder EW, Mayo-Wilson E, McDonald S, McGuinness LA, Stewart LA, Thomas J, Tricco AC, Welch VA, Whiting P, Moher D (2021) The PRISMA 2020 statement: an updated guideline for reporting systematic reviews. J Clin Epidemiol 134:178–189. https://doi.org/10.1016/j.jclinepi.2021.03.001

Price DJ de S (1961) Science since Babylon. Yale University Press, New Haven, CT

Ren D, Lee B, Brehmer M (2019) Charticulator: Interactive Construction of Bespoke Chart Layouts. IEEE Trans Vis Comput Graph 25(1):789–799. https://doi.org/10.1109/TVCG.2018.2865158

Rodgers C (2002) Seeing student learning: Teacher change and the role of reflection. Harvard Educ Rev 72(2):230–253

Santagata R, Zannoni C, Stigler JW (2007) The role of lesson analysis in pre-service teacher education: An empirical investigation of teacher learning from a virtual video-based field experience. J Math Teach Educ 10(2):123–140. https://doi.org/10.1007/s10857-007-9029-9

Sherin M, van Es E (2008) Effects of video club participation on teachers' professional vision. J Teach Educ 60:20–37. https://doi.org/10.1177/0022487108328155

Small H (1973) Co-citation in the scientific literature: A new measure of the relationship between two documents. J Am Soc Inform Sci 24(4):265–269

Star JR, Lynch KH, Perova N (2011) Using video to improve mathematics' teachers' abilities to attend to classroom features: A replication study. In: M G Sherin, V R Jacobs, R A Philipp (eds) Mathematics teacher noticing: Seeing through teachers' eyes, New Yor, pp. 117–133

Star JR, Strickland SK (2008) Learning to observe: Using video to improve pre-service mathematics teachers' ability to notice. J Math Teach Educ 11(2):107–125. https://doi.org/10.1007/s10857-007-9063-7

Ulusoy F, Çakıroğlu E (2021) Exploring prospective teachers' noticing of students' understanding through micro-case videos. J Math Teach Educ 24(3):253–282. https://doi.org/10.1007/s10857-020-09457-1
van Es EA (2011) A framework for learning to notice student thinking. In: Sherin MG, Jacobs VR, Philipp RA (eds) Mathematics teacher noticing: Seeing through teachers' eyes, New York, pp. 134–151
van Es EA, Sherin MG (2002) Learning to notice: Scaffolding new teachers' interpretations of classroom interactions. J Technol Teach Educ 10(4):571–596
van Es EA, Sherin MG (2008) Mathematics teachers' "learning to notice" in the context of a video club. Teach Teach Educ 24(2):244–276. https://doi.org/10.1016/j.tate.2006.11.005
van Es EA, Sherin MG (2010) The influence of video clubs on teachers' thinking and practice. J Math Teach Educ 13(2):155–176. https://doi.org/10.1007/s10857-009-9130-3
Wallin AJ, Amador JM (2019) Supporting secondary rural teachers' development of noticing and pedagogical design capacity through video clubs. J Math Teach Educ 22(5):515–540. https://doi.org/10.1007/s10857-018-9397-3
Yang Y, Ricks TE (2012a) How crucial incidents analysis support Chinese lesson study. Int J Lesson Learn Stud 1:41–48. https://doi.org/10.1108/20468251211179696
Yang Y, Ricks TE (2012b) Chinese lesson study: Developing classroom instruction through collaborations in school-based teaching research group activities. In: Y Li, R Huang (eds), How Chinese teach mathematics and improve teaching, New York, pp. 51–65

Author contributions
YW carried out data collection, analysis, visualization, and composition of this article; MC and JG reviewed the article's references and formatting requirements; and QZ conceptualized the whole framework, revised meticulously the essential knowledge content, and, ultimately, granted approval for the publication of the final version.

Competing interests
This study is free from conflicts of interest of the investigators, ethics committee members, guardians of the subjects, and those associated with the disclosure of research results. All authors declare that there is no conflict of interest.

Ethical approval
This article does not contain any studies with human participants or animals performed by any of the authors. Ethical approval is not required.

Informed consent
This article does not contain any studies with human participants or animals performed by any of the authors. Informed consent is not applicable.

Additional information
Correspondence and requests for materials should be addressed to Qiaoping Zhang.

Reprints and permission information is available at http://www.nature.com/reprints

Publisher's note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.