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ARTICLE

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Examining the superiority of the Sharpe single-index model of portfolio selection: A study of the Indian mid-cap sector

Janki Mistry [1]✉ & Ritesh Ashok Khatwani [2]

The purpose of the article was to examine the superiority and efficacy of Sharpe's single-index model of portfolio optimisation. The study has attempted to build an optimal portfolio of Indian mid-cap companies using William Sharpe's single-index model. The methodology is also known as the Market model. A portfolio was selected from the Nifty mid-cap 100 index of the NSE. MS-Excel 365 has been used for the analysis. The optimal portfolio returns during the fixed period of analysis were compared with the returns of the benchmark market portfolio. The return of the optimal portfolio using Sharpe's model was found to be considerably higher than the benchmark market portfolio and the risk of the same was found to be much lower. Hence it could be established that in the five years of the study period, the optimal portfolio outperformed the benchmark market portfolio—the Nifty mid-cap 100 index. The selected optimal portfolio was also found to be well diversified comprising 11 securities and eight sectors. The limitation of this model is that it was based on historical data and hence, in case of extreme market conditions, the optimal portfolio could fail to give superior returns. Sharpe's model resolves most of the technical difficulties of the earlier portfolio models and can very well be used by individual investors as well as portfolio managers worldwide to build optimal portfolios. The study has focussed on the mid-cap sector which is riskier than the large-cap sector. This is one of the first studies which has shown the efficacy of the single-index model for mid-cap companies in India.

[1] Department of Business and Industrial Management, Veer Narmad South Gujarat University, Surat, India. [2] Symbiosis Institute of Business Management, Pune, Symbiosis International (Deemed) University, Pune, India. ✉email: jdmistry@vnsgu.ac.in

Introduction

Harry Markowitz laid down the foundation of the Modern Portfolio theory in 1951. Although his model was theoretically sound, it had certain limitations which were later rectified by his own protégé William Sharpe through his single-index model[1]. The original model given by Markowitz was based on the premise that there are gains from diversification. He propounded through his theory, a method for diversification of securities. Grouping securities with negative relationships given by covariance or correlation coefficients was imperative to his optimisation method (Markowitz, 1952). This model required a large number of inputs. For n securities, the number of inputs according to the Markowitz model would be $\frac{n^2-n}{2}$. So, if the portfolio manager wanted to select a portfolio from the total number of listed securities in the Indian markets like the NSE[2] (having around1800 listed securities), or the BSE (with around 4000 listed securities), the number of estimates would run into millions. The quality of these inputs would also affect the optimum portfolio. For example, a classic failure of the model would be when there was a portfolio with three securities A, B, and C with weights 1, 1, and −1 and each having a standard deviation of 20 percent. In this situation, the portfolio variance would be −200—an absurd result as risk can never be a negative number. Another problem was the concept of selecting securities having negative covariances. In the real world, securities tend to move together and hence are found to have positive covariances. Since markets are often moved by sentiments, securities tend to move together in one direction. Empirical evidence also showed that portfolios with securities having positive covariances outperformed Markowitz's optimum portfolios (Sharpe, 1963). Owing to the limitations discussed and the contrary empirical evidence, the need for a simpler method for portfolio selection had become imperative. William Sharpe was a doctoral student at UCLA majoring in economics and finance. When the time came for Sharpe to write his thesis, Fred Weston suggested that he should meet Markowitz. Thus, Markowitz became Sharpe's unofficial thesis advisor. Markowitz put him to work and asked him to find a simpler method for portfolio selection and optimisation. Sharpe simplified the model which we now know as the 'market model' or the single-index model (Varian, 1993). Sharpe said that instead of comparing each security with another security and trying to find negative covariances and correlations between individual securities for portfolio selection, securities should be compared to some common index. This gave birth to the concept of the market index. Sharpe reasoned that common economic factors such as business cycles, interest rates, technology changes, cost of labour, raw material, inflation, weather conditions, etc., affected the performance of all firms. Unexpected changes in these variables would cause unexpected changes in the prices and returns of all the stocks in the market. Sharpe proposed that all the economic factors could be summarised by one macroeconomic indicator which would move the entire market. Further, it was assumed that all other uncertainties in stock returns were firm-specific, i.e., there was no other form of correlation between the securities. Firm-specific events such as profits, management quality, new inventions, etc., would only affect the fortunes of individual firms and not the whole market or the broad economy in any significant way. Thus, Sharpe proposed the concept of a single market index as the surrogate for all the other individual securities in the market (Sharpe, 1963). Markowitz and Sharpe were awarded the Nobel Prize for their contributions to Modern Portfolio theory.

Review of literature

The Markowitz model has stood out to be a substantive contribution to the theory of individual asset demand under uncertainty. However, it does have limitations at the axiomatic level. Tobin's extension of the introduction of the risk-free rate to the model given by Markowitz made it even more convincing. Its limitations were resolved to a large extent by the improvisations of Sharpe (1963), Lintner (1975), and Mossin (1966). Harris (1980) pointed out that the model had several applications and had been subjected to rigorous empirical testing which led to important propositions about the risk and its effect on the pricing of the assets. The CAPM and the single-index model received more validation in 1982 when an article published in the Harvard Business Review validated that the model provided a methodology for quantifying risk and translating that risk into estimates of expected return on equity (Mullins, 1982). Kaplan and Seigel in the year 1995 defended the mean-variance concept of Markowitz and brought out that even though the model might not be very practical, it should be understood in terms of a broad perspective. Later, Kaplan went on to create a very generalised functional model based on the pillars of the Markowitz model which he aptly renamed Markowitz 2.0. In Kaplan's model, the user could select a measure of return and a measure of risk and have a wide choice of return distribution models (2017). Despite getting worldwide acclaim as a breakthrough model, the problems of real-world investment constraints such as cardinality and floor-ceiling drawbacks, meta-heuristic techniques were used by many economists to build optimal portfolios. However, the mean-variance model has usually been the base model which would then be modified to build further models with better optimisation such as the mean-semivariance portfolio selection model (Yahaya, 2010). Studies selecting portfolios based on the fundamentals of stocks such as book-to-market ratio or certain investment indices have also been used by investors to build optimal portfolios (Khatwani, 2021). However, these portfolios again were based on stock variables like balance sheet data and not flow variables.

Several studies in India and abroad (Saravanan and Natarajan, 2012; Dharmalingam and Gurunathan, 2021; Mahmud, 2020; Mandal, 2013; Mohith et al., 2017) have embraced the Sharpe single-index model to prove portfolio efficacy as the model is versatile and it can accommodate changes based on the study being conducted (Lal and Rao, 2016). It is a very simple model as compared to the Markowitz model because it requires very few inputs (Bodie et al., 2020). Based on a study conducted in the Indian pharma sector, an optimal portfolio of pharma companies was created using the Sharpe single-index model. It was found that this portfolio outperformed the Nifty index. A portfolio of 12 stocks was created, and the intrinsic value of the shares selected in the portfolio was calculated. The final portfolio was selected of stocks that showed progressive intrinsic value (S. Sangeetha et al., 2021). Another Indian study that used the mean-variance design to optimise portfolios using Sharpe, Sortino, and Calmar Ratio took samples from six important sectors from the NSE of India. The study, which took place between January 2017 and December 2020, sought to discover the ratio that produced the highest cumulative returns for both the training and test periods for the majority of industries (Sen and Dutta, 2022). A similar Indian study compared 11 sectors using the Sharpe single-index model. This was a novel study wherein the Sharpe model was used to give sectoral weights in a portfolio rather than finding the weights of individual stocks. This study reinforces the

versatility of the model (Lal and Rao, 2016). The sectors with the highest cumulative returns for the same ratio were also determined. A Nigerian study used the Sharpe single-index model to find an optimal portfolio of five stocks which decreased the risk of the original 20-stock portfolio (Yahayah and Ikani, 2020). Another such study conducted in Bangladesh for 178 companies listed on the Dhaka Stock Exchange helped build an optimal portfolio of 54 companies and this portfolio had a better risk-return combination than the index as well as individual companies and outperformed both (Mahmud, 2019). An interesting approach to the Sharpe model was brought about by two Indian researchers who analysed the fundamentals of the securities selected by the model. The model basically selected securities based on the yields/returns. But at times the high yields could be on account of bubbles due to insider trading. A more refined approach was adopted by checking the stock fundamentals of the model-selected portfolio (Yadav and Sharma, 2020). Several studies across India examined the efficacy of the single-index model for the stocks listed on the BSE. Nalini (2014) studied 15 Indian stocks and selected four stocks to form an optimal portfolio using the single-index model from the S&P BSE index. Gupta (2008) examined daily market data from April 1997 to April 2007 on a sample of ten industry sectors chosen at random and discovered that investors could significantly improve their reward to risk when compared to market returns. The Sharpe ratio of the optimised portfolio rose from 0.527 to 0.994 (for the S&P Nifty index). Many such sectoral studies have also been conducted for Indian companies listed on the BSE and NSE (Ahuja, 2017; Anithadevi and Mallikharjunarao, 2017; R. and Reddy, 2022; Shriguru and Bagrecha, 2022).

Just as there are enough studies to prove that Sharpe's model works, there have been studies that prove the contrary. One such study by the Lahore School of Economics has empirically shown that Sharpe's model does not actually build an optimal portfolio. However, the final verdict of the same study was that since in reality the true market portfolio cannot be observed, it is impossible to disregard the model (Naqvi, 2000).

The current study was taken up to check the efficacy of this model for mid-cap stocks which are riskier than large-cap stocks. Usually, it is more difficult to build a mid-cap stock portfolio than a large-cap stock portfolio due to the volatility of the securities as well the irregularity in the cashflows of companies. Several studies have been found on the stocks listed on the Bombay Stock Exchange and the National Stock Exchange in the Indian context. However, the studies which were reviewed for this paper mostly focussed on large-cap companies (Saravanan and Natarajan, 2012; Ahuja, 2017; Anithadevi and Mallikharjunarao, 2017; Dharmalingam and Gurunathan, 2021; Gupta, 2008; Lal and Rao, 2016; Mandal, 2013; Mohith et al., 2017; Nalini, 2014; S. Sangeetha et al., 2021; Sen and Dutta, 2022; Shriguru and Bagrecha, 2022). The studies conducted in other countries such as Bangladesh also focussed on A group (large cap) companies listed on the Dhaka and the Chittagong stock exchanges (Mahmud, 2019, 2020). Similarly, the study conducted in Nigeria also focussed on large-cap companies listed on the Nigerian stock exchange (Yahayah and Ikani, 2020). Hence, in reviewing the literature, no studies on the mid-cap sector could be identified in India or abroad. Therefore, this study becomes important for investors who might have a preference for investing in the mid-cap sector in India. This study was taken up to incorporate companies listed on the NSE. Many studies have used the model to build optimal portfolios, but the comparative analysis of portfolios constructed with and without the use of the single-index model was seen to be lacking in the studies. This study was taken up to build an optimum portfolio in the mid-cap sector and then compare the risk and return of that portfolio with the risk and return of the benchmark index i.e., the NSE mid-cap 100 index.

Methodology

The study is based on using the Sharpe single-index model to build an optimal portfolio of mid-cap stocks. Since it was established that diversification is a valid strategy for return optimisation, every investor would want to build a portfolio of investments instead of holding individual stocks. This diversification could be across instruments/assets, industries, or even economies. However, retail investors would want to build diversified portfolios across industries and companies. *This study has tried to evaluate if a mid-cap stock portfolio constructed using Sharpe's single-index model outperforms the benchmark index.* The study was based on secondary data of adjusted closing prices collected from the website of the NSE of India. Yearly data for 5 years were considered for the study. The stocks were selected from the Nifty mid-cap index of the top 100 mid-cap companies listed on the NSE. The NIFTY mid-cap 100 Index is useful for understanding market movement in the mid-cap segment. It includes 100 stocks listed for trading on the National Stock Exchange (NSE) of India. The index is calculated using the free float market capitalisation method, with the level of the index reflecting the total free float market value of all the stocks in the index relative to a specific base market capitalisation value. The NIFTY mid-cap 100 index can be used for a variety of purposes, including fund portfolio benchmarking, the launch of index funds, ETFs[3], and structured products (NSE India, 2022). Mid-cap stocks carry greater risk than large-cap stocks. However, they have the potential to yield extremely high returns. Selection of good mid-cap stocks at the right time can be compared to sitting on an undiscovered gold mine. A unique feature of mid-cap stocks is that they would be re-rated if they came under the radar of institutional buyers. That is the time when the value of these stocks could skyrocket. Good mid-cap stocks would certainly outperform large-cap stocks as well as the market index in the bull run. However, they could also plummet very fast during the bear run. Nevertheless, with the proper stock-selection skills and investment disposition, these stocks have the potential to generate very attractive returns. Despite the fact that mid-caps have a long history of strong returns, not all mid-caps are profitable investments. Therefore, an investor seeking to profit from mid-caps must exercise extreme caution. Usually, the value of mid-cap stocks is derived from their growth potential, but this growth potential does not always materialise. This can result in significant value loss. Compared to large-cap stocks, mid-cap stocks represent relatively young companies. Mid-cap investors must be extremely patient, as these companies are typically in the early stages of a business cycle and can take a long time to realise their full potential (10 Important Facts about Indian Mid-Cap Stocks, 2022). However, with the right stock selection, investors could amass a modest fortune by investing in mid-cap stocks. Therefore, it becomes even more important to construct an optimal portfolio with mid-cap stocks.

Owing to these unique characteristics of mid-cap companies, an optimal portfolio was created using Sharpe's single-index model. Microsoft Excel 365 was used to analyse the data. Microsoft Excel is a spreadsheet programme developed by Microsoft Inc. that is available for Windows, macOS, Android, and iOS. It includes calculating or computation skills, graphing tools, pivot tables, and Visual Basic for Applications, a macro programming language (VBA). Excel is a part of the Microsoft Office software suite.

Steps to calculate the optimal portfolio using the Sharpe single-index model:

i. The yearly returns of all stocks in the Nifty 100 mid-cap index were calculated using the logarithmic method. This was done using the log function in MS[4] Excel given by Eq. (1).

$$\text{Log}\, P_1/P_0 \tag{1}$$

where P_1 is the closing price of the stock in year 1 and P_0 is the closing price of the stock in year 0.

ii. The Betas of all the stocks were calculated. Beta represents the relationship between the risk of each stock and the market risk. Beta is a coefficient and a measure of systematic risk of security. Beta in this study shows the rate of change in the mid-cap stock due to a unit change in the benchmarked market index. Betas were calculated using the regression function in MS Excel. The regression function is a part of the Data analysis tool pack add-in available in MS Excel.

The systematic risk and unsystematic risk of the stock were calculated using Eq. (2) as follows:

$$\begin{aligned}\text{Systematic risk of mid} - \text{cap security} &= \text{Beta}^{2}* \\ \text{variance of index} &= \beta^{2}*\sigma_{\text{m}}^{2}\end{aligned} \tag{2}$$

iii. The unsystematic risk of each stock in the market was calculated. The unsystematic risk is due to the firm-specific factors of the mid-cap stocks under consideration and is a random error term. It was calculated as a difference between the total risk of the security and the market-related risk. The following formula given by Eq. (3) was inserted in MS Excel to calculate unsystematic risk.

$$\begin{aligned}&\text{Unsystematic risk of mid} - \text{cap security} = \\ &\text{Total variance of security return} - \text{Systematic risk} \\ &\sigma_{\text{ei}}^{2} = \sigma_{\text{i}}^{2} - \beta^{2}*\sigma_{\text{m}}^{2}\end{aligned} \tag{3}$$

where $\sigma_{\text{i}}{}^2$ is the total risk of the mid-cap stock and $\sigma_{\text{m}}{}^2$ is the variance of the NSE 100 mid-cap index.

iv. The risk-free rate R_{f} was taken as 6.1 percent which was the average 10-year G-sec bill rate in India in the year 2021.

v. The securities which had negative returns during the period of study were removed.

vi. The market return was calculated using the closing price data of the Nifty 100 mid-cap index using the logarithmic method in MS Excel.

vii. The excess return over beta ratio was calculated using the formula given by Eq. (4):

$$\frac{R_{\text{i}} - R_{\text{f}}}{\beta} \tag{4}$$

where R_{i} is the security return and R_{f} is the risk-free return

viii. The stocks were rearranged in descending order from highest to lowest values of excess-return-over-beta ratios. This was accomplished using the 'sort' function in MS Excel.

ix. The market return for the period of five years from January 1, 2017 to December 31, 2021 was found to be 11.37 percent, and the market risk was calculated and found to be 9.81 percent.

x. The cut-off rate was calculated using the formula for cut-off rate using Eq. (5):

$$C = \frac{\sigma_{\text{m}}^{2}\sum_{t=1}^{j}\frac{(R_{\text{i}}-R_{\text{f}})\beta_i}{\sigma_{\text{ei}}^{2}}}{1+\sigma_{\text{m}}^{2}\sum_{t=1}^{j}\frac{\beta_i^{2}}{\sigma_{\text{ei}}^{2}}} \tag{5}$$

where $\sigma_{\text{m}}{}^2$ = market variance; $\sigma_{\text{ei}}{}^2$ = stock variance. The above formula was manually entered in MS Excel to get the cut-off rate.

i. The excess return over beta ratios was compared with the cut-off rates.

ii. The securities which had excess return over beta ratio higher than the cut-off rate were selected in the portfolio and others were rejected.

iii. The weights of individual securities were calculated using Eqs. (6) and (7), and an optimal portfolio was built.

$$X_{\text{i}} = \frac{Z_{\text{i}}}{\sum_{j=1}^{n} Z_j} \tag{6}$$

$$Z_{\text{i}} = \frac{\beta_{\text{i}}}{\sigma_{\text{ei}}^{2}}\left(\frac{R_{\text{i}} - R_{\text{f}}}{\beta_{\text{i}}}\right) - C^{*} \tag{7}$$

X_{i} is the weight of each security; ΣZ_j is the summation of all the Z_is; C^* is the highest cut-off rate selected

The returns of the optimal portfolio over the five-year period were compared to the returns of the market portfolio for the same period.

Assumptions of the Sharpe single-index model:

i. Investors have homogenous expectations with respect to return and risk.
ii. A uniform holding period is considered for calculating the risk and return of every security.
iii. Investors can borrow and lend at a risk-free rate of return.
iv. Price movements of securities are influenced by prevailing economic conditions.
v. The index selected is a proxy of the market.

Limitations of the study:

i. The beta of individual securities is assumed to be constant but, it fluctuates daily.
ii. Only the quantitative aspects in terms of risk and return are being considered but, security prices are affected by infinite reasons.
iii. The risk-free rate of return is also assumed to be constant, but it could change with the review of monetary policy.
iv. The market condition is always uncertain; the result reflects the market of that period.
v. Assumed values would vary from one investor to another investor.

Results and findings

All the securities of the Nifty mid-cap 100 index were analysed and their risks, returns, and betas were calculated. The securities data is given in Table 1:

The companies with negative returns were removed from the list and the excess return over beta and the cut-off rate were calculated after arranging the companies in ascending order of excess return over beta. Table 2 shows the ranking of securities. The list has been furnished in Appendix 1.

The cut-off rates were calculated, and the highest value of C was selected as the cut-off benchmark. The value of the cut-off was 40.535 percent. It has been emboldened in the above Table 2. All the values of C are depicted in Table 3.

The securities with an excess return over beta higher than the cut-off rate were selected to form the optimal portfolio according to the Sharpe single-index model. Table 4 shows the securities that were selected in the optimal portfolio.

Based on the criteria of selection, only eleven companies were selected as part of the optimal portfolio according to the single-index model. The portfolio of the 11 companies has included securities from various sectors such as Healthcare, Information Technology, Fast Moving Consumer Goods, Capital Goods,

Table 1 Calculation of risk, return and beta.

No.	Company name	R_i	σ_i	β_i	σ_{ei}^2
1	ABB	19.86	34.35	1.13	541.25
2	AUBANK	12.79	12.91	0.21	117.72
3	AARTIIND	42.76	19.49	0.71	156.87
4	ABBOTINDIA	33.87	19.98	−0.81	779.87
5	ATGL	183.6	127.33	5.99	4701.58
7	ABFRL	16.72	29.06	0.4	631.82
8	APLLTD	12.01	36.2	−0.27	1509.22
9	ALKEM	18.59	21.37	0.66	221.87
10	APOLLOTYRE	7.03	25.98	0.96	274.31
11	ASHOKLEY	8.81	22.48	0.86	197.22
12	ASTRAL	63.17	33.61	1.03	552.52
13	AUROPHARMA	10.14	47.69	0.16	2127.09
14	BALKRISIND	40.27	46.35	1.55	969.98
16	BATAINDIA	34.34	25.86	−0.42	898.81
17	BEL	16.97	40.26	1.53	637.60
18	BHARATFORG	12.49	29.74	1.15	340.72
20	CANBK	0.12	37.27	1.17	665.24
21	CLEAN	19.45	23	0.87	209.24
22	COFORGE	76.81	25.59	0.85	297.61
23	CONCOR	14.29	30.01	0.78	499.89
24	COROMANDEL	25.42	39.22	0.76	1008.98
25	CROMPTON	21.16	26.2	0.73	362.47
26	CUMMINSIND	6.8	31.44	1.05	446.88
27	DALBHARAT	27.85	38.1	1.87	390.27
28	DEEPAKNTR	97.75	61.92	2.39	1480.26
29	DIXON	102.17	96.97	2.06	5892.31
31	EMAMILTD	5.03	30.37	1.14	368.13
32	ESCORTS	48.78	51.46	1.67	1230.42
34	FEDERALBNK	4.27	24.29	0.71	300.15
35	FORTIS	15.56	39.61	1.09	836.20
37	GODREJPROP	46.82	38.39	0.91	868.06
38	GUJGASLTD	223.66	389.02	−6.46	204,659.06
39	GSPL	16.97	24.6	0.77	290.58
40	HAL	5.04	26.88	1.13	249.47
41	HINDPETRO	2.57	29.27	1.01	374.88
42	HINDZINC	3	19.55	0.68	165.87
43	ISEC	25.53	42.55	1.2	947.29
46	INDIAMART	92.52	84.45	−1.33	9505.72
48	IEX	70.73	96.34	3.78	3511.53
49	INDHOTEL	13.9	24.31	0.32	448.20
50	IRCTC	83.97	77.11	3.74	1633.82
51	IGL	25.89	38.03	1.05	768.93
52	IPCALAB	35.89	35.48	−0.47	1607.26
53	JSWENERGY	71.41	133.91	4.16	8667.68
54	JINDALSTEL	48.07	62.58	1.93	1905.03
55	L&TFH	2.77	38.9	0.75	994.93
56	LTTS	54.38	50.87	1.05	1645.88
57	LICHSGFIN	2.88	25.77	0.63	383.76
58	LAURUSLABS	86.53	156.52	1.78	19,337.18
59	MRF	8.64	17.3	0.38	184.20
60	M&MFIN	3.72	37.36	1.4	558.19
62	MFSL	14.71	24.29	0.51	371.98
63	MAXHEALTH	60.75	85.41	1.58	4887.44
64	METROPOLIS	53.09	15.91	0.51	118.96
65	MPHASIS	48.54	42.62	1.2	951.60
66	NATIONALUM	17.74	59.82	2.02	1600.30
67	NAVINFLUOR	61.22	58.72	1.13	2269.06
68	NAM-INDIA	20	62.38	−0.1	4014.62
69	OBEROIRLTY	24.86	25	0.86	274.35
70	OIL	4.88	42.04	1.35	829.24
71	OFSS	5.67	19.82	0.76	152.88
72	POLICYBZR	10.1	28.3	1.43	203.68
73	PAGEIND	27.37	31.59	1.2	392.75
74	PERSISTENT	73.19	89.11	2.84	3751.51
75	PETRONET	4.18	19.77	0.12	345.69
76	POLYCAB	65.58	55.35	1.65	1533.78
78	PRESTIGE	33.29	50.03	1.4	1317.40
80	SRTRANSFIN	8.24	25.7	0.78	325.74
81	SONACOMS	15.41	30.75	0.99	442.60
82	SUNTV	6.57	43.18	1.35	896.19
83	SYNGENE	22.7	38.89	−0.02	1527.73
84	TVSMOTOR	17.49	44.39	1.46	904.05
85	TATACHEM	19.25	42.17	1.29	871.14
86	TATACOMM	30.85	72.59	1.05	3879.98
87	TATAELXSI	69.67	87.87	2.86	3577.64
88	TATAPOWER	38.91	78.29	2.55	2838.17
89	RAMCOCEM	8.51	15.31	0.44	120.86
90	TORNTPOWER	26.54	29.09	1.13	324.17
91	TRENT	35.71	15.6	0.48	118.62
92	TRIDENT	96.4	166.65	5.12	13,554.27
94	UBL	15.95	18.98	0.32	250.93
95	VBL	37.65	12.82	0.43	73.99
97	VOLTAS	35.28	38.16	1.3	645.52
98	WHIRLPOOL	22.81	39.17	−0.08	1596.39

Consumer Services, Chemicals and Pharmaceuticals, Consumer Durables, and Realty. Hence, it can be said that the portfolio of mid-cap stocks selected by the model was fairly diversified.

The weights of individual securities selected were found and are depicted in Table 5.

According to the model, it could be observed that the maximum investment of around 31.07 percent should be recommended in Metropolis Healthcare Ltd., followed by Varun Beverages at around 26.8 percent. 17 percent of the investment amount should be placed in Coforge Ltd., 12.02 percent in Trent Ltd., and around 7 percent in Aarti Industries Ltd. About 3.89 percent of the portfolio should comprise the stock of Astral Ltd. These six stocks would comprise around 97.9 percent of the portfolio. The remaining 2.1 percent is divided into the remaining five stocks namely Navin Fluorine International Ltd., Dixon Technologies, Laurus Labs Ltd., Godrej Properties Ltd., and Metropolis Healthcare Ltd.

The return of the market portfolio was then calculated using the weights of the selected securities given by Eq. (8).

$$R_P = \sum_{i=1}^{n} X_i R_i \tag{8}$$

where X_i is the weight of security i and R_i is the return of security i.

The return of the optimum portfolio using the Sharpe single-index model was found to be 50.76 percent during the 5 years between January 2017 and December 2021. The return of the benchmark index Nifty mid-cap 100 index was found to be 15.61 percent during the same period.

The risk of both portfolios was also calculated using Eq. (9):

$$\sigma_P = \sqrt{\left(w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + \ldots .W_n^2\sigma_n^2 + 2w_1w_2\mathrm{Cov}_{1,2} + \ldots .2w_{n-1}w_n\mathrm{Cov}_{n-1,n}\right)} \tag{9}$$

The risk of Sharpe's optimal portfolio was 12.861 percent whereas the risk of the market portfolio was 23.02 percent.

Discussion

Market indices are benchmark portfolios that represent the performance of all the companies in the market. Hence, the index can be considered the proxy of the market portfolio which is the combination of all the risky securities in the market. In this case, the Nifty mid-cap 100 index represents the mid-cap segment of the NSE. According to the capital market line of the Capital Asset Pricing Model, the market portfolio is also a part of the risk-return line depicting all the fairly priced optimum portfolios of the market. This would mean that all the portfolios on the capital market line which lie to the left of the market portfolio would have a lower return and risk combination than the market portfolio and all the portfolios which lie to the right would have a higher risk-return combination than the market portfolio. However, the Sharpe single-index model helps the investor to identify a portfolio that has a higher return than the market portfolio with a lower risk than the market portfolio during the same period. The optimal portfolio using the Sharpe model has a mean return of 50.76 percent per year and a risk of 12.861 percent, whereas the index portfolio has a return of 15.61 percent and a very high risk of 23.02 percent. Hence, it can be inferred that investors would be better off investing in a portfolio suggested by the Sharpe model rather than investing in the market index made up of the top 100 mid-cap companies. It also gives a perspective on over-diversification. The Nifty mid-cap 100 index gave lower returns for a higher level of risk. One reason for that could be over-diversification which leads to a decrease in returns. Theoretically speaking, according to the CAPM, if an investor wants to move to

Table 2 Ranking securities and calculation of cut-off rate.

No.	Company	$(R_i-R_f)/\beta_i$	Rank	$(R_i-R_f)*\beta_i$	$(R_i-R_f)\beta_i/\sigma_{ei}^2$	$\Sigma(R_i-R_f)\beta_i/\sigma_{ei}^2$	$(\beta_i^2)/\sigma_{ei}^2$	$\Sigma(\beta_i^2)/\sigma_{ei}^2$
64	METROPOLIS	92.137	1	23.965	0.201	0.201	0.002	0.002
22	COFORGE	83.188	2	60.104	0.202	0.403	0.002	0.005
95	VBL	73.372	3	13.567	0.183	0.587	0.002	0.007
91	TRENT	61.688	4	14.213	0.120	0.707	0.002	0.009
12	ASTRAL	55.408	5	58.782	0.106	0.813	0.002	0.011
3	AARTIIND	51.634	6	26.029	0.166	0.979	0.003	0.014
67	NAVINFLUOR	48.779	7	62.286	0.027	1.006	0.001	0.015
29	DIXON	46.636	8	197.904	0.034	1.040	0.001	0.015
56	LTTS	45.981	9	50.694	0.031	1.071	0.001	0.016
58	LAURUSLABS	45.185	10	143.165	0.007	1.078	0.000	0.016
37	GODREJPROP	44.747	11	37.055	0.043	1.121	0.001	0.017
28	DEEPAKNTR	38.347	12	219.044	0.148	1.269	0.004	0.021
76	POLYCAB	36.048	13	98.142	0.064	1.333	0.002	0.023
65	MPHASIS	35.367	14	50.928	0.054	1.386	0.002	0.024
63	MAXHEALTH	34.589	15	86.347	0.018	1.404	0.001	0.025
2	AUBANK	31.857	16	1.405	0.012	1.416	0.000	0.025
94	UBL	30.781	17	3.152	0.013	1.428	0.000	0.026
5	ATGL	29.633	18	1063.225	0.226	1.655	0.008	0.033
7	ABFRL	26.550	19	4.248	0.007	1.661	0.000	0.034
32	ESCORTS	25.557	20	71.276	0.058	1.719	0.002	0.036
24	COROMANDEL	25.421	21	14.683	0.015	1.734	0.001	0.036
13	AUROPHARMA	25.250	22	0.646	0.000	1.734	0.000	0.036
49	INDHOTEL	24.375	23	2.496	0.006	1.740	0.000	0.037
74	PERSISTENT	23.623	24	190.536	0.051	1.790	0.002	0.039
86	TATACOMM	23.571	25	25.988	0.007	1.797	0.000	0.039
97	VOLTAS	22.446	26	37.934	0.059	1.856	0.003	0.042
87	TATAELXSI	22.227	27	181.810	0.051	1.907	0.002	0.044
14	BALKRISIND	22.045	28	52.964	0.055	1.961	0.002	0.046
69	OBEROIRLTY	21.814	29	16.134	0.059	2.020	0.003	0.049
54	JINDALSTEL	21.746	30	81.002	0.043	2.063	0.002	0.051
50	IRCTC	20.821	31	291.234	0.178	2.241	0.009	0.060
25	CROMPTON	20.630	32	10.994	0.030	2.271	0.001	0.061
78	PRESTIGE	19.421	33	38.066	0.029	2.300	0.001	0.063
9	ALKEM	18.924	34	8.243	0.037	2.337	0.002	0.065
51	IGL	18.848	35	20.780	0.027	2.364	0.001	0.066
90	TORNTPOWER	18.088	36	23.097	0.071	2.436	0.004	0.070
73	PAGEIND	17.725	37	25.524	0.065	2.501	0.004	0.074
92	TRIDENT	17.637	38	462.336	0.034	2.535	0.002	0.076
48	IEX	17.098	39	244.301	0.070	2.604	0.004	0.080
62	MFSL	16.882	40	4.391	0.012	2.616	0.001	0.080
43	ISEC	16.192	41	23.316	0.025	2.641	0.002	0.082
53	JSWENERGY	15.700	42	271.690	0.031	2.672	0.002	0.084
21	CLEAN	15.345	43	11.615	0.056	2.728	0.004	0.087
39	GSPL	14.117	44	8.370	0.029	2.756	0.002	0.090
88	TATAPOWER	12.867	45	83.666	0.029	2.786	0.002	0.092
1	ABB	12.177	46	15.549	0.029	2.815	0.002	0.094
27	DALBHARAT	11.631	47	40.673	0.104	2.919	0.009	0.103
23	CONCOR	10.500	48	6.388	0.013	2.932	0.001	0.104
85	TATACHEM	10.194	49	16.964	0.019	2.951	0.002	0.106
81	SONACOMS	9.404	50	9.217	0.021	2.972	0.002	0.108
35	FORTIS	8.679	51	10.311	0.012	2.984	0.001	0.110
84	TVSMOTOR	7.801	52	16.629	0.018	3.003	0.002	0.112
17	BEL	7.105	53	16.631	0.026	3.029	0.004	0.116
59	MRF	6.684	54	0.965	0.005	3.034	0.001	0.117
66	NATIONALUM	5.762	55	23.513	0.015	3.049	0.003	0.119
18	BHARATFORG	5.557	56	7.349	0.022	3.070	0.004	0.123
89	RAMCOCEM	5.477	57	1.060	0.009	3.079	0.002	0.125
11	ASHOKLEY	3.151	58	2.331	0.012	3.091	0.004	0.128
72	POLICYBZR	2.797	59	5.720	0.028	3.119	0.010	0.139
80	SRTRANSFIN	2.744	60	1.669	0.005	3.124	0.002	0.140
10	APOLLOTYRE	0.969	61	0.893	0.003	3.127	0.003	0.144
26	CUMMINSIND	0.667	62	0.735	0.002	3.129	0.002	0.146
82	SUNTV	0.348	63	0.635	0.001	3.130	0.002	0.148
71	OFSS	−0.566	64	−0.327	−0.002	3.127	0.004	0.152
70	OIL	−0.904	65	−1.647	−0.002	3.125	0.002	0.154
40	HAL	−0.938	66	−1.198	−0.005	3.121	0.005	0.159
31	EMAMILTD	−0.939	67	−1.220	−0.003	3.117	0.004	0.163
60	M&MFIN	−1.700	68	−3.332	−0.006	3.111	0.004	0.166
34	FEDERALBNK	−2.577	69	−1.299	−0.004	3.107	0.002	0.168
41	HINDPETRO	−3.495	70	−3.565	−0.010	3.098	0.003	0.171
55	L&TFH	−4.440	71	−2.498	−0.003	3.095	0.001	0.171
42	HINDZINC	−4.559	72	−2.108	−0.013	3.082	0.003	0.174
20	CANBK	−5.111	73	−6.997	−0.011	3.072	0.002	0.176
57	LICHSGFIN	−5.111	74	−2.029	−0.005	3.066	0.001	0.177
75	PETRONET	−16.000	75	−0.230	−0.001	3.066	0.000	0.177
8	APLLTD	−21.889	76	−1.596	−0.001	3.065	0.000	0.177
38	GUJGASLTD	−33.678	77	−1405.438	−0.007	3.058	0.000	0.178
4	ABBOTINDIA	−34.284	78	−22.494	−0.029	3.029	0.001	0.178
52	IPCALAB	−63.383	79	−14.001	−0.009	3.020	0.000	0.179
46	INDIAMART	−64.977	80	−114.939	−0.012	3.008	0.000	0.179
16	BATAINDIA	−67.238	81	−11.861	−0.013	2.995	0.000	0.179
68	NAM-INDIA	−139.000	82	−1.390	0.000	2.995	0.000	0.179
98	WHIRLPOOL	−208.875	83	−1.337	−0.001	2.994	0.000	0.179
83	SYNGENE	−830.000	84	−0.332	0.000	2.994	0.000	0.179

Table 3 Selection of securities with excess return to beta higher than cut-off rate.

No	Company	Rank	$(R_i - R_f)/\beta$	C
64	METROPOLIS	1	92.137	16.017
22	COFORGE	2	83.188	26.884
95	VBL	3	73.372	33.521
91	TRENT	4	61.688	36.334
12	ASTRAL	5	55.408	38.049
3	AARTIIND	6	51.634	39.825
67	NAVINFLUOR	7	48.779	40.025
29	DIXON	8	46.636	40.209
56	LTTS	9	45.981	40.355
58	LAURUSLABS	10	45.185	40.384
37	GODREJPROP	11	44.747	**40.535**
28	DEEPAKNTR	12	38.347	40.267
76	POLYCAB	13	36.048	40.042
65	MPHASIS	14	35.367	39.839
63	MAXHEALTH	15	34.589	39.763
2	AUBANK	16	31.857	39.680
94	UBL	17	30.781	39.579
5	ATGL	18	29.633	37.843
7	ABFRL	19	26.550	37.778
32	ESCORTS	20	25.557	37.179
24	COROMANDEL	21	25.421	37.035
13	AUROPHARMA	22	25.250	37.032
49	INDHOTEL	23	24.375	36.971
74	PERSISTENT	24	23.623	36.388
86	TATACOMM	25	23.571	36.314
97	VOLTAS	26	22.446	35.617
87	TATAELXSI	27	22.227	35.054
14	BALKRISIND	28	22.045	34.488
69	OBEROIRLTY	29	21.814	33.914
54	JINDALSTEL	30	21.746	33.528
50	IRCTC	31	20.821	31.975
25	CROMPTON	32	20.630	31.742
78	PRESTIGE	33	19.421	31.491
9	ALKEM	34	18.924	31.162
51	IGL	35	18.848	30.931
90	TORNTPOWER	36	18.088	30.302
73	PAGEIND	37	17.725	29.753
92	TRIDENT	38	17.637	29.481
48	IEX	39	17.098	28.921
62	MFSL	40	16.882	28.828
43	ISEC	41	16.192	28.620
53	JSWENERGY	42	15.700	28.347
21	CLEAN	43	15.345	27.866
39	GSPL	44	14.117	27.585
88	TATAPOWER	45	12.867	27.255
1	ABB	46	12.177	26.915
27	DALBHARAT	47	11.631	25.709
23	CONCOR	48	10.500	25.548
85	TATACHEM	49	10.194	25.296
81	SONACOMS	50	9.404	25.000
35	FORTIS	51	8.679	24.807
84	TVSMOTOR	52	7.801	24.480
17	BEL	53	7.105	23.975
59	MRF	54	6.684	23.869
66	NATIONALUM	55	5.762	23.513
18	BHARATFORG	56	5.557	22.991
89	RAMCOCEM	57	5.477	22.783
11	ASHOKLEY	58	3.151	22.253
72	POLICYBZR	59	2.797	20.942
80	SRTRANSFIN	60	2.744	20.716
10	APOLLOTYRE	61	0.969	20.286
26	CUMMINSIND	62	0.667	19.977
82	SUNTV	63	0.348	19.725
71	OFSS	64	−0.566	19.253
70	OIL	65	−0.904	18.984
40	HAL	66	−0.938	18.383
31	EMAMILTD	67	−0.939	17.990
60	M&MFIN	68	−1.700	17.599
34	FEDERALBNK	69	−2.577	17.409
41	HINDPETRO	70	−3.495	17.095
55	L&TFH	71	−4.440	17.028
42	HINDZINC	72	−4.559	16.702
20	CANBK	73	−5.111	16.461
57	LICHSGFIN	74	−5.111	16.342
75	PETRONET	75	−16.000	16.335
8	APLLTD	76	−21.889	16.325
38	GUJGASLTD	77	−33.678	16.271
4	ABBOTINDIA	78	−34.284	16.046
52	IPCALAB	79	−63.383	15.988
46	INDIAMART	80	−64.977	15.908
16	BATAINDIA	81	−67.238	15.822
68	NAM-INDIA	82	−139.000	15.820
98	WHIRLPOOL	83	−208.875	15.815
83	SYNGENE	84	−830.000	15.814

Bold values indicates the highest value of c and shall be selected as cut-off rate.

Table 4 Securities selected for the optimal portfolio.

No.	Company	Sector
1	Metropolis Healthcare Ltd.	Healthcare
2	Coforge Ltd.	Information Technology
3	Varun Beverages Ltd.	Fast-Moving Consumer Goods
4	Trent Ltd.	Consumer Services
5	Astral Ltd.	Capital Goods
6	Aarti Industries Ltd.	Chemicals and Pharmaceuticals
7	Navin Fluorine International Ltd.	Chemicals
8	Dixon Technologies (India) Ltd.	Consumer Durables
10	Laurus Labs Ltd.	Healthcare
11	Godrej Properties Ltd.	Realty

a point to the right of the market index, he/she would only be able to do so by borrowing at the risk-free rate of return, which would mean short selling. However, the Sharpe single-index model can give an optimal portfolio without short sales.

The Sharpe model resolves most of the problems of the Markowitz mean-variance model. The calculation of the optimal portfolio becomes easy. The optimal portfolio in this study was found to be a well-diversified portfolio of 11 securities comprising eight sectors. However, the limitation of this model is that it is based on historical data. There would be many instances when the optimal portfolio does not perform as predicted. The portfolio selection by this model is a static process whereas no other market is as dynamic as the stock market. The assumption of constant beta is also a flawed one as is the assumption of a constant risk-free rate of return.

The study has been able to prove that within a fixed time period, Sharpe's optimal portfolio outperforms the market index. In this study, the optimal portfolio has a mean return that far exceeds the index return. Hence, it can be inferred that diversification using a linear mathematical model such as the Sharpe single-index model can lead to higher returns at lower risk.

The same model can be applied to portfolio optimisation for small-cap companies or inter-sector comparison of portfolio risk and return. The Sharpe model can also be used to make a portfolio of different asset classes such as equity, debt, mutual funds,

Table 5 Calculation of weights for optimal portfolio.

Rank	Company	R_i	σ_i	β	Z_i	X_i	%
1	Metropolis Healthcare Ltd.	53.09	15.91	0.51	0.2212	0.3107	31.0713
2	Coforge Ltd.	76.81	25.59	0.85	0.1218	0.1711	17.1096
3	Varun Beverages Ltd.	37.65	12.82	0.43	0.1908	0.2680	26.8033
4	Trent Ltd.	35.71	15.6	0.48	0.0856	0.1202	12.0220
5	Astral Ltd.	63.17	33.61	1.03	0.0277	0.0389	3.8941
6	Aarti Industries Ltd.	42.76	19.49	0.71	0.0502	0.0706	7.0552
7	Navin Fluorine International Ltd.	61.22	58.72	1.13	0.0041	0.0058	0.5766
8	Dixon Technologies (India) Ltd.	102.17	96.97	2.06	0.0021	0.0030	0.2996
9	Laurus Labs Ltd.	54.38	50.87	1.05	0.0035	0.0049	0.4880
10	Godrej Properties Ltd.	86.53	156.52	1.78	0.0004	0.0006	0.0601
11	Metropolis Healthcare Ltd.	46.82	38.39	0.91	0.0044	0.0062	0.6202

etc. This model can easily be used by all types of investors for designing optimal portfolios. Although the model is a few decades old, it is still in use in various modified forms. It can be said that the model will have significant use for portfolio managers and investors in the future.

Data availability

All data generated or analysed during this study are included in this published article [and its supplementary information files].

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Notes

1 The model will be referred to as the Sharpe single-index model or market model or simply Sharpe model in the paper. They all refer to the same model.
2 National Stock Exchange
3 Exchange Traded Funds
4 MS- Microsoft

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Author contributions

The authors jointly and equally contributed to the study.

Competing interests

The authors declare no competing interests.

Ethical approval

This article does not contain any studies with human participants performed by any of the authors.

Informed consent

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Correspondence and requests for materials should be addressed to Janki Mistry.

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