Humanities & Social Sciences Communications

ARTICLE

https://doi.org/10.1057/s41599-023-01667-1 OPEN

The rationality of adaptive decision-making and the feasibility of optimal growth planning

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Rationality, the premise of economics, is an ideal behavioral norm. In the real world, however, intertemporal decision-making is based on adaptive behavioral principles from companies to individual households. It bases on managerial accounting procedures, whereby action plans are formulated and implemented, differences from actual results are recognized, and revisions accumulate over time. We take the intertemporal decision-making problem of households' consumption/saving (investment) planning in this paper. And we compare the validity of rationality and adaptability as decision-making principles. First, rational decision-making in the optimal growth model leads to a unique path. However, optimal growth planning is practically unstable on the saddle-point path and can only realize if it assumes rationality leading to perfect foresight. On the other hand, the growth paths guided by budget-controlled adaptive decision-making are diverse and distributed in the myriad around the optimal growth path. This redundancy creates stability in the management and operation of the plan. Because through the trial-and-error process of planning and actual comparison, we can implement a more advantageous plan while allowing for multiple next-best goals, including the optimal growth path. Moreover, the numerical results show that the sequentially adaptive consumption/investment planning is comparable to the optimal growth plan on a social welfare basis calculated by accumulating consumption utility and is practically manageable. For example, paths that exceed 0.9 as a ratio to the optimal growth plan are reachable from the initial planning stage at a ratio of 0.58. Based on the above results, we can now analyze intertemporal economic problems with this realistic, practical, and simple method, replacing dynamic optimization ones.

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Introduction

Rationality, the premise of economics, is an ideal decision-making criterion as a behavioral norm. Rational expectations, which support this behavioral principle, are expected to improve in accuracy due to the dramatic progress in information technology. However, even if the accuracy of probabilistic forecasting improves, the scheduled harmony that underlies the consequences of rational expectations remains a fiction for the behavioral principles that deal with future events.

In the real world, on the other hand, intertemporal decision-making is based on adaptive behavioral principles, from companies to individual households. In other words, it bases on managerial accounting procedures, whereby action plans are formulated and implemented, differences from actual results are recognized, and revisions accumulate over time. However, even if such budgetary control-type behavior is only adaptive, it is rational at the time of decision-making because the proposed plan is the result of selecting the most advantageous alternative at that time based on existing information. That is, it is sequentially rational. At the same time, it has an affinity for the full benefits of information technology development in the future.

Therefore, in this paper, we take up the intertemporal decision-making problem of households' consumption planning. And we compare the validity of rationality and adaptability as decision-making principles. Below, in the section "Principles of practical decision-making", we introduce the universality of practical budget-controlled decision-making based on the existing literature and the results of field surveys. In the section "Principles of rational decision-making", we adopt the Ramsey-type optimal growth model based on rational decision-making as the analytical framework for the intertemporal consumption/investment planning covered in this paper. In the section "Principles of sequentially rational decision-making based on budgetary control", we introduce a budget-controlled intertemporal consumption/investment planning model and compare the nature of its growth path with the optimal growth path using a phase diagram. Then, in the section "Social welfare evaluations", by numerical calculations, we compare the budget-controlled growth path with the optimal growth plan under the social welfare criterion and evaluate its level and operational management potential.

Principles of practical decision-making

Budgetary control behavior. Actual companies make decisions by repeatedly contrasting plans with results based on management accounting. First, the organization gathers and analyzes information accumulated to date, from internal management accounting information to business partners, competitors, and the external environment, and then considers profitability, risks, and other factors to formulate and implement the most rational plan. Then, companies periodically monitor the deviation of the execution results from the initial plan, i.e., the difference between the set achievement criteria and the actual cost and profit information, review the initial plan, and formulate a new one. In other words, companies make decisions sequentially and adaptively, adjusting for differences between plans and performance (Emmanuel et al., 1990; Otley, 2006). A survey of 597 executives at Deloitte (2014) found that "Corporate PBF (planning, budgeting, and forecasting) procedures and capabilities are common remarkably across organizations, regardless of size or industry."

Moreover, a wide range of organizations practices this sequential and adaptive decision-making process, not only firms but also households and even independent-financing government organizations (Caplan, 2012).

In the case of government organizations, they practice the decision-making concepts specified in budget management in corporate behavior as Results Based Management. Introducing a results-based approach aims to improve management effectiveness and accountability by "defining realistic expected results, monitoring progress toward the achievement of expected results, integrating lessons learned into management decisions and reporting on performance" (UNDP, 2002).

In household consumption/savings behavior, also, there is a behavioral norm of contrasting plans and actual results and adjusting the differences sequentially. Based on the household budget, each household successively implements a long-term consumption/savings plan ranging from daily income and expenditures to housing construction and life planning.

For example, according to the economic theory of self-control, consumers simultaneously have the desire for "immediate expenditure of income" and the desire for "long-term planning and investment," which requires some self-control or restraint to prevent overconsumption (Hernandez et al., 2014; Jonker, 2016). Therefore, in implementing budget management, consumers need to develop a budget plan, record and monitor their spending against the plan (Heath and Soll, 1996), and set a budget regularly to sustain self-control (Hernandez et al., 2014). In other words, "The concept of self-control is incorporated in a theory of individual intertemporal choice by modeling the individual as an organization" (Thaler and Shefrin, 1981).

The reality of budgeting behavior in households. We can see practical actions through household budget management in actual survey results. First, let's introduce the University of Michigan's Monthly Consumer Survey conducted in November and December 2001. 79% of consumers have a household budget or control their spending, 80% have a savings account, 39% are saving for long-term goals such as education, a car, or a home, and 63% have some retirement account (Hilgert et al., 2003).

Second, generational differences in attitudes toward saving for retirement indicate that households practice decisions about spending and savings allocation over time. According to the 2018 Report on the Economic Well-Being of American Households, there are notable differences in savings awareness and savings performance between generations, with older adults generally being more likely than younger adults to view holding retirement savings as smooth. For example, the percentage of people who perceive their savings to be on track for retirement increases with each generation: 26% at age 18~29, 35% at age 30~44, 42% at age 45~59, and 45% from age 60 (Board of Governors of the Federal Reserve System, 2019).

Third, empirical studies confirm that households implement consumption/savings plans adaptively, checking their deposits and budgets and responding to changes in their age, experience, and external environment. Hilgert et al. (2003) found that experience and knowledge of oneself, others, and the external environment contributed statistically significantly to improved consumption/saving and investment activities.

Finally, there are examples of government efforts to educate and support such household budgeting behavior. For example, the Federal Trade Commission distributes simple calculation sheets to fill out for each item of income and expense to help households manage their monthly budgets (Federal Trade Commission, 2012). Indeed, according to Hernandez et al. (2014), "Traditionally, Dutch households that need to cut down expenses have been advised by organizations like NIBUD (National Institute for Family Finance Information) to record all their payments to realize how much they spend and on what expenses they might save."

PDSA cycle as a budgetary control procedure. The Plan-Do-Study-Act (PDSA) cycle is a practical concept proposed by W.A. Deming (2018) in his lecture in Japan concerning quality control activities (QC). On the other hand, it is widely practiced beyond QC in corporate as a means of practical and adaptive decision-making procedures based on the above-mentioned budgetary control (Ohnishi and Fukumoto, 2016). Procedures by the PDSA cycle are as follows. (1) Plan: Formulate management plans based on evidence such as accounting transaction information. (2) Do: Implement plans. (3) Study: Verify the difference between the plan and actual results. (4) Act: Review and revise the plan for the next period. The PDSA cycle procedures can then be applied to practical budgetary control in households and governmental organizations (Hilgert et al., 2003; UNDP, 2002).

The following field study confirms that companies widely adopt the procedure equivalent to the PDSA cycle in the decision-making process of companies. For example, according to an ACCA and KPMG survey (2015), at the (1) Plan stage, 84% of the respondents said that if they could expand the scope of forecasting through the external data, they would significantly benefit in terms of forecasting accuracy. Next, according to a Deloitte (2014) study, concerning the (3) Study stage, 69.7% of companies have planned versus actual revenue variances in the plus or minus 10%. As a result, during the (4) Act stage, more than 90% of the companies will take action on this difference, 42.1% of them directly and immediately, 31.2% will address the difference from the forecast in planning, and 18.8% will consider feasible re-planning depending on the situation.

Principles of rational decision-making[1]

Optimal growth model. The optimal planning problem of consumption and savings allocation has been pioneered by Ramsey (1928) and completed by Uzawa (1964, 1965), Cass (1965), and Koopmans (1963). The optimal growth model essentially leads to a unique macroeconomic growth path when there is no change in the level of technology attributed to the mechanism of the production function, whether based on omniscient central planning or in a decentralized market economy. The core element of economic activity that leads to such an optimal growth path is a decision-making mechanism that allows rational expectations and judgments over an infinite future (Iwai, 1994).

On the other hand, the optimal growth model has evolved into the overlapping generation model by Diamond (1965) and others, which avoids the constraint of rational decision-making for the infinite future. That is, based on the theoretical imperative that consumption planning should base on more realistic behavioral hypotheses of a finite period, the model was developed and refined as a standard framework in which the generation responsible for consumption/savings planning in each period receives an inheritance from the previous generation and passes it on to the next generation (Iwai, 1994).

Furthermore, the optimal growth model has evolved into such models as the endogenous growth model by Romer (1986) and others, which incorporates technological progress that increases the productivity of society as a whole through the accumulation of knowledge capital through R & D into the optimal growth model.

Intertemporal rational decision-making structure. Assuming a single domestic economy, if the current capital balance is K_t, the labor supply is L_t, and the production function is $F(K_t, L_t)$, we can express the gross domestic product as $Y_t = F(K_t, L_t)$. Here, gross domestic product is assumed to be equal identically to gross domestic income. In addition, let C_t denote household consumption expenditure, S_t be savings, and I_t be a corporate capital investment. The production function $Y_t = F(K_t, L_t)$ is assumed to be a constant return to scale (first-order homogeneous), and if $y_t = Y_t / L_t$ and $k_t = K_t / L_t$ in terms of quantity per capita, we can express the function as $y_t = f(k_t)$.

Assume that $f(k_t)$ satisfies the Inada condition.

$$f(0) = 0, f'(k_t) > 0, f''(k_t) < 0, f'(0) = \infty, f'(\infty) = 0$$

In the following, we will also express consumption expenditures and savings in per capita levels, denoted c_t and s_t, respectively.

In this paper, we assume that there is no change in the level of technology, i.e., the structure of the production function defined above is constant. If long-term economic growth is the policy goal, the decision criterion is the comparative balance of the utility provided by current and future consumption. In other words, out of the income level y_t obtained at time t in the current period, the current level of utility will improve if the current consumption amount c_t expands. But on the other hand, the savings amount s_t, which will be the source of capital investment in the next period, will decrease. As a result, the growth of production and consumption level in the next period $t + 1$ and beyond will become constrained.

In the following, we will use a discrete system as an example to concisely understand the relationship between each state variable over time. First, let us confirm the rational decision-making structure in the optimal growth model. The optimal growth model assumes that an omniscient central planning agency formulates a growth plan leading to an infinite future or that household following eternal future acts on rational expectations. Under this assumption, the allocation of current and future consumption, i.e., the combination of consumption and savings, is determined to maximize the sum of the discounted present value of consumption utility determined every period as $r > 0$ (see Fig. 1). As a result, the existence of such a path would result in an optimal consumption/investment plan that maximizes consumption utility over the entire period from the present to eternity.

Rational decision-making consumption/investment planning models and optimal growth path. We construct a continuous system model of optimal growth planning in this section. Assume a constant rate of growth $\dot{L}_t / L_t = n > 0$ in terms of household labor supply. If household savings become the source of capital investment by firms and determine capital accumulation, i.e., $\dot{K}_t = I_t = S_t$, we obtain the following state transition equation for capital accumulation.

$$\begin{aligned} \dot{k}_t = d(K_t/L_t)/dt = \dot{K}_t/L_t - (K_t/L_t)(\dot{L}_t/L_t) = s_t - nk_t \\ = y_t - c_t - nk_t \end{aligned}$$

$$\dot{k}_t = f(k_t) - c_t - nk_t \tag{1}$$

The optimal growth plan comes down to the problem of determining the path of consumption and capital accumulation that maximizes the social welfare function of summing (integrating) the consumption utility of representative households from the present to the infinite future, subject to the above Eq. (1) and the initial value $k_0 > 0$ as constraints.

$$\text{Max} \int_0^\infty u(c_t) \mathrm{e}^{-\rho t} dt$$

Here, $\rho > 0$ represents the subjective discount rate. We assume the following conditions for the utility function $u(c_t)$.

$$u'(c_t) > 0,\ u''(c_t) < 0, u'(0) = \infty$$

Then we can solve this optimization problem by the Maximum Principle. We define the below Hamilton functions with λ_t as the

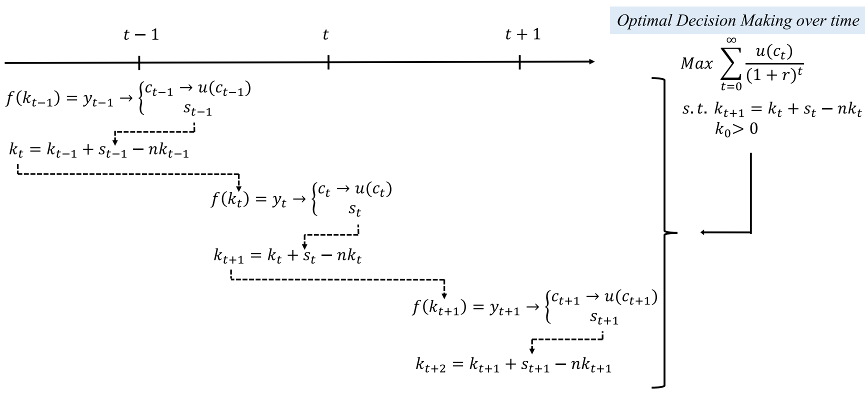

Fig. 1 Decision-making structure over time in the optimal growth model.

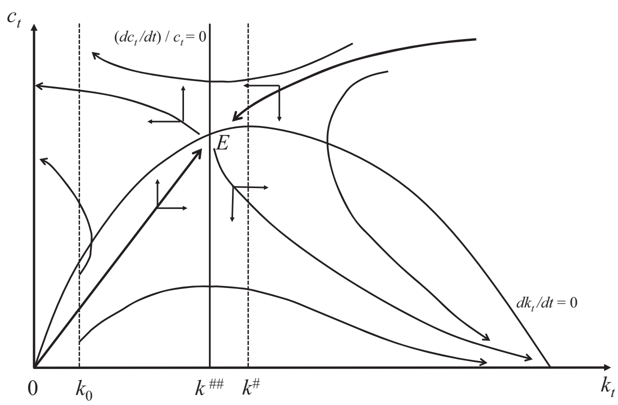

Fig. 2 Optimal growth path.

Lagrangian multiplier.

$$H_t = u(c_t)e^{-\rho t} + \lambda_t(f(k_t) - c_t - nk_t)$$

According to the Maximum Principle, the dynamic optimization problem is given by the following three conditions on the Hamiltonian function, in addition to the previously presented constraints consisting of Eq. (1) for capital accumulation and initial values.

$$\partial H_t/\partial c_t = u'(c_t)e^{-\rho t} - \lambda_t = 0$$

$$\dot{\lambda}_t = -\partial H_t/\partial k_t = -\lambda_t(f'(k_t) - n)$$

$$\lim_{t\to\infty} \lambda_t k_t = 0$$

The third expression above is a transversality condition requiring that the imputed present value of capital in the infinite future be zero. Now, substituting the first expression above into the second expression yields the Keynes–Ramsey Rule (2).

$$\varepsilon_t(\dot{c}_t/c_t) + \rho = f'(k_t) - n \qquad (2)$$

Here, $\varepsilon_t = -c_t\ u''\ (c_t)/u'\ (c_t) > 0$ represents the elasticity of marginal utility.

Ultimately, an optimal growth plan that maximizes consumption utility per household over time starts from an initial condition $k_0 > 0$ for capital and finally comes to satisfy the transversality condition in a growth path guided by the equation for capital accumulation (1) and the Keynes–Ramsey Rule (2). We can represent the optimal growth path in the phase diagram in Fig. 2 and as a saddle-point path toward the steady-state equilibrium E where the consumption and capital steady-states intersect. There is always only one such steady-state equilibrium E other than the origin at the Modified Golden Rule level. Since the Modified Golden Rule level $k^{\#\#}$ satisfies $f'(k^{\#\#}) = n + \rho > n = f'(k^{\#})$, the consumption level is lower than that of the Golden Rule level $k^{\#}$.

Principles of sequentially rational decision-making based on budgetary control

Economic behavior and adaptive decision-making. In this section, we construct a model of consumption-savings planning over time with replicator dynamics. As a prerequisite, we will review the validity of the model's behavioral principle, adaptive decision-making, within the context of traditional economics and related research fields.

As introduced in the section "Optimal growth model", traditional economics generally deals with optimization planning problems over time under rational decision-making principles. On the other hand, psychology has traditionally, and behavioral economics has recently taken a critical view of rational behavior, pointing out that people do not necessarily behave by maximizing a utility function because their decision-making involves incomplete information, limited cognitive resources, and decision biases (Knoll, 2010). Rosati and Stevens (2009) have also pointed out that human behavior has a context that depends on each social institution, custom, culture, and historical dependency. And they have pointed out that seemingly irrational decision-making is evolutionary and adaptive in various choice settings. In addition, people act adaptively, especially in the face of environmental changes and planning over time (Payne and Bettman, 1988). For example, long-term savings planning also depends on behavioral economic or psychological factors such as self-control, emotions, and choice architecture. That is, it depends on the decision context (Knoll, 2010).

On the other hand, adaptive decision-making behavior has also been attracting attention in economics. Lucas (1986) has focused on adaptive behavior, which has been gaining attention in psychology, applying the concept of "adaptability" to the trial-and-error process in behavioral models. In addition, Day (1983) notes that various adaptive processes govern economic activities, such as feedback control, behavioral rules, trial-and-error search, suboptimization with feedback, and other sequential decision procedures. In particular, Day's argument is that the adaptive action principle moves away from the conventional static steady-state analysis to an evolutionary view of the economic entities' activities, rules of behavior, and organization development. And he argues that it looks at dynamic analysis as a complex, nonlinear system with the scope of structural changes (phase change) and disequilibrium processes in the economic system.

It has also recently received attention as a hypothesis that decision-making based on bounded rationality can substitute for rational expectations through adaptive learning (Evans and

Table 1 Payoff matrix at the time *t*.

	Strategies	Strategies Selection rates	Counterparties (Customers, Competitors, etc.) Q1 y^1_t	Q2 y^2_t	Q3 y^3_t	…	Qm y^m_t
Organization	P1	x^1_t	(U^{11}_t, V^{11}_t)	(U^{12}_t, V^{12}_t)	(U^{13}_t, V^{13}_t)	…	(U^{1m}_t, V^{1m}_t)
	P2	x^2_t	(U^{21}_t, V^{21}_t)	(U^{22}_t, V^{22}_t)	(U^{23}_t, V^{23}_t)	…	(U^{2m}_t, V^{2m}_t)
	P3	x^3_t	(U^{31}_t, V^{31}_t)	(U^{32}_t, V^{32}_t)	(U^{33}_t, V^{33}_t)	…	(U^{3m}_t, V^{3m}_t)
	…	…	…	…	…	…	
	Pn	x^n_t	(U^{n1}_t, V^{n1}_t)	(U^{n2}_t, V^{n2}_t)	(U^{n3}_t, V^{n3}_t)	…	(U^{nm}_t, V^{nm}_t)

McGough, 2020). According to Evans and McGough (2020), the adaptive learning approach models economic agents in dynamic and stochastic environments as adaptive learners, forming expectations and making decisions based on forecasting rules that are updated in real-time as new data become available. The model makes forecasts by autoregression of observed exogenous and endogenous variables and updates estimates over time. In Evans and McGough (2020), the central issue discussed is whether adaptive learning will converge over time to a specified rational expectation equilibrium (REE), in which case we say the REE is stable under adaptive learning. Incidentally, in the model in this paper, the assumptions involved in the forecast estimation correspond to the procedures for developing the next period plan based on the PDSA (see the section "Budget-controlled decision-making process for intertemporal consumption/investment planning").

On the other hand, the replicator dynamics approach adopted in this paper differs from the methodology of Evans and McGough (2020). However, we can say that they share the same perspective in that they attempt to capture the process of decision-making through adaptive learning while sequentially updating accumulated information as the core behavioral principle that defines socioeconomic activities. Although there are applied studies of replicator dynamics, such as Safarzynska and van den Bergh (2011) on technology development investment, Sakaki (2004) on economic growth with technological innovation, and Cantner et al. (2019) on industrial organization, it is not yet mainstream as an economics methodology.

However, we have applied replicator dynamics originally in the area of population genetics. Therefore, the fact that we base the population state transitions on the random matching hypothesis has limited its interpretation and application in social domains such as economic activity and organizational behavior (Deguchi 2004). Deguchi (2004) reconstructed the interpretation of replicator dynamics as agent-based learning dynamics through Markov processes, providing a theoretical framework broadly applicable to socioeconomic decision-making problems. Furthermore, by interpreting the internal constitutive equation of payoffs that govern the state transitions of replicator dynamics as a mechanism that allows each agent to update its stored information over time and cross-reference among agents, it is possible to construct a model for flexible institutional design through indirect control that he mentions. In addition, Sakaki (2018) presents a comprehensive agent-based methodology for institutional design in social groups through inter-organizational management.

Based on the above previous studies on real-world decision-making behavior, this section will model the consumption and savings decision-making process in replicator dynamics according to the PDSA cycle, a practical procedure for sequentially adaptive decision-making behavior, in the following. We discuss the relationship between practical decision-making procedures based on budgetary control and replicator dynamics in the section "Relationship between budgetary control-based decision-making procedures and replicator dynamics". In the section "Budget-controlled decision-making process for intertemporal consumption/investment planning", we reconstruct consumption and investment (saving) planning over time with replicator dynamics based on this decision-making process, and in the section "Budget-controlled intertemporal consumption/investment planning model and growth path", we create and analyze a phase diagram of the growth path of the model concerned.

Relationship between budgetary control-based decision-making procedures and replicator dynamics. We can model the decision-making process based on budgetary control as a PDSA cycle. In other words, if we view business planning as a payoff matrix that depends on the external environment, business decisions can be defined as dynamic actions that select strategies on the payoff matrix throughout each period using the PDSA cycle as a procedure. Let us reconstruct the PDSA cycle on the payoff matrix as follows.

(1) Plan: We can summarize the plans (strategies) P in Table 1 as a payoff matrix that the organization concerned can formulate for the time t in the current period. The payoff matrix corresponds with the business partner's plan (strategy) Q, which is assumed to be the external environment. As shown below, for each Q^j of m types of proposed plans for which the organization can collect information for business partners, the organization can formulate n types of possible plan P^i.

$$P \equiv \left\{P^1, P^1, \cdots, P^n\right\}, Q \equiv \left\{Q^1, Q^1, \cdots, Q^m\right\}$$

(2) Do: The organization concerned estimates the expected payoff U^{ij}_t for each plan P^i that the organization can select in the current period t based on the external environment situation (the selection ratio y^j_t of each plan Q^j by the business partners). And the organization adopts and implements the best one among them.

$$\mathrm{Max}_{P^i \in P} E\left[u_t\left(P^i\right)|Q\right] = \mathrm{Max}_i\left\{\sum_{j=1}^m y^j_t U^{ij}_t | i \in \mathrm{N}\right\}$$

(3) Study: Recognizing the difference between the plan and the actual result after the implementation, the organization concerned collect information on the external environment regarding the revised draft plan of the business partners and updates the payoff matrix for the next period $t+1$.

(4) Act: In the following period $t+1$, based on the updated payoff matrix, the organization concerned estimates the expected payoffs of all possible plans P^i, develops the optimal plan option and implements it. The organization repeats this process over time.

On the other hand, the replicator dynamics allow us to derive the demographics described on the payoff matrix regarding the selection ratio of each plan proposal adopted by each population. In other words, the adoption rate of each plan, which is the result of decision-making by each group, can be interpreted as a social selection rate and derived as a state transition variable over time.

Therefore, based on the decision-making process (1)–(4) over time, using the PDSA cycle described above as a procedure, we define the state transition of each plan proposal: x^i_t (the ratio that the organization concerned selects strategy P^i) by replicator dynamics as follows (Weibull, 1995). We can construct a state transition equation in which the organization concerned adopts the plan P^i with a relatively high expected payoff from among all the possible plans at the time and the ratio x^i_t changes over time. As a result, the selection ratio of the plan proposal with the highest expected payoff in each period rises the most in the current period t.

$$\Delta x^i_t/x^i_t = \left(E\left[u_t\left(P^i\right)|Q\right] - E\left[u_t(P)\right]\right)/E\left(u_t(P)\right)\ \text{(Discrete systems)} \tag{3}$$

$$\dot{x}^i_t/x^i_t = E\left[u_t\left(P^i\right)|Q\right] - E\left[u_t(P)\right]\ \text{(Continuous systems)} \tag{4}$$

$$E\left[u_t\left(P^i\right)|Q\right] = \textstyle\sum_{j=1}^m x^i_t y^j_t U^{ij}_t$$
$$E\left[u_t(P)\right] = \textstyle\sum_{i=1}^n E\left[u_t\left(P^i\right)|Q\right] = \sum_{i=1}^n \sum_{j=1}^m x^i_t y^j_t U^{ij}_t$$

Let us assume that the payoff matrix itself in each period is also dynamically updated each period, reflecting the actualization of the planning proposals by the organizations concerned. In this case, the replicator dynamics will independently select the optimal plan from among the available plan alternatives in the current period within each period. On the other hand, the replicator dynamics do not lead to optimal planning over the eternal future. But it provides adaptive decision-making across two periods so that the optimal plan can be adopted in the current period while reflecting the actual results up to the previous period. And it connects continuously and sequentially this process to the selection in the next period and beyond.

In addition, the above actions of an adaptive nature cannot directly predict and correct uncertainties in the subsequent period. However, in replicator dynamics, the results of decisions made in the current period are stored as information. And this updated information is the basis for decisions in the subsequent period. The above results show that if we can capture all the options subject to decision-making in a payoff matrix, we can derive the state transitions of sequentially rational decision-making outcomes by replicator dynamics with the PDSA cycle as the implementation procedure for budget management.

Furthermore, the PDSA cycle is a practical procedure that enables micro-decision makers to adapt to the macro external payoff environment over time. Replicator dynamics is a population dynamics that interconnects micro and macro through this sequentially rational decision-making procedure.

Budget-controlled decision-making process for intertemporal consumption/investment planning. Below, under the framework of the optimal growth model, let us structure a budget-controlled decision-making process for intertemporal consumption/investment planning using the PDSA cycle procedure, sequentially contrasting and evaluating the planning and actual results. The model in this paper assumes that households are responsible for intertemporal decisions related to consumption/savings planning and that firms plan and implement production by making capital investments entirely from household savings.

Procedure 1: Plan, Do. Companies produce $f(k_t)$ for the current period under the current period capital stock level k_t planned in the previous period $t-1$. Households implement the consumption for the current period c_t of the previous period's plan level from the income earned from this production. Based on this result, in the current period t, companies plan the capital stock level k_{t+1} and production $f(k_{t+1})$ for the next period $t+1$.

$$k_{t+1} = k_t + f\left(k_t\right) - c_t - nk_t \tag{5}$$

Procedure 2: Study. In management accounting practice, households generally recognize the difference between the current period consumption level $c^e{}_t$ planned in the previous period $t-1$ and the actual consumption level c_t in the current period t and formulate a revised plan c_{t+1} for the next period $t+1$. However, in the model in this paper, households actualize firstly the consumption level $c^e{}_t$ planned in the previous period for the current period as in Procedure 1. And they plan the next consumption amount c_{t+1} by comparing the actual production amount $f(k_t)$ in the current period with the discounted present value $f(k_{t+1})/(1+r)$ of the next production plan and recognizing the difference (advantageous difference) between them[2].

First, based on the discrete system definition (3), we implement a sequentially adaptive process to recognize the advantageous difference between the above two amounts using the replicator dynamics (6). Equation (6) allows households to plan the allocation ratio x_{t+1} concerning production in the current and next term.

$$x_{t+1} = x_t f\left(k_t\right)/\left[x_t f\left(k_t\right) + (1-x_t) f\left(k_{t+1}\right)/(1+r)\right] \tag{6}$$

Procedure 3: Action. Next, based on the difference between the intertemporal allocation ratios x_t and x_{t+1} of production derived in Procedure 2 and the difference between the actual production value $f(k_t)$ for the current period and the planned production value $f(k_{t+1})$ for the next period, the household formulates a revised consumption plan c_{t+1} from the current period to the next period through the following definition[3].

$$c_{t+1} = c_t + \Delta c^e_t \tag{7}$$
$$\Delta c^e_t \approx \left(x_{t+1} - x_t\right) f\left(k_t\right) + x_t\left[f\left(k_{t+1}\right) - f\left(k_t\right)\right]$$

Based on the results planned according to the above procedures 1–3 in the current period t, a consumption/investment plan is continued to be implemented in the next period $t+1$, i.e., a plan to allocate the production amount $f(k_{t+1})$ to consumption amount c_{t+1} and savings amount s_{t+1}. Budget-controlled decision-making realizes by sequentially repeating the above process in each subsequent period (see Fig. 3). Therefore, we need not assume, as in optimal growth planning, that an omniscient and omnipotent central planning authority formulates a growth plan leading to an infinite future or that households following an eternal future act on rational expectations.

Budget-controlled intertemporal consumption/investment planning model and growth path. Now, we reconstruct the intertemporal consumption/investment planning model of a discrete system derived from the procedure of the PDSA cycle in the previous section by the following differential equations of a continuous system.

$$\dot{x}_t = x_t\left(1 - x_t\right)\left[f\left(k_t\right) - e^{-\rho}\left\{f\left(k_t\right) + f'\left(k_t\right)\dot{k}_t\right\}\right] \tag{8}$$

$$\dot{c}_t = \dot{x}_t f\left(k_t\right) + x_t f'\left(k_t\right)\dot{k}_t \tag{9}$$

$$\dot{k}_t = f\left(k_t\right) - c_t - nk_t \tag{10}$$

However, we use the following approximate formula in Eq. (8).

$$\dot{x}_t = x_t\left(1 - x_t\right)\left[f\left(k_t\right) - e^{-\rho} f\left(k_{t+dt}\right)\right];\ \left(k_{t+dt}\right) \approx f\left(k_t\right) + f'\left(k_t\right)\dot{k}_t$$

To analyze the trajectories of the dynamical systems (8)–(10), let us examine the steady state of each variable and construct a phase diagram (see Appendix A in Supplementary information

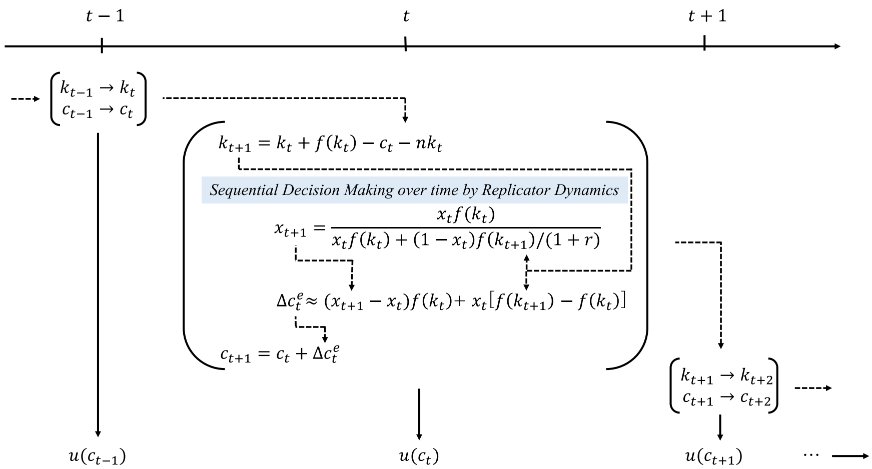

Fig. 3 Decision-making structure over time in the budgetary control model.

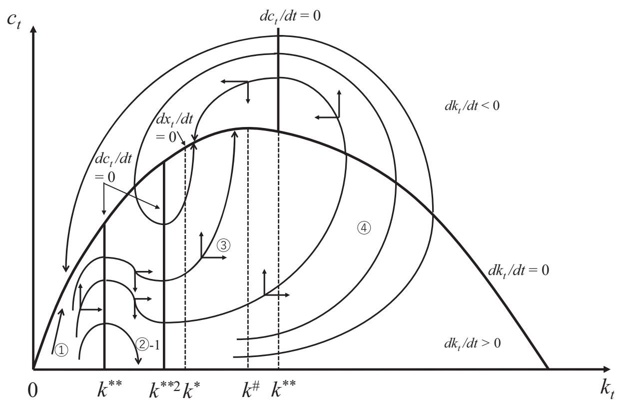

Fig. 4 Budget-controlled growth path in the case of $k^\# < k^{**}\left(\dot{k}_t < 0\right)$.

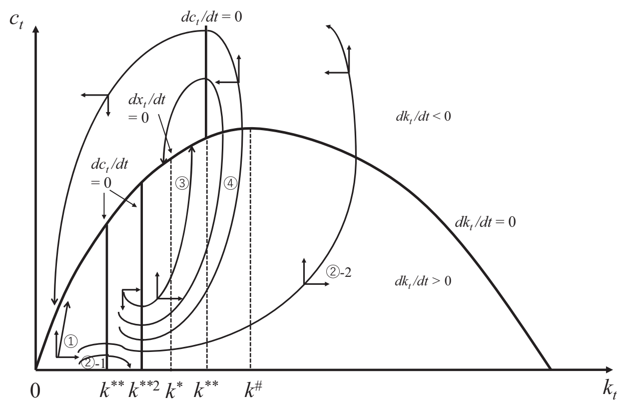

Fig. 5 Budget-controlled growth path in the case of $k^{**}\left(\dot{k}_t < 0\right) \le k^\#$.

for details). First, the boundary line representing the stationary state in the state transition equation of capital accumulation (10) is the same as the optimal growth path. In the region below the boundary, k_t increases with time and decreases in the above.

On the other hand, the state transition Eqs. (8) and (9) for the relative allocation ratio for production in the current and following periods: x_t and consumption c_t, respectively, contain terms for $\dot{k}_t$ and $\dot{x}_t$, which are generally not steady-state. However, it is possible to determine the relative position of Golden Rule level $k^\#$ and each curve (actually a vertical line) representing the steady state of the $\dot{x}_t = 0$ and $\dot{c}_t = 0$ under certain conditions.

So let us examine the direction of the time path representing capital and consumption for each boundary separated by a curve representing the steady state of each state variables x_t and c_t. First, the boundary line that satisfies the steady state of Eq. (8), $\dot{x}_t = 0$, can be obtained independently of the level of consumption and is expressed as $k_t = k^*$. The k^* exists only when $\dot{k}_t > 0$. Also, x_t decreases when $k_t < k^*$ and increases when $k^* < k_t$ over time.

On the other hand, the boundary that satisfies the steady state $\dot{c}_t = 0$ in Eq. (9) is obtained independently from the consumption level, and we denote it by $k_t = k^{**}$. Since there can be more than one capital level in this steady state when $\dot{k}_t > 0$, we denote them by k^{**} and k^{**2}. In this case, the relationship $k^{**} < k^{**2} < k^*$ exists. Consumption c_t increases when $k_t < k^{**}$, decreases when $k^{**} < k_t < k^{**2}$, and increases when $k^{**2} < k_t$ with time. On the other hand, when $\dot{k}_t < 0$, c_t decreases when $k_t < k^{**}$ and increases when $k^{**} < k_t$ with time.

Let us examine the budget-controlled growth path under the above phase structure. The growth path in the model in this paper starts from the initial conditions, replaces the Keynesian–Ramsey rule in the optimal growth model, and follows the dynamic Eqs. (9) and (10) for consumption and capital accumulation through Eq. (8), which is decision-making based on the PDSA cycle. We will now discuss the steady-state positional relationship of each state variable.

Among the phase diagrams, Fig. 4 shows a typical growth path for the following cases.

$$k^{**}\left(\text{when } \dot{k}_t > 0\right) < k^{**2}\left(\text{when } \dot{k}_t > 0\right) < k^* < k^\#,\ k^\# < k^{**}\left(\text{when } \dot{k}_t < 0\right)$$

On the other hand, Fig. 5 shows a typical growth path for the following cases:

$$k^{**}\left(\text{when } \dot{k}_t > 0\right) < k^{**2}\left(\text{when } \dot{k}_t > 0\right) < k^* < k^\#,\ k^{**}\left(\text{when } \dot{k}_t < 0\right) \le k^\#$$

The difference in the phase diagrams in Figs. 4 and 5 is the difference in the position of the steady state of $\dot{c}_t$ for $\dot{k}_t < 0$, i.e., $k^{**}\left(\text{when } \dot{k}_t < 0\right)$, relative to the position of the Golden Rule level $k^\#$. Both are identical in the qualitative nature of the trajectories drawn by the growth paths. However, in areas that pass along the pathway, the former is more likely to see capital accumulation exceed the Golden Rule, and the growth path spans a wider area.

In both Figs. 4 and 5, innumerable paths converge to the following two regions (11) and (12), and there are four types of pathways to the steady state that belong to these two regions, depending on the initial conditions. However, the steady-states guided by the model in this paper are derived by decision-making based on the PDSA cycle, successively repeating the contrast between the plan and actual result over time. Therefore, since they are not the results of an optimal consumption/investment plan planned for eternity or with perfect foresight, there is no criterion corresponding to the transversality condition, and it is impossible to eliminate dynamic inefficiency in principle. However, even in this case, it is still possible to sustain a sequential consumption/ investment plan while reducing capital overaccumulation. In the first and third of the following four paths, it is possible to mitigate dynamic inefficiency by reducing the excess accumulation of capital.

$$0 < k_t < k^{**}\left(\text{when } \dot{k}_t > 0\right) \text{ on the } \dot{k}_t = 0 \tag{11}$$

$$k^{**2}\left(\text{when } \dot{k}_t > 0\right) \le k_t \le k^{**}\left(\text{when } \dot{k}_t < 0\right) \text{on the } \dot{k}_t = 0 \tag{12}$$

First, it is a path that monotonically converges to a steady state of a low welfare level in the Eq. (11) domain, starting with a low initial capital level. This pathway can mitigate dynamic inefficiencies, but it is impossible to approach or reach the Modified Golden Rule level (Path ① in Figs. 4 and 5).

In the second path, after some growth with a similarly low initial capital level, the consumption level declines, and the consumption level locks in at zero in the region of $k^{**}\left(\text{when } \dot{k}_t > 0\right) \le k_t < k^{**2}\left(\text{when } \dot{k}_t > 0\right)$ (path ②-1 in Figs. 4 and 5). In this case, capital accumulation may continue while restraining consumption due to this capital constraint and avoiding lock-in. Then, after excess accumulation over the Golden Rule level, the capital stock is disposed of and finally converges to the low-level region or 0 in Eq. (11). The second path is also inefficient (Path ②-2 in Fig. 5).

In the third path, capital and consumption levels converge in the region of Eq. (12) while uniformly improving when capital levels have accumulated to a certain extent to around k^{**2} after the initial point (path ③ in Figs. 4 and 5). In the case of Fig. 4, where $k^{\#} < k^{**}\left(\text{when } \dot{k}_t < 0\right)$, there is also a case where the amount of capital at a steady state reaches the Golden Rule level. On the other hand, if $k^{**}\left(\text{when } \dot{k}_t < 0\right) \le k^{\#}$ in Fig. 5, the steady state is below the Golden Rule level. In both cases, however, it is possible in principle to implement a growth plan that reaches the Modified Golden Rule level in the optimal growth path in a steady state. Moreover, unlike the fourth growth path described below, the third path can mitigate dynamic inefficiencies as it is possible to avoid excessive capital accumulation, i.e., reach an outcome similar to the optimal growth path.

The fourth path can occur when the capital level accumulates to some extent after the initial point and reaches a level near k^{**2} and when capital accumulation proceeds with the suppressed consumption level compared to the third case. In this fourth growth path, the amount of capital accumulated beyond the Golden Rule level may result in a rapid increase in consumption levels and then convergence into the realm of Eq. (12) while discarding over-accumulating capital. Alternatively, if the capital amount is further over-accumulated, the pathway will eventually converge to the low or zero region of Eq. (11) (path ④ in Figs. 4 and 5). In this case, there is a possibility of convergence to a steady state that includes the Modified Golden Rule level. But due to the overaccumulation of capital, this path is dynamically inefficient.

The above results show that the myriad convergence paths can become steady states in the region (5) to (12) containing the Modified Golden Rule level. They are consistent with one of Nicholas Kaldor's stylized facts that growth paths do not converge to a single steady state and that the aggregate output and labor productivity are diverse at the steady states.

Moreover, the budget-controlled growth path does not lead to optimal growth and does not guarantee the maximization of social welfare levels. However, the third path ③ above is efficient as it avoids excessive capital accumulation. In other words, it is not optimal but sufficiently efficient. And it can be reached stably in planning and management without the instability of walking a tightrope as in the saddle-point path in the optimal growth model.

Social welfare evaluations

Numerical model. In this section, we examine the degree to which differences in decision-making principles affect the dominance of social welfare by numerical calculations using Mathematica12. We evaluate the social welfare level by measuring the utility level from per capita consumption accumulated over time (see Appendix B in Supplementary information).

Now, consider the Cobb–Douglas type Eq. (13) as a production function that satisfies the Inada condition. We assume a relatively mature, low-growth economy with an initial capital level of $k_0 = 100$, a labor supply growth rate of $n = 0.01$, and a subjective discount rate of $\rho = 0.01$, and calculate the growth path with a parameter of $a = 0.5$. In this case, the level of the Golden Rule is $k^{\#} = 2500$ and $c^{\#} = 25$, and the level of the Modified Golden Rule is $k^{\#\#} = 625$ and $c^{\#\#} = 18.75$.

$$y_t = f\left(k_t\right) = k_t^a; 0 < a < 1 \tag{13}$$

If we define the utility function per capita as constant elasticity of substitution (CES) type according to Oyamada (2012), we can express the social welfare evaluation equation over time as Eq. (14). We can then denote the Keynes–Ramsey Rule, derived from the conditions for optimal growth planning based on rational decision-making, by the following differential Eq. (15).

$$\int_0^\infty u\left(c_t\right)\mathrm{e}^{-\rho t}\mathrm{d}t = \int_0^\infty \left[\left(c_t^{1-\sigma} - 1\right)/(1-\sigma)\right]\mathrm{e}^{-\rho t}\mathrm{d}t \tag{14}$$

$$\dot{c}_t = \left(c_t/\sigma\right)\left[f'\left(k_t\right) - n - \rho\right] \tag{15}$$

Here σ is the elasticity of marginal utility and is a positive constant value since the utility function is of the CES type. Also, σ is the inverse of the intertemporal elasticity of consumption substitution. In this paper, we set the reciprocal σ of intertemporal consumption substitution elasticity at 1/1.4, which is close to the estimate for Japan and the United States based on Yagihashi and Katano (2020).

Preliminary considerations. As a preliminary discussion of the main topic, we will review the relationship between the initial level of capital accumulation and the attainable level of social welfare. First, we estimated the initial value of per capita consumption, $c_0 = 3.94697$, by numerical calculation. This condition is that the initial consumption level must satisfy to realize the optimal growth path when the initial value of the capital level is in an economic environment of size $k_0 = 100$.

Now, in a relatively mature economic environment with $n = 0.01$ and $\rho = 0.01$, if the initial capital balance starts from a low level that is not commensurate with this environment (e.g., $k_0 = 1$), the pathway will not lead to an optimal growth path under practical budget management. However, even in an economy with such a low initial capital level, it is possible to approach the optimal growth path through budgetary control

Table 2 Optimal growth path for $k_0 = 100$ and each evaluation indicator for the budget-controlled (PDSA) growth path.

	Optimal	Budgetary control management (PDSA)						
c_0	***3.94697***	0~	2.377~	2.4103~	3.46~3.48	3.65	***3.94697~***	9~10
k_{1000}	***625.006***	274.9~	752.379	748.97	642.978	***624.978***	595.279	~ 0.0034
c_{1000}	***18.7499***	0.04~	19.9065	19.8776	18.927	***18.7496***	18.4453	~ 0.1582
W^{Optimal}	***357.615***	-	-	-	-	-	-	-
W^{PDSA}	-	5.058~91.654	303.11~347.152	347.17~	***357.522***	357.471	357.193~	302.734~222.556
$W^{\mathrm{PDSA}}/W^{\mathrm{Opt.}}$	-	0.0141~0.2563	0.8476~0.9707	0.9708~	***0.999738***	0.999598	0.99882~	0.8465~0.6223
Path type		②-1	④	③	③	③	③	①

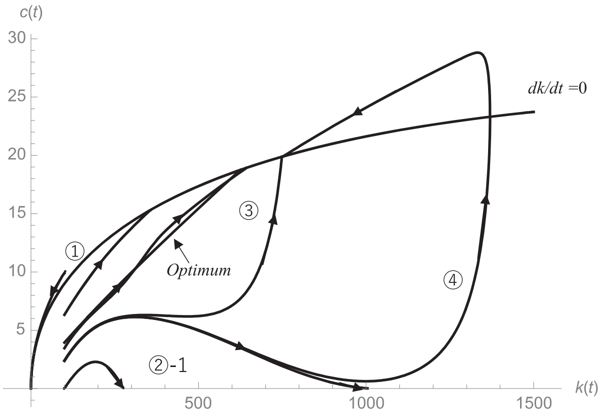

Fig. 6 Budget-controlled growth paths and the optimal growth path.

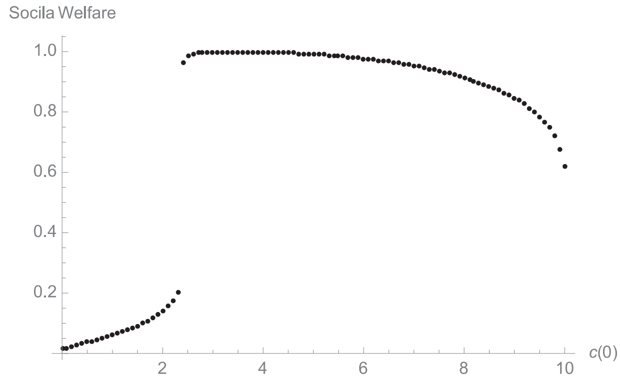

Fig. 7 Social welfare level (c_0 = 0~10).

management if the economic environment has relatively high growth potential with $n = 0.07$ and $\rho = 0.07$.

In sum, to develop consumption/investment plans realizing a sufficient social welfare level through practical budget management, a prerequisite is that an initial capital level must accommodate the specific economic environment. In the case of economies not meeting these conditions, we need external compensatory measures in advance, such as capital improvements through public financial assistance.

Comparison of social welfare levels. In this section, we will examine below how close a budget-controlled (PDSA) growth path, in which decisions are made sequentially by contrasting plans and actual results, can approach an optimal growth path derived from rational decision-making and how feasible it is (see Appendix B in Supplementary information).

Table 2 shows the results of calculating the level of social welfare after 1000 periods of decision-making while contrasting plans and actual results by setting the initial value of the capital level $k_0 = 100$, the production allocation plan $x_0 = 0.99$, and the consumption level c_0 in the range of 0~10 feasible under the initial production level. Figure 6 shows examples of numerical results for the optimal growth path in Fig. 2 and the budget-controlled growth path shown in Fig. 5. Figure 7 shows the respective ratios of the social welfare level to the optimal growth path, varying the initial consumption values by 0.1 in the range of 0~10.

The leftmost column in Table 2 shows, from top to bottom, the initial value of consumption, the level of capital, consumption, and each social welfare indicator after 1000 periods, and the type of pathway at the bottom. For the case of the optimal growth path in the next column and the case of the budget-controlled ones in each subsequent column, we calculated the above values after 1000 periods for each region of the initial consumption values.

As shown in Table 2 and Fig. 6, each of the budget-controlled growth paths examined in the section "Budget-controlled intertemporal consumption/investment planning model and growth path" changes in the order of ②-1, ④, ③, and ① as the initial consumption plan increases from 0 to 10 under an initial production allocation plan (0.99).

First, let's examine the social welfare level starting from the case where the initial value of the consumption level belongs to $0 \le c_0 < 2.377$. In this case, the consumption level converges to 0 for any growth path (②-1), and the ratio of the social welfare level to the optimal growth path is only 0.0141~0.2563 ($c_0 = 0$~2.376).

Next, when $2.377 \le c_0 < 2.4103$, any growth path becomes dynamically inefficient due to the overaccumulation of capital at the initial periods (④). However, at this time, the ratio of the social welfare level to the optimal growth path diverges to 0.8476~0.9707 ($c_0 = 2.377$~2.4102) (see Fig. 7).

Third, for a wide range of initial consumption levels of $2.4103 \le c_0 < 9$, all growth paths converge uniformly in the region containing the Modified Golden Rule level (③). As initial consumption levels rise, the growth path overaccumulates capital initially but eventually approaches the optimal growth path and then deviates from the Modified Golden Rule level as capital accumulation levels decline. The social welfare level and the steady-state levels of capital and consumption reach the most equivalent levels to the optimal growth path when $c_0 = 3.46$~3.48 and $c_0 = 3.65$, respectively, but they are not identical. With the same initial value of $c_0 = 3.94697$ as the optimal growth path, the steady-state levels of consumption and capital decline to some extent, but the level of social welfare remains high at 0.9988.

Finally, capital accumulation stagnates as the initial consumption planning level further increases. In a growth path with an initial consumption level of $c_0 > 9$, capital and consumption decline uniformly (①). As a result, the economic growth level remains very low. The ratio of the social welfare level to the optimal growth path rapidly declines to 0.8465~0.6223 ($c_0 = 9$~10) (see Fig. 7).

From the results of the above numerical calculations, when the initial value of the consumption level is in the range of $2.4103 < c_0 < 9$, all the growth paths based on budget control (PDSA cycle) converge uniformly in the region around the Modified Golden Rule level. Then, the ratio of the social welfare level to the optimal growth path exceeds 0.9 at $c_0 = 2.4103$~8.2, approaches 0.99 at $c_0 = 2.5$~5.1, and reaches 0.999 at $c_0 = 3.1$~3.9. We can evaluate the feasibility of each of the social welfare level ratios above 0.9 as 0.58, 0.26, and 0.08 in order when estimated as the ratio of each area to the possible initial planned consumption range $c_0 = 0$~10 concerning the amount of production. That is, those paths are sufficiently manageable in practical terms.

In sum, we can start with a wide range of initial plans and make decisions based on budget control while sequentially modifying the plan to deal with environmental changes that may occur constantly. In other words, consumption/investment planning based on budget control is more realistic and practical than optimal growth planning, which requires strict operation on the unstable saddle point path.

Concluding remarks

Conclusions. Economics generally assumes optimal decision-making based on rational expectations. In management practice, however, decisions are made based on the principle of budgetary control, i.e., successive revisions of plans by contrasting plans with actual results through PDSA cycles. Not to mention businesses, households, and government agencies have also adopted this practical approach widely. In this paper, we have modeled and examined the economic growth problem based on consumption/investment planning of households making such budgetary control-type decisions employing replicator dynamics.

This paper clarifies that myriad growth paths can approach the optimal growth plan while mitigating dynamic inefficiencies through budget-controlled decision-making. The growth path guided by budget-controlled decision-making can provide substantially more stable control and achieve sufficient efficiency, in contrast to optimal growth planning, where even a few error variations can lead to failure. Of course, budget-based decision-making does not assume omniscient central planning or rational expectations, as in optimal growth planning, because the plan is modified sequentially to adapt to changes in the external environment.

Furthermore, the numerical results show that if the national economy reaches a capital size commensurate with external environmental conditions, budgetary control-type decision-making can provide enough performance and manageability. For example, the growth paths could achieve a ratio of 0.9 or more on the social welfare criterion to the optimal growth plan, with a rate of 0.58 of the possible initial consumption range.

Rational decision-making leads to the optimal and unique growth path. However, optimal growth planning leads to an unstable saddle-point path and can only realize if it assumes the very rationality associated with perfect foresight. On the other hand, the growth paths guided by budget-controlled adaptive decision-making are diverse and redundant, distributed in myriad ways around the optimal growth path.

However, this redundancy creates administrative stability in the planning because the plan is carried out through the trial-and-error process of forecast versus actual, allowing for multiple suboptimal targets that include the optimal growth path. Moreover, the numerical results show that the budget-controlled consumption plan is comparable to the optimal growth plan on a social welfare basis, calculated by accumulating long-term consumption utility, and is sufficiently manageable in practice. For example, a path that exceeds 0.9 as a ratio to the optimal growth plan is practically reachable at the initial planning stage at a ratio of 0.58. Based on the above results, we can analyze intertemporal economic events with this realistic, practical, and simple method, replacing dynamic optimization ones.

In addition, advances in computer science and information technology, such as Deep Learning and Big-Data will improve the accuracy of estimating the mechanisms and causal relationships of complex behaviors hidden in existing data. As a result, the sequentially adaptive decision-making to correct differences between plans and performance will be more consistent with those advances and make it easier to approach optimal planning.

Remaining issues. The PDSA cycle is a standard of conduct that consumers, businesses, and government agencies have widely adopted as procedures for a practical guide. And the replicator dynamics are widely applicable to the analysis of socioeconomic phenomena, corresponding to the PDSA cycle procedures, with sequentially adaptive decision-making as the behavioral criterion. However, PDSA cycles and replicator dynamics also have limitations beyond the scope of this paper.

First, as introduced in the section "PDSA cycle as a budgetary control procedure", the PDSA cycle was initially conceived as a practical procedure for quality control reflecting the results of management engineering and has been widely adopted, mainly in business management decision-making, in conjunction with various management accounting methods. However, in domains where it is difficult to provide adequate services through business management based on pure market transactions, the PDSA cycle is likely inconsistent with operational procedures onsite. Such domains of incompatibility include the operations of healthcare, welfare, and education, which we have traditionally viewed as examples of market failure. In the above areas, the PDSA cycle may not be compatible with the goal of quality improvement in principle, which we cannot capture in efficiency in day-to-day operations in the field.

In particular, while attempts to improve the safety and efficiency of medical practice through PDSA have been actively under recommendation in the healthcare sector (NHS England and NHS Improvement, 2022), inherent problems have been identified with PDSA in terms of quality improvement (QI) of healthcare. According to Reed and Card (2016), concerns exist in the medical field about the appropriateness of adopting the PDSA for improving healthcare, as the strict application of the PDSA has traditionally had aspects that undermine the act of learning onsite and require complex procedures in practice. Onsite where the implementation of medical treatment (Do) is the highest priority, it is essential to invest heavily in leadership, expertise, and human resources for organizational change in a way adapted to each healthcare service to improve healthcare service through PDSA. But they point out that this is still insufficient in reality.

In addition, according to Knudsen et al. (2019), previous research findings indicate that methodological problems frequently occur in planning for healthcare QI through PDSA and that there is no clear link between PDSA implementation and improved clinical implementation and patient healing outcomes. Then they used PubMed, Embase, and CINAHL to search for articles on QI projects using PDSA published in 2015~2016 to investigate the actual evaluation of the project's results. As a result, even though most of the QI projects reported improvements, low adherence to key methodological features (iterative cyclic method, continuous data collection, small-scale testing, and use of a theoretical rationale) in individual projects left a challenge as to whether PDSA would truly improve QI. Based on these results, they suggest that methodologies addressing QI in healthcare need continuous improvement.

Second, the replicator dynamics that model sequentially adaptive decision-making with PDSA as a dynamic system has the limitation that it does not explicitly capture expectations for future uncertainty, especially in the deterministic model structure of this paper, even when applied to the market-based intertemporal economic choice problem. The intertemporal choice problem that is the target of this paper is the optimal savings one between current consumption utility and future production expansion, which Ramsey pioneered. On the other hand, financial researchers have conducted many studies on the optimal consumption/investment problem originating from the so-called Merton's portfolio problem.

For example, as a recent study on intertemporal consumption choices, Shigeta (2022)'s numerical and mathematical analyses show that unlike in the constant relative risk aversion utility, present bias in the Epstein–Zin utility causes economically significant over-consumption, maintaining a plausible attitude toward risks. Chen and Li (2020) also numerically find that time-inconsistent preferences lead agents to more consumption–wealth ratio.

Of particular interest in this area of research is the real option, especially the option to expand, which is the most common option in practice and shares the same behavioral principle as the sequentially adaptive decision-making model in this paper. For example, in Rambaud and Sánchez (2017), real options, and more specifically, the option to expand, are assumed to be included in the project information in addition to the expected cash flows. And they determine the present value of projects with the option to expand the production capacity by a given percentage within one generally within n years. The results also show that option values increase concerning option expiration, indicating the possibility of extending the framework of this paper as a decision-making model that captures the risk management of long-term planning.

Third, even though the scope of this paper is the long-term economic growth planning of consumption and savings, the replicator dynamics in this paper discard its major factor, technological progress. However, concerning the endogenous growth model introduced in the section "Optimal growth model", it is immediately possible to reconstruct this decision-making process in terms of replicator dynamics, the sequential behavioral principle of this paper. As a result, we can probably approach the growth path guided by the endogenous growth model in a sequentially adaptive manner at each phase of diminishing, constant, and increasing social marginal productivity of knowledge capital.

On the other hand, replicator dynamics have the disadvantage that in the evolutionary growth process, only adaptive behavior function that causes the selective phenomenon of technological innovation by the market, and a mechanism to generate the mutation phenomenon does not function, which is the generative element of technological innovation. Particularly in economic growth theory, the growth planning assumed at a certain technological level in this paper is undeniably unrealistic concerning the long-term plan of optimal growth.

Safarzynska and van den Bergh (2011) devised an evolutionary growth model of innovation for replicator dynamics with genetic mutation and recombination in biology. This model is due to the widely shared perception that in innovation, the recombination of existing ideas, products, or technologies has become a core mechanism for diversity creation in both the economy and technology. It attempts to capture the diffusion process of a finite number of n alternative technologies with constant mutation and recombination rates.

In contrast to the above approach, it is not the primary task of economics to predict the specific areas that we cannot target in advance, such as what scientific findings will be to practical use and what technological innovations will occur through the marketplace. As a well-known anecdote, it would have been impossible to predict the practical application of the automobile by extending the technology in the horse-drawn carriage era. It would also have been difficult to predict in the 1990s that the personal digital assistant (PDA) would evolve into the modern smartphone by recombining technologies such as mobile communication, Internet environments, packet communication, and graphical user interfaces.

However, even if economists cannot predict the specific new technological areas that will drive the economy, it is possible to take an approach that answers the evolutionary economic growth problem within the framework of answering the allocation problem of scarce resources, such as the extent to which we can allow trial and error to discover completely unknown technological areas while promoting improvements in existing technological ones to ensure stable growth. Sakaki (2004) quantitatively answers this resource allocation problem by performing numerical calculations in a deterministic model using replicator dynamics.

For replicator dynamics, the above constraints exist, and extensions are also necessary. However, intertemporal planning based on the sequentially adaptive decision-making principle and one of the modeling methods, replicator dynamics, use all the information accumulated from the past to adaptively deal with differences between forecasts and actual results by sequentially repeating optimal decisions at each period. We can also practically deal with future uncertainty by replicator dynamics.

Received: 12 January 2023; Accepted: 31 March 2023;
Published online: 17 April 2023

Notes

1 Based on Iwai (1994), we present an outline of the optimal growth model.

2 To be consistent with the optimal growth model, we should adopt a decision-making principle where households plan consumption levels based on the advantageous difference in consumption amounts between the current and following terms. The reason for adopting the difference in production value as a decision-making criterion is to obtain an analytical result using the approximate Eq. (8). However, as confirmed by the numerical calculations in the section "Social welfare evaluations", even the decision-making criteria in this paper yield result comparable to the optimal growth on social welfare criteria.

3 Assuming that x_t is correlated with, but not equal to, the average propensity to consume, we approximate the change in consumption by the changes in x_t and $f(k_t)$.

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Competing interests

The author declares no competing interests.

Ethical approval

This article does not contain any studies with human participants performed by any of the authors.

Informed consent

This article does not contain any studies with human participants performed by any of the authors.

Additional information

Supplementary information The online version contains supplementary material available at https://doi.org/10.1057/s41599-023-01667-1.

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