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A balance for fairness: fair distribution utilising physics

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The problem of 'uneven distribution of wealth' accelerated during the COVID-19 pandemic. In the chaotic modern society, there is an increasing demand for the realisation of true 'fairness'. In this study, we propose a fair distribution method 'using physics', which imitates the Greek mythology, Themis, having a 'balance of judgement' in her left hand, for the profit in games of characteristic function form. Specifically, we show that the linear programming problem for calculating 'nucleolus (a solution for the fair distribution)' can be efficiently solved by considering it as a physical system in which gravity works. In addition to significantly reducing the computational complexity, the proposed scheme provides a new perspective to open up a physics-based policymaker that is adaptable to real-time changes. We will be able to implement it not only in liquid systems but also in many other physical systems, including semiconductor chips. The fair distribution problem can be solved immediately using physical systems, which should reduce disputes and conflicts based on inaccurate information and misunderstandings, eliminating fraud and injustice.

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Introduction

The Buddha taught that the 'goodness' that embodies 'righteousness' stands in contrast to duties and to personal ties. In other words, the definition of 'good' is what will be in the interest of oneself, in the interest of others and in the interest of those to be born in the future. Humans feel happy when they are needed within a community and when they play a role that benefits others and the whole community. This core aspect of human nature is often forgotten in actual social activities, seen as a 'beautiful thing' at a distance from the practical.

In a modern society driven by neoliberalism, only the more 'primitive' desires of man are judged essential. Because neoliberalism assumes that individual negligence is the source of poverty, disparity and poverty are the incentives for hard work rather than goodness or spiritual insight (Jinno 2010). We have come to believe that market-based competition brings efficiency and optimisation to society. As a result of this, not only the destruction of the natural environment but also the destruction of the human environment has been promoted as a means of achieving growth. The result has been a loss of trust and bonds between humans due to excessive competition.

Today, humanity is facing a serious crisis not only because of the COVID-19 but also because of the lack of trust between people. In order to defeat infectious diseases, people need to trust scientists, citizens need to trust public institutions and countries need to trust each other. New viruses can occur everywhere, and if they do occur, the important question will be whether the government can immediately implement a city blockade. Unfortunately, we must include the economic loss of the blockade as a variable. In such a case, the timing of the blockade may be delayed without international assistance (Harari 2020).

Nevertheless, it has been proven in socialist countries that easy solutions to poverty, such as equal distribution, do weaken incentives for hard work. Also, the fact that acting alone and selfishly within an altruistic community can produce great benefits for the individual (in the short term) seems to be an obstacle to formulating a powerful critique of neoliberal ideology. The question that remains is whether it is not possible to build a system that mobilises digital and new analogue technologies so that people can do 'good' with confidence?

The problem of 'uneven distribution of wealth' (Piketty 2014) has been accelerated in the midst of the COVID-19 pandemic (Pastreich 2019, 2020). The 'universal basic income', which includes wealth redistribution, solving social security problems and combating poverty, is being actively discussed and experimented with. In the year 2020 Spain became the first country in the world to implement a 'basic income' for the needy. In this chaotic 'new turn of capitalism', there is an urgent need to establish a firm and blameless 'fairness' in order to avoid inequities and social justice for everyone. However, there are different kinds of difficulties for the realisation as follows.

1. Fairness requires proper judgement and the processing of various factors impacting the current situation. The calculation often cannot keep up with the changing dynamic system.
2. The elements critical to determining fairness that depends on human interpretation are difficult to formalise.
3. As Arrow's impossibility theorem (Arrow 1951; Saeki 1980) shows, we do not even know whether there is a 'fair solution' or not.

In this study, we propose a physical method to realise a 'fair distribution of profit' within the restricted domain of games. Therefore, we only have to focus on how to overcome the computational difficulty for 'fairness' calculations as long as we treat the restricted domain of games called 'games in characteristic function form'.

Previously, Kim proposed a flexible analogue computation scheme that overcomes the computational difficulties of digital computing by taking advantage of natural (physical) properties, such as conservation laws, continuity and fluctuations (Kim et al. 2016, 2017). In particular, he extracted an efficient decision-making scheme (reinforcement learning) called the 'tug-of-war principle' (Kim et al. 2015), and he and coworkers created 'self-decision-making physical systems' by applying it to quantum dots (Kim et al. 2013), single photons (Naruse et al. 2015), atomic switches (Kim et al. 2016), semiconductor lasers (Naruse et al. 2017), ionic devices (Tsuchiya et al. 2018), resistive memories (Kim et al. 2019) and internet of things (IoTs) (Ma et al. 2019).

The physics-based computations are useful in the calculation of 'fairness'. Themis, the Greek goddess of justice, installed in courthouses and law offices, is considered the 'guardian of justice and order' (see Fig. 1). She wears a blindfold as a sign of her objectivity and selflessness, holding a sword in her right hand to protect society from vice and a 'balance of justice' in her left hand to measure right and wrong. In other words, she leaves the impartial judgment to nature (physical phenomena) rather than perceptions and attachments. Although that balance is a symbol, the 'use of nature (physics)' is thought to have the effect of eliminating artificiality and gaining consensus (concessions) from the people concerning issues.

The direct use of physics in computation is not a particularly new concept (Steinhaus 1960; Levi 2009). Rather, before the advent of the digital computer, everything was analogue. For example, suppose that Fig. 2 has three villages with 50, 70 and 90 elementary school students living in each village. How can we minimise the total distance travelled for all students' to school when we build an elementary school? As shown in Fig. 2, the centre of gravity is the answer (as to where the elementary school should be built) made by hanging weights determined by the number of students in each village. If we consider the brute-force search for possible locations, the calculation would be intractable, however, by 'designing' the problem in this way, we can leave the calculation (or even part of it) to physics.

The role of computation in our society is increasing daily. Humanities and social sciences are no exceptions, especially as computation, including artificial intelligence (AI), is transforming our society in many ways. For example, studies on computational policymaking, using AI and multi-agent simulations, have already begun (Lempert 2002; Zheng et al. 2020). Attempts to create a fair voting system are being promoted (Brandt et al. 2016; Endriss 2017). That includes voting theory, such as the computational complexity of the winner determination and manipulation in elections, algorithms for fair allocation and coalition formation, such as matching and hedonic games. Moreover, studies are also underway on building reciprocal relationships that people need to move from competition to co-operation (Nowak et al. 2006; Yamamoto et al. 2020).

Despite the importance of computation, there are serious limitations to the existing computational methods, including AI. When it comes to our ability to computing, we are almost powerless, especially when considering solving societal problems. On the basis of four simple examples, this study shows that the calculation of fair distribution can be realised directly by exploiting physics. If it is implemented on a semiconductor chip (using analogue resistance values (Kim et al. 2019), it could be linked to GPS and big data, so that the distance from your current location to your home is immediately known, helping us decide in an instant how much to spend on a taxi in case of riding with friends (taxi problem), enabling us to make our own decisions. The distribution of the bankruptcy problem can also be calculated immediately, which should reduce disputes and conflicts based on

Fig. 1 Themis, the Greek goddess of justice, installed in courthouses and law offices, is considered the 'guardian of justice and order'. She wears a blindfold as a sign of her objectivity and selflessness, holding a sword in her right hand to protect society from vice and a 'balance of justice' in her left hand to measure right and wrong. This photograph was reprinted with the permission of the copyright holder, the Nippon Catalogue Shopping Co. Ltd. http://www.catalog-shopping.co.jp.

inaccurate information and misunderstandings, eliminating fraud and injustice.

Games in characteristic function forms. In this study, we propose a physical method to realise a 'fair distribution of profit' within the games in characteristic function form (Neumann and Morgenstern 2007; Gilles 2010). Two types of games called the 'taxi problem' and the 'bankruptcy problem' are introduced.

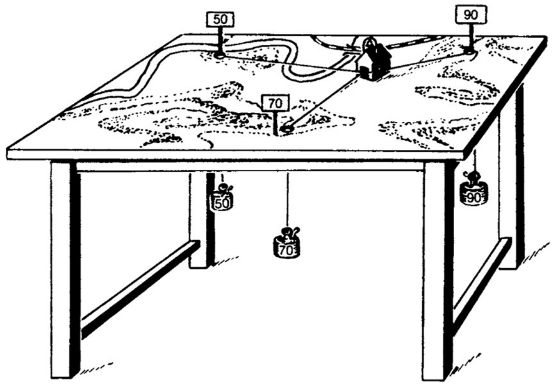

Fig. 2 An example for physically solving computational problems. There are three villages with 50, 70 and 90 elementary school students living in each village. How can we minimise the total distance travelled for all students' to school when we build an elementary school? The answer is the centre of gravity (as to where the elementary school should be built). This figure was reprinted with the permission of the copyright holder, the Oxford University Press. H. Steinhaus: Mathematical Snapshots, Oxford Univ. Press (1960).

A cooperative game wherein the players can cooperate with each other can be an effective means of determining how the profit obtained through cooperation can be fairly distributed (Yokoo et al. 2013; Kishimoto 2015). Cooperative games have been applied to market analysis, voting analysis and cost-sharing analysis in economics, as well as to market design, such as auction theory and matching theory, the theme of the 2020 Nobel Prize in Economics (Kishimoto 2015).

Let us define $N = \{1, 2, \ldots, n\}$ as the set of n players. We call the non-empty subset of N 'coalitions'. If R is a set of real numbers, we call the real value function v: $2^N \to R$ on 2^N (the set of the subset of N) 'a characteristic function'. For each coalition S ($\subseteq N$), $v(S)$ represents the profit that can be gained by the members of the coalition S by cooperating. More concretely, it is the difference between the cooperative case and the non-cooperative case that is critical. We call (N, v), which is the set of players N and a characteristic function v, a 'game of characteristic function form'.

Taxi problem. Consider a situation in which three persons (1, 2, 3) finish eating at a restaurant and go home by taxi. If they return home alone, the charges are Person 1: \$20, Person 2: \$21 and Person 3: \$25, respectively. In addition, between each of the two houses, the following taxi fees are charged: \$10 between 1 and 2, \$14 between 1 and 3 and \$20 between 2 and 3. In this case, the shortest route is (2–1–3), which costs \$45.

In this example, the characteristic functions are the followings.

$$v(1,2,3) = (20 + 21 + 25) - 45 = 21, \quad (1)$$

$$v(1,2) = 41 - 30 = 11, \quad (2)$$

$$v(1,3) = 45 - 34 = 11, \quad (3)$$

$$v(2,3) = 46 - 41 = 5, \quad (4)$$

$$v(1) = v(2) = v(3) = 0. \quad (5)$$

where a characteristic function $v(x, y)$ is the profit if x and y have cooperated, that is the difference between the total amount of fares for going home alone and the fare for going home in a taxi by the shortest route.

The question is, 'How should the profit of $21 be divided 'fairly' among the three?' In general, this problem can be solved using the concept of 'nucleolus (Schmeidler 1969)' as follows. First, in the games of characteristic function form (N, v), we define the 'complaint' $C(S, x)$ of the coalition S with respect to the allocation x as follows.

$$C(S,x) = v(S) - \sum_{i \in S} x_i \tag{6}$$

That is the profit of the coalition S minus the 'total allocation (distribution) of those who participated in the coalition S'. The 'nucleolus' is the allocation that minimises the largest complaint and is generally known to be uniquely determined (for simplicity, we avoid an exact mathematical definition of 'nucleolus' here).

In the case of the taxi problem, the complaint against each coalition is as follows

$$C(\{1,2\},x) = v(1,2) - x_1 - x_2 = 11 - x_1 - x_2, \tag{7}$$

$$C(\{1,3\},x) = v(1,3) - x_1 - x_3 = 11 - x_1 - x_3, \tag{8}$$

$$C(\{2,3\},x) = v(2,3) - x_2 - x_3 = 5 - x_2 - x_3, \tag{9}$$

$$C(\{1\},x) = v(1) - x_1 = -x_1, \tag{10}$$

$$C(\{2\},x) = v(2) - x_2 = -x_2, \tag{11}$$

$$C(\{3\},x) = v(3) - x_3 = -x_3. \tag{12}$$

Using the total rationality $x_1 + x_2 + x_3 = v(1,2,3) = 21$, we rewrite the complaints of the two-person coalitions.

$$C(\{1,2\},x) = x_3 - 10, \tag{13}$$

$$C(\{1,3\},x) = x_2 - 10, \tag{14}$$

$$C(\{2,3\},x) = x_1 - 16. \tag{15}$$

Since the nucleolus is the allocation that minimises the maximum constraint, we suppose the above six complaints Eqs. (10), (11), (12), (13), (14) and (15) are less than or equal to a certain value T, then we minimise T. In other words, consider the following linear programming problem.

Minimise(T):

$$-T \le x_1 \le T + 16, \tag{16}$$

$$-T \le x_2 \le T + 10, \tag{17}$$

$$-T \le x_3 \le T + 10, \tag{18}$$

$$x_1 + x_2 + x_3 = 21, \tag{19}$$

$$x_1 \ge 0, \tag{20}$$

$$x_2 \ge 0, \tag{21}$$

$$x_3 \ge 0. \tag{22}$$

In order for there to be x_1, x_2, x_3 that satisfies the constraints, the following formulas must hold from Eqs. (16), (17), (18) and the individual rationality $(x_i \ge v(i))$.

$$-T \le T + 16 \iff T \ge -8, \tag{23}$$

$$-T \le T + 10 \iff T \ge -5. \tag{24}$$

The above expressions can be transformed to the followings from Eqs. (16), (17), (18) and the total rationality Eq. (19).

$$-3T \le (x_1 + x_2 + x_3) = 21 \le 3T + 36 \tag{25}$$

$$\iff T \ge -7 and T \ge -5 \tag{26}$$

The minimum value of T that satisfies all these conditions is -5, which, when substituted, yields the following constraint condition,

$$-5 \le x_1 \le 11, \tag{27}$$

$$-5 \le x_2 \le 5, \tag{28}$$

$$-5 \le x_3 \le 5, \tag{29}$$

$$x_1 + x_2 + x_3 = 21 \tag{30}$$

$T = -5$ can be realised only with $x_1 = 11$, $x_2 = 5$ and $x_3 = 5$. That is, 'nucleolus' in the taxi problem is $(11, 5, 5)$ and according to the nucleolus's criterion, Person 1 would receive $11 and Persons 2 and 3 would both receive the allocation of $5. In general, in games of characteristic function form with n players, we must solve a linear programming problem to minimise the maximum complaint and if there is more than one allocation that achieves the smallest maximum complaint, the second-largest complaint in the allocation must be minimised. To find 'nucleolus', it is necessary to solve the linear programming problems at most $n - 1$ time. The computational complexity of linear programming is commonly known as at the most polynomial time (Yokoo et al. 2013; Karmarkar 1984).

Bankruptcy problem (estate distribution problem). In what can be considered a game of characteristic function form, there exists a famous problem known since long ago. If a person goes bankrupt leaving a debt of $D_1, D_2, \ldots, D_n$ to each of the n creditors $1, 2, \ldots, n$, how should the bankrupt's entire estate (M) be 'fairly' distributed to each creditor? At first glance, a proportional division according to the size of the debt seems to be 'fair'. However, if we consider a case in which the value of the entire estate is smaller than the amount of any debt we encounter a situation in which, because all creditors are similarly unable to fully recover their share, it is better to split the debt into equal parts.

In fact, this sort of bankruptcy problem is described in the Babylonian Talmud, a Jewish scripture more than 2400 years ago (Table 1). In Table 1, M represents the total estate and D represents the creditors and the amount of their debts. If the total estate is 300, the division is proportional, but if it is 100, it is an equal division. Even though for 200, the criteria are not as clear at first glance, there exists a clear criterion of 'nucleolus' which was first formulated in 1969. It is very surprising that the Sages of the Talmud more than 2400 years ago had clear ideas and criteria for the concepts of 'justice' and 'fairness' in this specific financial case. We had to wait until 1985 for a satisfactory explanation of this criterion of bankruptcy problem (Aumann and Maschler

Table 1 The bankruptcy problem in the Talmud.

	$D_1 = 100$	$D_2 = 200$	$D_3 = 300$
$M = 100$	$33 + 1/3$	$33 + 1/3$	$33 + 1/3$
$M = 200$	50	75	75
$M = 300$	50	100	150

If a person goes bankrupt leaving a debt of D_1, D_2 and D_3 to each of the creditors 1, 2 and 3, how should the bankrupt's entire estate (M) be 'fairly' distributed to each creditor? This table shows an answer to the allocations.

1985). That was accomplished by Aumann and Maschler's discussion in relation to game theory (Nakayama 1986; Funaki 1989).

Aumann and Maschler went on to read from the following description of other passages in the Talmud Mishnah (Baba Metzia 2a).

Two hold a garment; one claims it all, the other claims half. Two hold a garment; one claims it all, the other claims half. Then the one is awarded 3/4, the other 1/4.

This statement says that if there are those who demand the whole thing and those who demand half of it, it should be divided into 3/4 and 1/4 respectively. This is neither a proportional nor an equal division, but a 'residual equal division'. For some reason the latter allows 1/2 to the former, so the remaining 1/2 is to be divided equally.

Consider the case of two creditors. The amount that Creditor 1 is willing to admit to Creditor 2 is the greater of $(M - D_1)$ and 0, so $(M - D_1)_+$, defined as $x_+ = max(x, 0)$. Similarly, the amount that Creditor 2 is willing to allow to 1 is $(M - D_2)_+$. The shares of Creditors 1 and 2 (X_1 and X_2) are as follows

$$X_1 = (M - D_2)_+ + \frac{\{M - (M - D_1)_+ - (M - D_2)_+\}}{2}, \quad (31)$$

$$X_2 = (M - D_1)_+ + \frac{\{M - (M - D_1)_+ - (M - D_2)_+\}}{2}. \quad (32)$$

They called 'the Contested Garment principle' the distribution principle in a bankruptcy case with one bankrupt and two creditors. In the Talmudic bankruptcy problem (three creditors) the CG principle holds for any two creditors and is shown to be 'nucleolus' (Aumann and Maschler 1985).

Method

Physical method for fair distribution. In this section, we mainly describe how to solve a game of characteristic function form with 'gravity'. First, we have two cylinders A (blue) and B (yellow) filled with a liquid (incompressibility) such as Fig. 4. The adjusters of A and B are assumed to be interlocked: if one moves, the other moves the same amount in the opposite direction (see Fig. 3). By moving the adjusters in sequence according to the instructions as in Fig. 4, each liquid level represents the 'amount of complaint' of the corresponding coalition. The liquid levels E_1, E_2 and E_3 of the blue liquid correspond to the definitions of complaints $C(\{2, 3\}, x)$, $C(\{1, 3\}, x)$ and $C(\{1, 2\}, x)$, respectively. Finally, when the two cylinders are set up vertically at the same time to exert gravity, we find that gravity sets the liquid level in motion and that the final equilibrium liquid level will represent the 'fair distribution' (physical solution).

The procedure for the solution with physics is as follows

1. The valve is open so as to allow for the adjustment of the liquid level by moving the adjuster. For a cylinder of blue liquid A, divide equally the profit $v(1, 2, 3)$ that would be obtained if everyone had cooperated as shown in Fig. 4a. Here, $x_i = v(1, 2, 3)/3$.
2. For each i, add the profit of a two-person coalition other than i $(+v(j, k))$, and then subtract the profit that would be gained if everyone had cooperated $(-v(1, 2, 3))$ as shown in Fig. 4b.
3. For each i, we record the current liquid level and then set it to the origin of the upcoming gravitational change in the liquid level y_i. That is, each distribution will be $x_i = y_i + v(1, 2, 3)/3$, satisfying the requirements of the conservation law $y_1 + y_2 + y_3 = 0$. The current liquid level E_i expresses the complaint of the coalition (j, k). That is, $E_i \equiv C(\{j, k\}, x) = v(j, k) - (x_j + x_k) = x_i - v(1, 2, 3) + v(j, k)$.

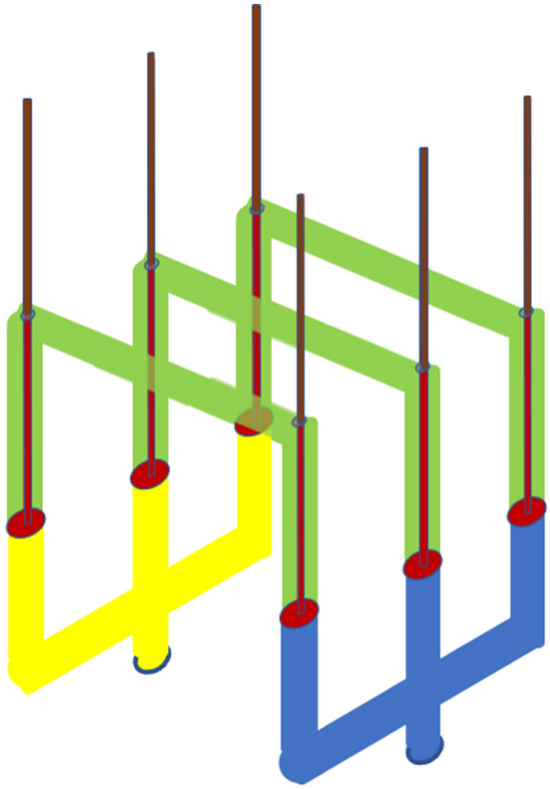

Fig. 3 An implementation for 'interlocked adjuster'. Each cylinder filled with green liquid is connecting blue and yellow cylinders. Due to the interlocking adjuster, if a blue liquid level E_i increases (decreases), the corresponding yellow liquid level C_i decreases (increases).

4. Then, for each i of yellow liquid cylinder B, add the profit generated by the coalition $i(+v(i))$, and then subtract $-v(1, 2, 3)/3$ as shown in Fig. 4c. The current liquid level C_i will express the complaint of coalition i. That is $C_i \equiv v(i) - x_i$.
5. All six complaints will thereby be expressed and the preparation will be complete. At this point, the valves are closed and the adjusters of cylinders A and B are interlocked as shown in Fig. 4d. That is, an increase in cylinder A's E_i results in a corresponding decrease in the corresponding liquid level C_i of cylinder B.
6. Stand the two-cylinder systems vertically at the same time so that gravity works (see Fig. 3).
7. Wait until the equilibrium state is attained and then record the final liquid level change y_i as shown in Fig. 4d.
8. Note that the liquid level change satisfies the following equation:

$$y_i \geq v(i) - v(1, 2, 3)/3. \quad (33)$$

9. The distribution is determined by the following equation:

$$x_i = y_i + v(1, 2, 3)/3. \quad (34)$$

Physical solutions on fair distribution problems

The physical solutions of the taxi problem and the bankruptcy problems (three ways) are shown in Fig. 5. We could find solutions to all the problems using physics.

Figure 5a shows the initial liquid levels of the taxi problem, which can be set by following the instructions stated in 'Method'. Although we only show the cylinder system filled with blue liquid, the other cylinder system filled with yellow liquid is supposed to

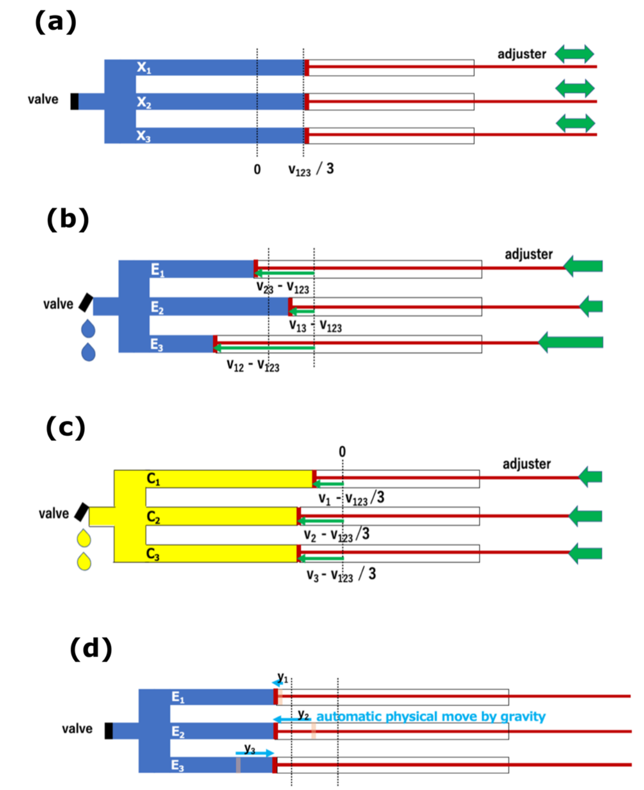

Fig. 4 Fair distribution method utilising physics in the game of characteristic function form. a Equally divide the profit of coalition with all ($v(1, 2, 3)$). That is, $x_1 + x_2 + x_3 = v(1, 2, 3)$. Here, x_i is the allocated profit for i. **b** For each i, add the profit of coalition $\{j, k\}$ ($+v(j, k)$) and subtract the profit of coalition with all ($-v(1, 2, 3)$). For each i, mark the present position as the initial position of displacements y_i. Here, $x_i = y_i + v(1, 2, 3)/3$, which means $y_1 + y_2 + y_3 = 0$ due to the volume conservation. Each E_i represents the amount of complaint about coalition $\{j, k\}$. Here, the complaint $E_i \equiv v(j, k) - (x_j + x_k) = x_i - v(1, 2, 3) + v(j, k)$. **c** In the yellow cylinder system, for each i, add the profit of coalition $\{i\}$ ($+v(i)$) and subtract the profit of coalition with all ($-v(1, 2, 3)/3$). Each C_i represents the amount of complaint about coalition $\{i\}$. Here, the complaint $C_i \equiv v(i) - x_i$. **d** Stand vertically this equipment for automatic physical moves caused by gravity. Owing to the interlocking adjuster, if y_i increases in the blue cylinder, the adjuster moves to the left-hand side ($-y_i$) in the corresponding yellow cylinder. For each i, read the move (displacement) y_i from the initial position. Note that each y_i must be satisfied $yi \geq (v(i) - v(1, 2, 3)/3)$. Finally, we can obtain the distribution $x_i = y_i + v(1, 2, 3)/3$.

be connected through an 'interlocking adjuster' (if one move, the other moves the same amount in the opposite direction). Each liquid level represents each complaint, that is, $E_1 = C(\{2, 3\}, x)$, $E_2 = C(\{1, 3\}, x)$, $E_3 = C(\{1, 2\}, x)$, $C_1 = C(\{1\}, x)$, $C_2 = C(\{2\}, x)$ and $C_3 = C(\{3\}, x)$, respectively. Then, we set up the cylinder system vertically so that gravity works (Fig. 3). There are various possibilities depending on whether the 'interlocking adjusters' in the figure are realised with other liquids or implemented mechanically. In any case, at each level of equilibrium, the tensile forces and the pressure inside the liquid (the sum of the statistical and gravitational pressures according to Pascal's principle) will be balanced.

In an ideal case, the 'equal liquid level' of every cylinder will be realised at equilibrium. However, this system cannot reach the ideal equilibrium state due to the constraints of the adjusters. The highest liquid level (maximum complaint) will gradually decrease, heading towards the equal liquid level. However, simultaneously, other liquid levels will increase due to interlocking, resulting in the highest liquid level eventually stop when it cannot further drop.

In the case of the taxi problem, the maximum complaint was $E_2 = E_3 = -3$ and the equilibrium liquid level was -5. Initial levels of the cylinder filled with blue liquid ($E_1 = -9$, $E_2 = -3$ and $E_3 = -3$) could reach the equilibrium level of -5 easily; whereas, those filled with yellow liquid ($C_1 = C_2 = C_3 = -7$) reached -11, -5 and -5, respectively. Therefore, the displacements y_1, y_2 and y_3 were $+4$, -2 and -2, respectively. As a result, we obtained the following physical solution $x_i (i = 1, 2$ and $3)$ from the equation $x_i = y_i + 7$ (see 'Method').

$$x_1 = 11, \tag{35}$$

$$x_2 = 5, \tag{36}$$

$$x_3 = 5. \tag{37}$$

According to the physical movements, Person 1 would receive \$11 and Person 2 and 3 would both receive the allocation of \$5, which were the same results from the calculation of the linear programming problem.

Figure 5b shows initial liquid levels of the bankruptcy problem ($M = 100$ cases). Here the characteristic functions are defined as $[(estate(M)) - (total\ debts\ except\ S)]_+$. We obtained $v(1) = v(2) = v(3) = v(1, 2) = v(1, 3) = v(2, 3) = 0$ and $v(1, 2, 3) = 100$. In this case, the maximum complaint was $C_1 = C_2 = C_3 = -100/3$ and the equilibrium liquid level was $E_i = -200/3$. Owing to the volume conservation of both liquids, the liquid levels of the cylinders stopped moving. Therefore, the displacements y_1, y_2 and y_3 were all 0. As a result, we obtained the following physical solution $x_i (i = 1$, 2 and 3) from the equation $x_i = y_i + 100/3$ (see 'Method').

$$x_1 = 100/3, \tag{38}$$

$$x_2 = 100/3, \tag{39}$$

$$x_3 = 100/3. \tag{40}$$

According to the physical movements, Creditors 1, 2 and 3 would receive the same allocation of 100/3 (equal division), which were the same values described in the Talmud (see Table 1).

Figure 5c shows initial liquid levels of the bankruptcy problem ($M = 200$ cases). From the definition of the characteristic function, we obtained $v(1) = v(2) = v(3) = v(1, 2) = v(1, 3) = 0$, $v(2, 3) = 100$ and $v(1, 2, 3) = 200$. In this case, the maximum complaint was $E_1 = -100/3$ and the equilibrium liquid level was $E_i = -100$. Although the maximum complaint E_1 gradually decreased, heading toward the equilibrium liquid level of -100, it stopped at -50 because C_1 reached -50 from the initial level of $-200/3$ due to the interlocking between E_1 and C_1. Therefore, the displacements y_1, y_2 and y_3 were $-50/3$, $25/3$ and $25/3$, respectively. As a result, we obtained the following physical solution $x_i (i = 1, 2$ and $3)$ from the equation $x_i = y_i + 200/3$ (see 'Method').

$$x_1 = 50, \tag{41}$$

$$x_2 = 75, \tag{42}$$

$$x_3 = 75. \tag{43}$$

According to the physical movements, Creditor 1 would receive 50 and Creditors 2 and 3 would both receive the allocation of 75, which were the same values described in the Talmud (see Table 1).

Figure 5d shows initial liquid levels of the bankruptcy problem ($M = 300$ cases). From the definition of the characteristic

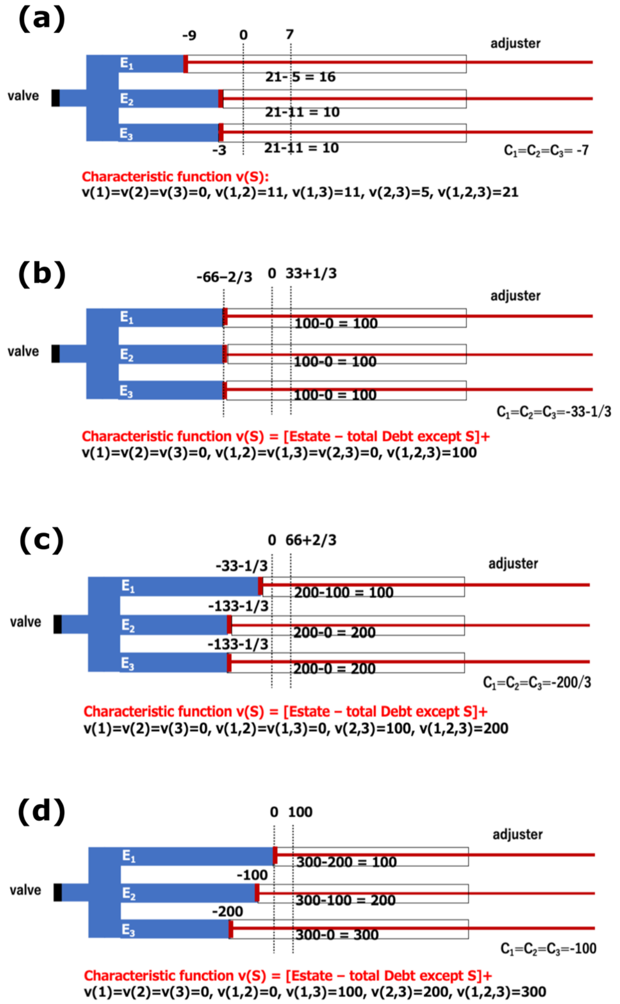

Fig. 5 Physical solutions for profit distribution in a game of characteristic function form. a The taxi problem: Initial liquid levels were $E_1 = -9$, $E_2 = -3$, $E_3 = -3$, $C_1 = -7$, $C_2 = -7$ and $C_3 = -7$, respectively (see 'Method'). Owing to gravity (stand it vertically), the system could reach equilibrium state $E_1 = E_2 = E_3 = -5$. Therefore, the displacements from each initial liquid level were $y_1 = +4$, $y_2 = -2$ and $y_3 = -2$, respectively. We obtained the allocations $x_1 = 11$, $x_2 = 5$ and $x_3 = 5$ from $x_i = y_i + 7$. Person 1 would receive \$11 and Persons 2 and 3 would both receive the allocation of \$5 in this case. **b** The bankruptcy problem ($M = 100$ cases): initial liquid levels were $E_1 = E_2 = E_3 = -200/3$ and $C_1 = C_2 = C_3 = -100/3$, respectively (see 'Method'). Owing to the volume conservation of both liquids, the liquid levels of the cylinders stopped moving. Therefore, the displacements y_1, y_2 and y_3 were all 0. We obtained the allocations $x_1 = x_2 = x_3 = 100/3$ from $x_i = y_i + 100/3$. Every creditor would receive the same allocation of 100/3 (equal division). **c** The bankruptcy problem ($M = 200$ cases): initial liquid levels were $E_1 = -100/3$, $E_2 = E_3 = -100 -100/3$ and $C_1 = C_2 = C_3 = -200/3$, respectively (see 'Method'). Although the maximum complaint E_1 gradually decreased, heading toward the equilibrium level of −100, it stopped at the level of −50 because C_1 also reached −50 from the initial liquid level of −200/3 due to the interlocking between E_1 and C_1. Therefore, the displacements y_1, y_2 and y_3 were −50/3, 25/3 and 25/3, respectively. We obtained the allocations $x_1 = 50$, $x_2 = 75$ and $x_3 = 75$ from $x_i = y_i + 200/3$. Creditor 1 would receive 50 and Creditors 2 and 3 would both receive the allocation of 75. **d** The bankruptcy problem ($M = 300$ cases): Initial liquid levels were $E_1 = 0$, $E_2 = 100$, $E_3 = 200$ and $C_1 = C_2 = C_3 = -100$, respectively (see 'Method'). Although the maximum complaint E_1 gradually decreased, heading toward the equilibrium level of −100, it stopped at the level of −50 because C_1 also reached −50 from the initial level of −100 due to the interlocking between E_1 and C_1. Therefore, the displacements y_1, y_2 and y_3 were −50, 0 and +50, respectively. As a result, we obtained the allocations $x_1 = 50$, $x_2 = 100$ and $x_3 = 150$ from the equation $x_i = y_i + 100$. Creditors 1, 2 and 3 would receive 50, 100 and 150, respectively (proportional division).

function, we obtained $v(1) = v(2) = v(3) = v(1,2) = 0$, $v(1,3) = 100$, $v(2,3) = 200$ and $v(1,2,3) = 300$. In this case, the maximum complaint was $E_1 = 0$ and the equilibrium liquid level was $E_i = -100$. Although the maximum complaint E_1 gradually decreased, heading toward the equilibrium liquid level of −100, it stopped at −50 because C_1 reached −50 from the initial liquid level of −100 due to the interlocking between E_1 and C_1. Therefore, the displacements y_1, y_2 and y_3 were −50, 0 and +50, respectively. As a result, we obtained the following physical solution $x_i (i = 1, 2 \text{ and } 3)$ from the equation $x_i = y_i + 100$ (see 'Method').

$$x_1 = 50, \tag{44}$$

$$x_2 = 100, \tag{45}$$

$$x_3 = 150. \tag{46}$$

According to the physical movements, the creditors would receive 50, 100 and 150, respectively (proportional division), which were the same values described in the Talmud (see Table 1).

Using four practical and simple examples, with $N=3$ persons, we showed that a fair distribution in games of characteristic function form can be physically obtained using the proposed method. We can also obtain the physical solutions in general N-person problems provided we successfully implement the interlocking adjusters. In general N-person problems, the number of variables (complaints) increases exponentially (2^N-2), that is, 6, 14, 30, 62, 126, … , and the physical correspondences become complicated and difficult to understand. Therefore, more complex examples will be dealt with in our future work.

Conclusions

In this study, we have shown that the fair distribution of profit in games of characteristic function form can be solved utilising physics. More specifically, the linear programming problem used to compute 'nucleolus' can be regarded as a physical system that responds to gravity, and the solution can be found efficiently.

Although it is only shown to be effective for solving relatively simpler problems with $N=3$, we can deduce that if we could physically construct the system for general N-person problems, we could always obtain a physical solution of the fair distribution. We plan to consider more complex examples in future. Identification of the necessary conditions, such as physical parameters and constraints, for this physical solution to work, is also future works.

In order for this approach to become a candidate, a viable method, for overcoming the computational difficulties in digital computers, it is necessary to at least address the general N-person problems, and how the solution should be presented. Nevertheless, the employment of such a physical solution may have the following implications and prospects:

1. Able to achieve computational reduction.
2. Able to make real-time decisions at the time of information updates, e.g., parameter changes during calculations (flexibility).
3. Able to interact with information between different species.
4. Able to interlock the distribution motion directly from physical calculations (water distribution, oil distribution) and
5. Eliminate artificiality by using natural phenomena, thus making it easier for people to understand and to reach consensus (compromise).

Real-world problems, such as the distance between two people in a taxi problem, must be addressed often without complete information available. We should be flexible and able to respond flexibly if the information is updated along the way. This system will become a candidate for overcoming the computational difficulties in such problems and will provide a new perspective to open up a physics-based policymaker that is adaptable to real-time changes.

Discussion and future developments

The role of computation in our society is increasing daily. Despite the importance of computation, there are serious limitations to the existing computational methods, including AI. For instance, consider the 19×19 board game of Go. The number of self-avoiding routes from one end to the bottom right diagonal end is greater than the number of atoms in the universe ($10^{87} > 10^{80}$). Surprisingly, this number could not be calculated until recently, when a new theory called 'the theory of counting' was developed (Iwashita et al. 2012). Even the things that lie in the ordinary course of our daily lives are full of things that we cannot calculate. When it comes to our ability to computing, we are almost powerless, especially when considering solving societal problems.

Humans are yet to come up with a computational alternative to the digital computer. Recently computational methods that exploit physical characteristics, such as quantum computing, have been proposed, but they have not been generalised or theorised.

Kim et al. have been trying to extract computational methods that exploit physical characteristics, such as fluctuations (randomness), conservation laws (correlation) and analogue values (cardinality of the continuum), not only in quantum physics but also in the framework of classical physics (Kim et al. 2016, 2015, 2013; Naruse et al. 2015; Kim et al. 2016). They have also developed devices that can be implemented using various physical systems, and in which the physics itself makes its own computational decisions (Naruse et al. 2017; Tsuchiya et al. 2018; Kim et al. 2019; Ma et al. 2019).

This paper illustrates based on four simple examples that the calculation of fair distribution can be realised directly by exploiting physics. Extending beyond $N=4$ is a challenge for future works, but if we can generalise it, we can implement it not only in the liquid systems but also in many other physical systems, which will impact humanity. For example, if it is implemented on a semiconductor chip (using analogue resistance values (Kim et al. 2019)), it could be linked to GPS and big data, so that the distance from your current location to your home is immediately known, helping us decide in an instant how much to spend on a taxi in case of riding with friends (taxi problem), enabling us to make our own decisions.

The distribution of the bankruptcy problem can also be calculated immediately, which should reduce disputes and conflicts based on inaccurate information and misunderstandings, eliminating fraud and injustice. Further, decisions can be made on the basis of 'information based on accurate calculations', and the resources of thought that were previously taken up by the problem itself can now be used for higher-order problems (e.g., meta-thinking, which considers even the reaction of the other party after the decision has been made for self-interests next time).

Natural intelligence (NI) (Kim et al. 2016, 2017) means that intelligent functions emerge from computations directly using the underlying physics and physical characteristics, such as fluctuations (randomness), conservation laws (correlation) and analogue values (cardinality of the continuum). The proposed method based on NI provides solutions and physical implementations, as well as a completely new perspective to social sciences, which is a totally different way of AI (digital computing). To date, the winners of the excessive competition have reaped short-term benefits, but in the end, thanks to their disdain for the overall benefit, the bill has returned to them. As the social implementation of NI progresses, we can see the negative consequences of our overly selfish behaviour by referring to the results of instantaneous physical computations when deciding our actions, and we will be able to change our strategy to one of cooperation rather than competition. Importantly, a change in strategic direction—co-operation rather than competition—will also come from self-interest (long-term) strategies because one can realise owing to NI that for self-interest in the long term, one cannot avoid taking into account the collective interests. Collective interests will be guaranteed as a system, enabling humanity to live in a higher order of thinking. It will be very interesting to see what new social problems will arise in such a case.

Data availability

We generated and analysed no data in this study.

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Author contributions

S.-J.K. invented the method. All authors analysed the results. S.-J.K. wrote the manuscript in discussion with T.T. and K.S.

Competing interests

The authors declare no competing interests.

Additional information

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