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Multifractal features of spatial variation in construction land in Beijing (1985–2015)

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ABSTRACT Over the last 30 years, Beijing has experienced rapid development. The city's spatial pattern and changes are extremely complicated. Therefore, to reveal the spatial patterns at different scales and levels, the construction land in Beijing within the 6th ring road in 1985, 1999, 2008, and 2015, was analyzed using global and local multifractal analyses. The global analysis shows that, over the past 30 years, the development of new construction land in this region experienced a process that transformed it from being a hierarchical spatial structure with a low-evolution intensity (1985) to being a spatial structure with a higher evolution intensity and multi-scale differential development (1999, 2008) with a hierarchical spatial system (one main urban area with a number of satellite towns) (2015). Local analysis further shows that the spatial pattern of the construction land in this region tended to be simplified over the past 30 years. The construction land in this region displays specific evolutionary characteristics with the increase in breadth (q) and intensity (D_q) and a multifractal dimension spectrum ($f(a) - a(q)$) and dominant spectrum at various spatial scales over a four-year period. In general, the influence of high-density areas on the spatial pattern of the construction land is decreasing, while the influence of low-density areas is increasing. Based on this, this paper also summarizes the theoretical value of multifractal analysis in economic geography research, highlighting its advantages, limitations, and application range, and presents the technical analysis process and other suggestions for future research.

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Introduction

Fractal analysis is an important method in the study of spatial evolution. Compared with traditional mathematical methods, which are not effective in spatial self-organization analysis, fractal analysis is an effective means of analyzing the spatial complexity of a study area. Fractal analysis can also provide a corresponding quantitative analysis to obtain an in-depth understanding of spatial patterns and evolution processes (Benguigui et al., 2000; Chen, 2008). For example, consider a metropolitan area with simultaneously existing outstanding fractal features, a diffusion of urban properties, and a range in the multi-functionality or adaptive adjustment of its rural areas. In particular, the suburban areas that are seemingly chaotic but actually belong to the spatial self-organizational evolution of both urban and rural properties are regions wherein the urban and rural dual-attribute interactions are the most complex, and they have already become a major concern (Wilson, 2008). Due to the impact of non-agriculturalization on suburban agricultural areas, the traditional suburban area presents a kaleidoscopic combination of various spatial developments, multiple industries and multiple types of land use, which greatly affects the development of the region.

It is no longer suitable to describe this type of urban and rural interaction process and the enhancement, weakness, or transformation of urban and rural attributes using the simple single evolutionary expansion/contraction scenario of traditional suburban areas. That is, simply using physical space indexes based on Euclidean geometry such as built-up areas, perimeters, and population size/density to describe the complexity and evolution of the city and its suburbs is no longer reasonable. Accordingly, complexity analysis based on non-Euclidean geometry such as chaos, fractal and complex network methods can more deeply reflect and accurately describe the characteristics of the spatial evolution of such areas (Chen, 2005, 2008, 2015; Zhang, 2011; Song et al., 2005, 2006). Although this type of knowledge has already aroused great attention to the disciplines of economic geography, urban planning, and land resource management, very few researchers have engaged in the analysis of the processes and characteristics of the multi-layered strong and weak changes and the suburban adaptive evolution of metropolitan development modalities.

The two objectives of this study are as follows. First, the study adopts multifractal theory to reveal the spatial pattern of the newly increased construction land in Beijing within the 6th ring road area where the greatest changes have occurred with regard to Beijing's urban and rural attributes. In addition, the study considers the spatial characteristics of the region's newly increased construction land to determine the changes in the urban and rural spatial patterns generated by the impact of a variety of processes and dominant multifractal complexity characteristics in this metropolitan city. Second, through hierarchical changes in Beijing's newly increased construction land, the study aims to obtain a deeper and more accurate understanding of the basic path of urban development in Beijing over the past 30 years, which includes not only the significant developmental achievements but also anomalous periodic processes or abnormal phenomena.

It should be noted that because there has always been outward expansion and internal filling of the construction land within the 6th ring road area, the above elucidation may seem to be only an analysis of the newly increased construction land. However, to a great extent, this study reflects changes in the spatial patterns of the whole area, especially changes in the urban construction land caused by the city's urban and rural development. Therefore, the innovation of this paper lies in its multifractal perspective and its mathematical analysis process (the analysis of global and local indicators and related diagrams) to highlight the changes in the regional spatial pattern, especially the change of construction land caused by the evolution of the urban-rural development function, in the region within Beijing's 6th ring road, which is dominated by an urban-rural fringe. Moreover, this paper tries to provide a useful analytical paradigm for the study of spatial coupling and complexity in economic geography by using the multi-year spatial multifractal method based on a quantitative analysis.

Based on the above, in methods section, this paper introduces the study's analysis methods including multifractal technology and its two sets of index systems. In the section of case study, the development background and basic situation of Beijing's six-ring inner region are introduced. The global index Analysis section analyzes the overall spatial evolution process of the region in the past three decades using global multifractal indicators for the years 1985, 1999, 2008, and 2015. The section of local index analysis focuses on the analysis of the local and detailed evolutionary characteristics of each year in the study area using local multifractal indicators in the four-year period. On the basis of the above analysis, the conclusions section summarizes the specific features of the Beijing six-ring internal space system in this development process and seeks to discover the internal rules of its spatial evolution. Finally, the discussion outlines the theoretical problems involved in this study.

Methods

The scattered yet extensive, complex patches of different land types in the geographical physical space are ideal and seemingly kaleidoscopic fractal research objects (Frankhauser, 2008; Chen, 2011, 2010). Specifically, a spatial multifractal system tends to be heterogeneous due to its internal structure, the strength of various components, and the fluctuation of each other. The spatial evolution characteristics of the system can often be revealed by a variety of indices depending on the overall hierarchical (q) change, local singularity ($a\ (q)$) and a variety of linearized fitting curves.

Current spatial multifractal studies mainly focus on the analysis of the numerical fluctuation of spatial data and the grid fractal dimension and the radius fractal dimension of the physical space (Appleby, 1996; Ariza-Villaverde et al., 2013; Ihlen, 2012; Kantelhardt Jan. et al., 2002; Sémécurbe et al., 2016; Thomas et al., 2007, 2008). The purpose of this approach is to determine the circumstances of the change in the overall and local characteristics of a spatial system along with the D_q value of the generalized correlation dimension to clarify its evolutionary structure and developmental trend. Scholars have gradually introduced multifractal technique to the field of economic geography based precisely on these relatively advantageous characteristics in the analysis of space, time and hierarchy of complex systems. As far as this article is concerned, the allometric growth of a metropolitan area in all respects, levels and elements can always produce, by various degrees in many fields, expansion (corresponding to the scenario of repeated fractal stacking adopted by this paper) or contraction (corresponding to the scenario of gradual fractal segmentation such as agricultural land contraction) (Zhang, 2011), thereby resulting in the counterbalancing of the space of different land-use types and objectively promoting changes in land structure, spatial structure, and the regional evolutionary process. However, multifractal parameters can be obtained by analyzing the "3S" data-based grid fractal dimension and the radius fractal dimension, thereby performing the complexity change analysis of the research object.

In the current study, the multifractal technique has two sets of index systems with which to analyze regional global spatial characteristics and local spatial characteristics, respectively.

Global indexes. Multifractal systems tend to be hierarchical due to the counter-balancing in space of the fractal units at all levels; the size of the hierarchy is often expressed by q. In the practical analysis process, with the change in the q value, certain specific characters of the geographical space and the corresponding changes of their mathematical singularity characteristics are often used to resolve the changes in parameters such as the fractal dimension of the geographical system at all levels, as well as the phenomena of the fractal units at all levels, for instance, the scope of influence and size in a large system (Chen, 2010). Specifically, the global index of multifractal systems includes the generalized correlation dimension (D_q) and the mass index ($\tau(q)$). The generalized correlation dimension (D_q) can also indirectly reflect the mass index ($\tau(q)$). That is, the global index has the consistency of certain index. Thus, D_q will produce a D_q–q map of the regional global spatial change along with the change in q. That is, when $q \to +\infty$, the details of the regional spatial structure will be clarified. When $q \to -\infty$, some details of the regional spatial structures will be ignored. The former tends to be the analysis of the characteristics of micro-evolution, while the latter tends to be the analysis of the characteristics of macro-evolution.

Local indexes. A multifractal system is a large system. The regional spatial evolution often contains a number of local subsystem changes. Therefore, regional spatial evolution can also be analyzed through the changes in different local indexes. However, due to differences in the "growth" ratio or other variables, the various localities of a fractal system will also change based on a certain singularity index ($a(q)$). Thus, the locality of various fractals will also have a corresponding fractal dimension, forming a multifractal dimension spectrum ($f(a)$) that changes along with ($a(q)$) to describe the corresponding changes in the subsystem. Therefore, a change in system locality can be understood through the interpretation of the fitting curves and parameters of $a(q)$–q, $f(a)$–q, $f(a)$–$a(q)$. More details of the above two index systems can be found in the relevant literature (Benguigui et al., 2000; Chen, 2005, 2008, 2015, 2011, 2010, Chen and Jiang, 2009; Chen and Wang, 2013; Ihlen, 2012; Frankhauser, 2008; Kantelhardt Jan. et al., 2002; Longley and Mesev, 2000; Thomas et al., 2007, 2008).

The above is a brief introduction to the two index systems from the process of mathematical analysis, which is relatively abstract. As the detail of the mathematical background of the two index systems have been descried in many literatures, it is no longer repeated in this paper. In general, speaking, the global index analysis is just as observing the overall shape of a mountain area with a telescope. Local analysis, however, is like using a microscope to observe the shape and structure of green space in a mountain photograph, or to appreciate the close-range view of flowers. The combination of the two index system will form a complete understanding of a complex system from general to detail.

From the above mentioned, therefore, multifractal techniques are effective in analyzing the spatial evolution of a region that seems chaotic but actually has regular features. Hence, the corresponding Dq, $\tau(q)$, $a(q)$, and $f(a)$ obtained using spatial multifractal analysis under specific situations can be used to judge the following features of the multifractal system: ① the main fractal dimension value, which can reflect the space of the non-integral existence of fractal objects from different angles; ② spectrum width. The wider range the multifractal spectrum, the greater complexity the fractal object; ③ spectrum deviation, which reflects fractal dimension spectrum deviation, indicates the change of the principal characteristics of the fractal object; ④ the shape of the fractal dimension spectrum, which reflects the regularity of the change of the system's self-organization; ⑤ spectral bias, which reflects the system's dominant component of change (Chen, 2005; Kantelhardt Jan. et al., 2002; Thomas et al., 2012). Therefore, with the change in the q value, the spatial characteristics of the fractal system will be gradually analyzed.

Case study

Research background. Beijing is an international metropolis. Benefiting from the rapid development of China since 1978 and its unique status as the nation's capital, Beijing has achieved rapid development over the past 30 years and attracted the attention of all disciplines. The social economic foundation development of Beijing can be divided into four phases, specifically, a low-level balanced development phase (1980s), a low-level unbalanced development phase (1990s), a high-level unbalanced development phase (2000s), and a high-level balanced tendency development phase (2010s) (Song and Liu, 2016, Song and Liu, 2011, Song et al., 2015a). The development of the first two phases is characterized by rapid high economic development, during which period the GDP growth rate is basically maintained at ~10%. During this period, the drive of economic interests caused Beijing's urbanization and industrialization levels to achieve rapid improvement, enhancing its spatial agglomeration (see the Spatial Pattern of 1985, 1999 in Fig. 1). By the year 2000, the non-agronomization of the rural areas, which had already occurred in the previous phase, had become the focus of Beijing's development, and its rapid urbanization continued. However, this trend of industrialization was relatively weakened and transferred to the suburbs, reaching Beijing's middle suburbs. In the 2010s, the "big city malaise" brought about by the comprehensive rapid development seriously affected Beijing's sustainable development, and the role of spatial planning and ecological environmental construction formulated in the previous phase began to emerge, and secondary centers in the surrounding areas linked to the main urban area through traffic corridors developed rapidly (see the Spatial Pattern of 2008, and 2015 in Fig. 1). Overall, the spatial development of Beijing has undergone corresponding changes due to non-agriculturalization, urbanization, and industrialization

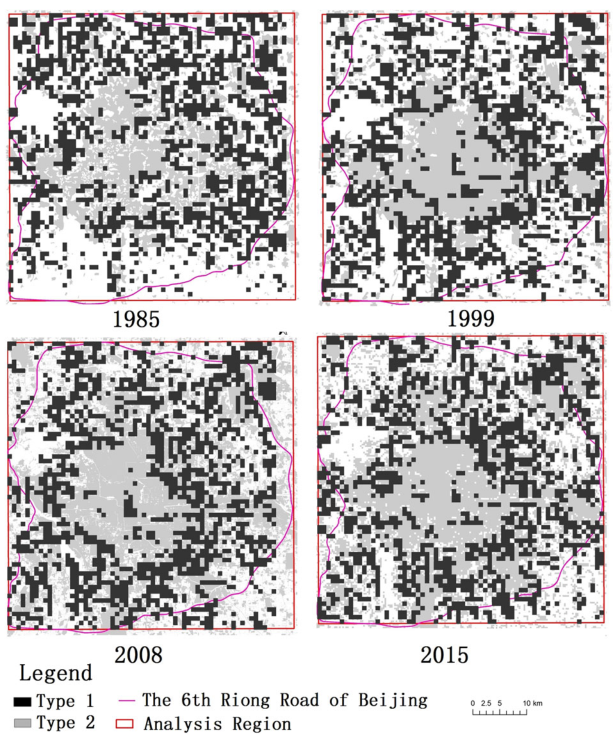

Fig. 1 The spatial patterns of the construction land within the 6th Ring Road of Beijing in four years

("three modernizations" hereinafter), resulting in the alteration of development functions due to various contradictions between urban and rural areas as well the selection of different development paths by related interested parties.

From the perspective of spatial development since 1978, Table 1 summarizes the circumstances of realization of the "three modernizations" and spatial differences of the urban fringes of the various periods and the near and mid-suburban belts during different periods (He et al., 2006; Song and Zhu, 2013a; Song and Liu, 2016). From the point of view of the spatial distribution of the research object of this paper (newly increased construction land within Beijing's 6th ring road), they mostly concentrate on the main urban areas and the near and middle suburbs of Beijing in various periods. That is, the research object of this paper is primarily situated in Beijing's main urban area and the suburban belts characterized by prominent urban and rural contradictions and significant changes in spatial patterns.

Study area. On the basis of Beijing's functional evolution of urban and rural development, the spatial pattern of its main urban areas and suburban areas of have also undergone correspondingly adaptive changes. Since 1978, the process of the three modernizations (non-agriculturalization, urbanization, industrialization) in this region brought about by reform and opening-up have not at all been synchronous. As a result, the extent of the realization of the functions of urban and rural development and the mode of dominance are not the same. However, all these characteristics can leave "imprints" on the whole regional space and its local space (the area of Fig. 1 including the main urban area and the suburban area), providing an objective basis for conducting multifractal analysis. It should be explained that the change in spatial patterns of the urban and rural properties involved the changes in construction land and changes in other land types, especially the agricultural land. However, in reality, the land types of the non-construction land in Beijing mainly include cultivated land, garden land, forest land, traffic land, and water areas, involving relatively complicated internal disputes. Precisely because the changes in the spatial patterns of the urban and rural areas in Beijing (especially in the areas within the 6th ring road, where the changes in land types are significant) are mainly dominated by construction land expansion, this paper focuses on the analysis of construction land type.

Since the founding of the People's Republic of China, Beijing's construction land has basically shown a non-homogeneous development mode around Tiananmen Square. The radius fractal dimension-based single-fractal analysis using data from 2006 showed that, due to the impact of the built-up area development functions (mainly from the commodity service sector, non-agricultural production, and domestic living, i.e., a variety of production functions of the arable land of Beijing's suburban areas outside the concentric circle ~23 km from the center of the city), generated or partially generated single-fractal characteristics (Song and Liu, 2012; Song et al., 2013b; Song and Yu, 2015b). The area and shape of rural settlements in different Beijing suburban areas (areas outside of the concentric circle ~19 km from the center of the city) also changed regularly, and some of them demonstrated single-fractal characteristics. Based on the above analysis, this paper generates a basic definition on the subject range of the urban and rural space of Beijing that causes these changes. Furthermore, to obtain an in-depth understanding of the construction land spatial expansion ability, the expansion process, and the "field density" (by analogy with the density field distribution of divergent physical systems) of Beijing's built-up land within the 6th ring road, this study area was then established the square area of 53.2 km side length within Beijing's 6th ring road according to Beijing's land-use status maps of 1985, 1999, 2008, and 2015 where the construction land is mainly concentrated (Fig. 1).

From introduction section and the above analysis, it can been seen that the real geographic fractal system of the region within Beijing's 6th ring road, which is composed of specific construction land patches, is a complex of land patches with various development attributes. This system fluctuations in space. Therefore, the extended dimension (D_q) in spatial multifractal analysis, the singularity ($a(q)$) in mathematics, and the fitting curves of D_q–q,$f(a)$–$a(q)$ formed by these indexes can be used to analyze the specific characteristics of the overall and local changes of the spatial system composed of construction land patches, which can improve our understanding of Beijing as a multi-scale spatial system.

Data processing. In this paper, LANDSAT 5 images and LANDSAT 8 are used as data sources. With the help of ENVI4.8, and eCognition software, an image of the construction land was extracted by remote sensing using an object-oriented classification method. The results show that, compared with the supervised classification and unsupervised classification methods, the object-oriented classification method has a higher accuracy. In order to ensure that the accuracy met the needs of the analysis, the extracted data were further manually corrected using the ArcGIS 10.2 software.

This study is based on grid-based fractal analysis. The construction land data in 30 m resolution was first extracted based on remote sensing images and then conducted, classified and sorted. Taking 1985 and 2015 as examples, Table 2 shows the multi-scale patch data. According to the data acquisition, the data in this study are actually divided into 10 levels (First Column), corresponding to different regional spatial recognition accuracies (Second and Third Columns), respectively. The 10th level data ($m = 9$, that is, the data scale of the patch side length is 103.91 m, which is observed to be an ~100 m level of data accuracy) are greater than the spatial scale threshold that is generally considered significant in studies (patch scale of ~30 m in side-length) (Thomas et al., 2012). Therefore, the above data at various

Table 1 Characteristics of the "three modernizations" during different periods in the past 40 years in Beijing

	Industrialization		Urbanization		Non-agriculturalization
	Region	Intensity	Region	Intensity	Intensity
1978–1980s	Near-suburb	Ordinary	Inner edge of the outer suburbs	Ordinary	Weak
1990s	Mid-suburb	Ordinary	Mid-suburbs	Relatively Strong	Ordinary
2000s	Enclave adjacent to the outer suburbs	Relatively strong	Outer edge of mid-suburbs	Relatively Strong	Strong
After 2010	Block mass + sphere tendency	Weak	Global multi-level, multi-center pattern	Strong	Strong

Table 2 Level and proportion of construction land within the 6th ring road of Beijing (1985, 2015)

Data level (*m*)	Regional spatial level	Grid length (*m*)	Grid count	1985		2015	
				Number of non-empty grid	Proportion (%)	Number of non-empty grid	Proportion (%)
0	Regional level grading	53200	1	1	100	1	100
1	↓	26600	4	4	100	4	100
2	District /county level	13300	16	16	100	16	100
3	↓	6650	64	64	100	64	100
4	Sub-district level	3325	256	250	97.66	256	100
5	Block level	1662.5	1024	904	88.28	1023	99.9
6	Road level	831.25	4096	2974	72.61	4066	99.27
7	Residential compound level	415.63	16384	9040	55.18	15847	96.72
8	↓	207.81	65536	27764	42.36	57249	87.36
9	Community level	103.91	262144	90485	34.52	193199	73.7

levels meet data accuracy requirements and can be used in multifractal analysis. The data show that in the block level ($m=5$), the space of $1.6625 \times 1.6625\,\text{km}^2$ without construction land has already appeared in this study area. The higher the precision (i.e., the greater the m), the greater the number and the higher the proportion of the spaces without construction land. Comparing the proportion of non-spaces over the total number of spaces at 5–9 levels, between 1985 and 2015, it can be found that the growth of the construction land within Beijing's 6th ring road is relatively obvious during the 30-year period. The higher the accuracy, the greater the change in the non-space ratio tends to be (Tanner et al., 2016). The number and proportion accuracy of the patches at the neighborhood level ($m=9$) are the highest, and the spatial layout and evolution of the patches are closest to the common sense of the residents.

It is therefore observed that the construction land has been expanded from the state of a small proportion in 1985 to the state of a large proportion in 2015, and the change in construction land within Beijing's 6th ring road has varied dramatically. Thus, the above data preparation provides a specific data basis for the subsequent disclosure of multifractal characteristics of the study area, especially for the disclosure of the spatial singularity evolution displayed by a variety of spectra.

Allometric growth analysis and shannon entropy analysis. The allometric growth of socioeconomic activities is often the source of spatial fractal features (Chen, 2008). Based on statistically consistent data from 1979 to 2009, the allometric analysis of major socioeconomic indicators in Beijing shows that in the process of industrialization, urbanization, and non-agriculturalization in the past 30 years, Beijing's various development functions have experienced relatively complex allometric growth and have produced spatial fractal features in many fields (Song and Liu, 2016). This development trend continues to this day.

Shannon entropy analysis was carried out on all regional spatial scales ($m=5$–7) in 1985, 1999, 2008, and 2015. The results (Table 3) show that the Shannon entropy of the space system within Beijing's 6th ring increased along a time series and with spatial accuracy, except for the year 2008 in which the Shannon entropy decreased. The former suggests that with the development of time, the spatial system within the 6th ring of Beijing tended to be increasingly chaotic, which may have produced the multifractal characteristics of spatial patch combinations. The latter shows that with the fragmentation of construction land patches, the stability of the spatial pattern within the 6th ring road of Beijing weakened enhancing the chaos

Table 3 Four-year shannon entropy analysis of the spatial pattern of construction land in the 6th ring road of Beijing

	1985	1999	2008	2015
$m=5$	6.461126	6.798970	6.741504	6.802389
$m=6$	7.669188	8.126874	8.075565	8.134571
$m=7$	8.843320	9.442068	9.392525	9.447101

of the main urban fringe and suburban space. Both conclusions support the subsequent spatial multifractal analysis. The temporary reduction in Shannon's entropy in 2008 could be due to the large-scale spatial remediation carried out at the time of the Beijing Olympic Games. During this period, the government carried out a large-scale cleanup of various illegal buildings and planned to build a number of Olympic venues and supporting facilities, thus reducing the "chaos" of the spatial pattern. However, this phenomenon has not hindered the overall spatial evolution of the region. Therefore, the basic analysis based on Shannon entropy is acceptable.

Basic mathematical analysis. Further, all parameter values to the global indicators and local indicators in three spatial scales ($m=5$ (block level), $m=6$ (road level), $m=7$ (life cell level)) in the four years are obtained using a standardized mathematical analysis program (Huang and Chen, 2018). In addition, the corresponding mathematical features were interpreted according to factors such as the size of each parameter value, the shape of the fitting curve, and the size of the goodness of fit (R^2) and were used as the basis for the mathematical analysis of global and local indicators. For example, Table 4 shows the main parameters Dq, $\tau(q)$, R^2, $a(q)$ for the block level ($m=5$) in 1985 taking the typical values such as the upper/lower limit value of q value, D_0, D_1, D_2 as the main reference. Due to space limitations, the subsequent related analysis will not be repeated.

It should be noted that in the process of program analysis, the range of q values for each spatial scale is limited to the range of [−2.0–13.0]. The lower limit of q is set at −2.0 to avoid the cross between the analysis of the next spatial scale (e.g., $M=7$) and that of the above level (e.g., $M=6$). A small q value (or a high density area of construction land) in a certain level of spatial scale may become a medium q value (medium density area) or even a large q value (small density area) in the upper level of the spatial scale. Thus, the mathematical characteristics of the spatial pattern of construction land within the 6th ring road of Beijing at different spatial scales can be distinguished and compared at the spatial scale of the block level ($m = 5$), the road level ($m = 6$), and the community level ($m = 7$).

In fact, there is also a specific analysis process for the selection of the range of m and the range of specific q values in a certain spatial scale, which requires the corresponding log-log numerical analysis before the above analysis. In general, complex space systems with self-organizing characteristics satisfy the bilogarithm phenomenon which is usually called the power law. A space system that simultaneously satisfies fractal law in horizontal space and Zipf law in vertical level is often a necessary condition for its existence of self-organizing characteristics. Therefore, this paper takes the $m = 5$ grid data of 1985 as an example to demonstrate the correlation analysis.

As shown in Fig. 2a, the log–log fitting of Shannon entropy ($I_1(r)$) and grid scale (r) formed an inclined curve with an acceptable goodness of fitting ($R^2 = 0.9996$), indicating that $m =$ 5–7 is in the scale-free zone of the spatial fractal system within Beijing's 6th ring. Therefore, the selected spatial systems $m = 5$ (block level), $m = 6$ (road level), and $m = 7$ (living community level) satisfy to multifractal analysis which corresponding to the fractal phenomenon in the horizontal space.

As the space resolution growths (the m value growths bigger), grids that contains the same patches would be more in the space system within the Beijing six ring, resulting a higher hierarchy. Therefore, taking the1985 data at $m = 5$ for example, the double logarithm curve between the number plaque and its serial number is shown in Fig. 2b from which the corresponding grid range to the multifractal feature and its q value can be roughly obtained. According to the analysis results, this region is roughly located in the sorted 355–750 grid (at the level of $R^2 = 0.9566$), which is basically in line with the status quo. This grid rank analysis based on the number of patches also supports the existence of multifractal phenomenon.

Table 4 Main quantitative indicators for a multifractal analysis of the space within the 6th ring of Beijing (taking the 1985 data at $m = 5$)

q	D_q	$\tau(q)$	R^2	$a(q)$	$f(a)$
−2	2.599704	−7.79911	0.99726	3.472876	0.833635
−1.7	2.505062	−6.76367	0.997932	3.405556	0.997343
−1.6	2.470945	−6.42446	0.998177	3.374123	1.009159
−1	2.248383	−4.49677	0.999495	3.004687	1.470982
0	1.971379	−1.97138	0.99983	2.119588	1.964036
1	1.859975	0	0.999788	1.863023	1.863219
2	1.802394	1.802394	0.999314	1.776693	1.739926
13	1.680295	20.16354	0.995953	1.658639	1.167326
13.1	1.679941	20.32729	0.995945	1.658396	1.164154

In fact, the above basic analysis of Beijing's space in four years is necessary. From the analysis results of the three spatial scales ($m = 5$–7) in the four years involved in this paper, the spatial data at all levels in the study area met the corresponding mathematical tests and supported the subsequent multifractal analysis. Therefore, in order to save space, similar analysis will not be described later.

Global index analysis

Global analysis mainly reflects the breadth (q value range), evolution intensity (magnitude of the generalized fractal dimension D_q) and other characteristics of the multifractal of regional space. The mass index of another indicator ($\tau(q)$), which reflects the overall non-homogeneity of the said space system, also has a great correlation with D_q and is consistent with it. In this paper, the three values were calculated based on the data of the construction land in Beijing within the 6th ring road in 1985, 1999, 2008, and 2015. The D_q–q fitting curves (Fig. 3a) were then obtained. Because $\tau(q)$ is helpful in determining the statistical properties of the spatial distribution of construction land within Beijing's 6th ring road (such as white noise, single fractal and multifractal system characteristics), Fig. 3b also shows the $\tau(q)$–q fitting curves. As observed from Fig. 3, the spatial systems constructed by the construction land within Beijing's 6th ring road all meet the multifractal characteristics in the global index system, which also verifies Chen and Wang (2013) judgment on the overall spatial evolution of Beijing's built-up area based on multifractal analysis (Chen and Wang, 2013).

The global analysis of the various years is intended to analyze the complex pattern of the spatial density field of non-spaces that carry with them the multiple development processes/multiple development functions (each space contains unequal number of patches) to reflect the overall situation of spatial development and the spatial characteristics of the main urban area and suburban area of Beijing in different periods. The detailed analysis is as follows:

The evolution breadth of construction land (q value range). The range of q values of the spatial system of the construction land within Beijing's 6th ring road in 1985, 1999, 2008 and 2015 is shown in Fig. 2a. Among them, the q value range of 1985 was the largest (−1.7–13.0), followed by 1999 (−0.5–13.0) and 2008 (−0.6–12.5), while the q value range of 2015 was the smallest (1.0–13.0). This result shows that, with the rapid development of Beijing since the reform and opening-up, the hierarchical structure of the construction land space system displayed a trend of shrinking, and it is mainly manifested as the disappearance of the agglomeration phenomenon of the construction land that is not entirely empty (mainly the marginal area of the main urban area).

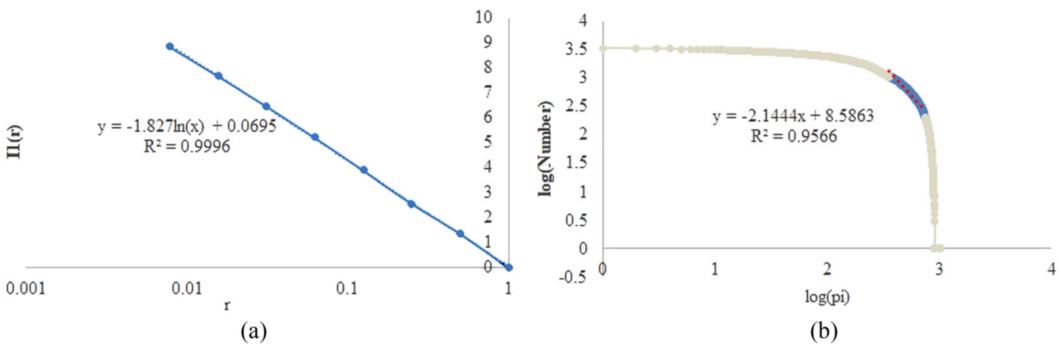

Fig. 2 Bilogarithmic fitting and testing of the data with multifractal characteristics (taking 1985 for example) (**a**) $I_1(r)$-r *Bilogarithmic fit curve*; (**b**) Bilogarithmic fit curve between the number of patches in a grid and its serial number

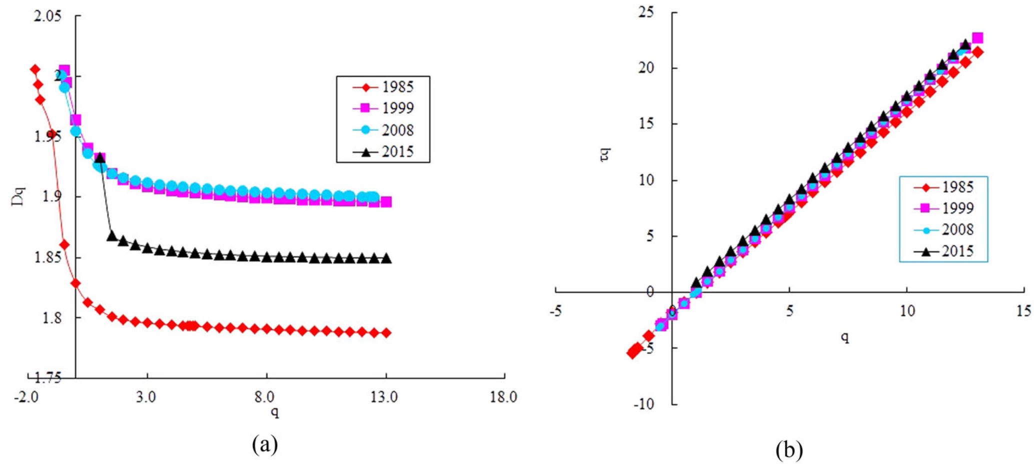

Fig. 3 Global index diagram of Beijing in 1985, 1999, 2008, and 2015. (**a**) D_q-q fitting curves; (**b**) $\tau(q)$ fitting curves

If it is reflected in the mathematical analysis, it is the degrading of the area where the q value is negative (mostly corresponding to high level, large area construction land distribution area, such as the distribution area of commercial service centers at all levels) (Chen and Wang, 2013). However, the q value of the area with diffuse patch growth (a type of diffusion) remains the same. Through the change of the q value range (reduced from 14.7 to 12.0), it is observed that, after 30 years of development, the hierarchy of high-level construction land within Beijing's 6th ring road has been weakened.

The reason should be that the comprehensive development of Beijing in many fields has already reduced the hierarchy of the various social and economic activities and is reflected by the construction land it carries. The commercial service centers, which are still clearly discernible in pictures from 1985, had, in the pictures of 2015, already been buried in the non-agricultural construction land that carries various development functions. The scientific explanation is that at a certain spatial scale when all regional spaces are of spaces/non-spaces, there are no multifractal characteristics because spatial differentiation phenomenon no longer exists, which means that most of the main urban areas are no longer included in the scope of multifractal analysis. Moreover, the existing mosaic distribution area may also be completely solidified into construction land in the next phase, thus losing its multifractal characteristics, which is also easy to understand because there is no longer any room for further growth in an area that is all construction land. Therefore, further reflected in this study are the complex spatial evolution rules of the suburban and urban areas of Beijing.

The geographical space of this study mainly refers to the suburban area or the marginal area of the main urban area. However, the actual situation is that the main urban areas of Beijing have shown increases or decreases in patches each year in different proportions and have become a spatial hierarchy with certain q values. Therefore, the entire study area within the box of Fig. 1 cannot simply be represented as a suburban area or the marginal area of the main urban area. Hence, to some extent, this study reflects the overall complex evolutionary characteristics of the construction land within Beijing's 6th ring road, which is also the most dramatic area of urban and rural property changes in Beijing.

Comparison of the evolution intensity of construction land (size of the D_q value). The growth of construction land in Beijing manifests as the spatial filling and expansion process in the main urban area and suburban area. Viewed from the evolution intensity of the four-year space system of Beijing as reflected by the D_q value (Fig. 2a), in 1985, the D_q values corresponding to the various q values are all smaller than that of the other three years. The D_q values in 1999 and 2008 are both in the high value zone and have no obvious difference, indicating that the evolution intensity of these two years is relatively strong. The D_q value in 2015 is in the intermediate state as a whole. The change of spatial evolution intensity of the construction land corresponds, respectively, to the conventional beginning phase of rapid development of Beijing in the 1980s, the relatively disorderly overly rapid development phase in the 1990s and 2000s, and the rapid development phase with regulation since the 2010s (Song et al., 2013b). Therefore, the change in the above D_q value in different periods actually corresponds to a multi-q level spatial growth process of filling of the construction land in the main urban area and outward expansion of the construction land in the suburban area. Identification of the size of the above specific D_q value can help enhance the quantitative understanding of the growth capacity of the construction land in the four years (Fig. 1).

Spatial system features analysis ($\tau(q)$). A single fractal is a special case of multifractal system. The shape of the $\tau(q)$-q fitting curve is the basis from which to distinguish whether a system is a single fractal system or a multiple fractal system. The single fractal $\tau(q)$-q fitting curve in Fig. 3 presents a straight line, while the multiple fractal presents a convex curve. Therefore, the possibility that the space system belongs to a white noise system is excluded based on Fig. 2b. It is necessary to use local indicators to determine whether it belongs to the multifractal system. It is observed that the D_q–q fitting curve in the global index analysis and the fitting curves of $a(q)$–q $f(a)$–q *and* $f(a)$–$a(q)$ in the local index analysis all exhibit a non-horizontal curve shape (see Figs. 3–8 in Section "Local index analysis" for details) rather than a single fractal horizontal line shape. Therefore, this paper further believes that the construction land system within Beijing's 6th ring road is a multifractal system. Under the conditions where other indicators (D_q, a_q, $f(a)$) of the four years in Beijing have already been determined to have met the fractal characteristics (see the section below), it also rules out the possibility that this space belongs to a white noise system.

In short, with the development of areas within Beijing's 6th ring road over the past thirty years, the cluster center of growth area of the construction land has gradually been weakened. The diffuse growth of the most grass-roots patches in the urban fringe area has basically remained unchanged and become the main part of the growth of construction land in Beijing. The changes in the global indexes such as the q value range, the D_q value size and the shape of the $\tau(q)$–q fitting curve can all reflect the overall era characteristics of the spatial evolution of Beijing in different periods (1980, 1990, 2000, 2010s), respectively.

Local index analysis

Analogous to the physical field density, the density field of the local system composed of different q-level fractal units will also be different. Therefore, *a(q)* is used to describe the local density distribution of the geographical space field. Moreover, as the local state of the fractal unit of space multifractal system usually changes along with *a(q)*, a singular spectrum *f(a)* is used to describe the fractal dimension value of the local system. One important basis for the multifractal analysis is the interpretation of the *f(a)*–*a(q)* fitting diagram.

In this paper, the multilayered *a(q)*–*q*, *f(a)*–*q, and f(a)*–*a(q)* fitting curves of the four years of the construction land within Beijing's 6th ring road were obtained based on the extracted spatial data of four years and mathematical analysis of the local index system. Correspondingly, to highlight the spatial changes closely related to the people's daily lives, in this paper, the multifractal characteristics of the construction land within Beijing's 6th ring road were analyzed on the macro scale (block level, $m = 5$), medium scale (road level, $m = 6$) and micro scale (community level, $m = 7$), respectively.

Local index analysis in 1985

Block level ($m = 5$). The numerical fitting diagram of various indexes of this scale is shown in Fig. 4. This scale, the largest scale spatial level in the diagram of this paper, generally refers to the spatial scope of the sub-district (block length $r = 1662.5$ m). Therefore, the analysis of the evolution of the spatial pattern of this scale is actually intended to reflect the growth conditions of the block (sub-district) level space (the lowest-level administrative division in China). In addition, the goodness of fit (R^2) of the *a (q)*–*q*, *f(a)*–*q*, and *f(a)*–*a(q)* fitting curves at the spatial scale is greater than 0.95, which is in line with the mathematical analysis requirements. The three fitting curves in the local index analysis further in this paper also satisfy this requirement. Therefore, the corresponding descriptions in the following will not be repeated.

The practical implication of the range (i.e., breadth) of the q value of multifractals is generally determined by the object of study. Specific to the changes of Beijing in the past 30 years, in this paper, the definition or determination of the range of the q value of the construction land is the depiction of its form of spatial growth. In other words, it is considered that the growth of high-density and large-scale construction land in the main urban area is described using the small q value-based parameter values, for example, the encroachment of urban green space by major venues. Conversely, it is described using the large q value-based parameter values, for example, the expansion of relatively small-scaled but large numbered rural households homestead to farmland. After this connotation is clarified, it is convenient to compare the range of the q value of the construction land for each year.

The global analysis shows that in the block level accuracy ($m = 5$) of Beijing in 1985, the q value range of spatial multifractal of the construction land within Beijing's 6th ring road was −1.7–13.0. Usually, in different spatial scales (m), the specific q value range may be different, and thus re-verification and analysis on each scale is necessary. Specific to this paper, the local q value range and the global q value range in the same spatial scale are exactly the same. Therefore, they are not further analyzed in our later analysis.

The *f(a)*-*a(q)* schematic diagram (Fig. 4c) shows a left-slanted bell shape, indicating that the high-density growth of the construction land in Beijing at the block level ($m = 5$) is stronger than the low-density growth in 1985. Specifically, the spatial overall development of Beijing at that time was that the agglomerative expansion (e.g., expansion of construction land in towns) of construction land of that period is stronger than the decentralized development (e.g., expansion of homestead to farmland at marginal countryside). This characteristic indicates that at the beginning of the reform and opening up, great changes occurred in the social economic foundation of the main urban area of Beijing (because it is the principal body of the construction land within the 6th ring road). The expansion of the urban construction land expanse to the agricultural and rural residential areas is obvious. If combined with the global indicator analysis, the overall grading is obvious in 1985 (Section "The evolution breadth of construction land"), and it can be considered that the social and economic development of Beijing at that time realized the expansion of the construction land at the block level in the form of radial growth in the suburbs and aggregated growth in the main urban area.

The multifractal spectral width of the construction land in 1985 was 1.747, which was the largest in the analysis in this paper, indicating that, during this period, the forms of agglomerative and radiative space growth of Beijing's construction land are diverse and in a more complex mode of evolution at the block level.

In combination with the actual situation, the above spatial evolution of construction land genuinely reflects the rapid and complex agglomeration of various developmental elements towards the built-up area and the spatial expansion of the construction land during the early stage of reform and opening-up caused by the rapid urbanization and industrialization development.

Road level ($m = 6$). The numerical fitting diagram of various indexes on the spatial scale at the road level (grid length $r = 831.25$ m) is shown in Fig. 5. In this spatial scale, the most representative geographic object is a certain road for residents to travel daily. Therefore, in this paper, this scale is referred to as the road level. At this level, social economic activities generally focus on a certain road because of planning and market guide. The growth of construction land will also radiate a certain distance to the surroundings along this road, thus forming construction land patches of a certain scale and shape. Therefore, the analysis of the growth of construction land at $m = 6$ actually corresponds to the road level (for example, a large-scale commercial street) spatial analysis.

At the road-level spatial scale, the q value of Beijing's construction land with multifractal characteristics is also between −1.7–13.0. Therefore, the regular growth of Beijing's construction land in 1985 is with multifractal characteristics at the road level and with a relatively larger range.

The *f(a)*–*a(q)* schematic diagram (Fig. 5c) shows that the multifractal spectrum is medium left-deviated, indicating that, on the spatial pattern at the road level, the construction land patches expand outwards in a radiative manner at a moderate intensity, which is similar to the spatial expansion at the block level ($m = 5$). In addition, the spectral width of the road level spatial multifractal in 1985 was 1.417, slightly less than the spatial complexity at the block level, which suggests that the spatial change within Beijing's 6th ring road at the road level is relatively simpler than that at the block level.

Community level ($m = 7$). This spatial scale (grid length $r = 415.63$ m) corresponds to the range of the main activities in the daily life of most residents. Figure 6 also reflects the growth

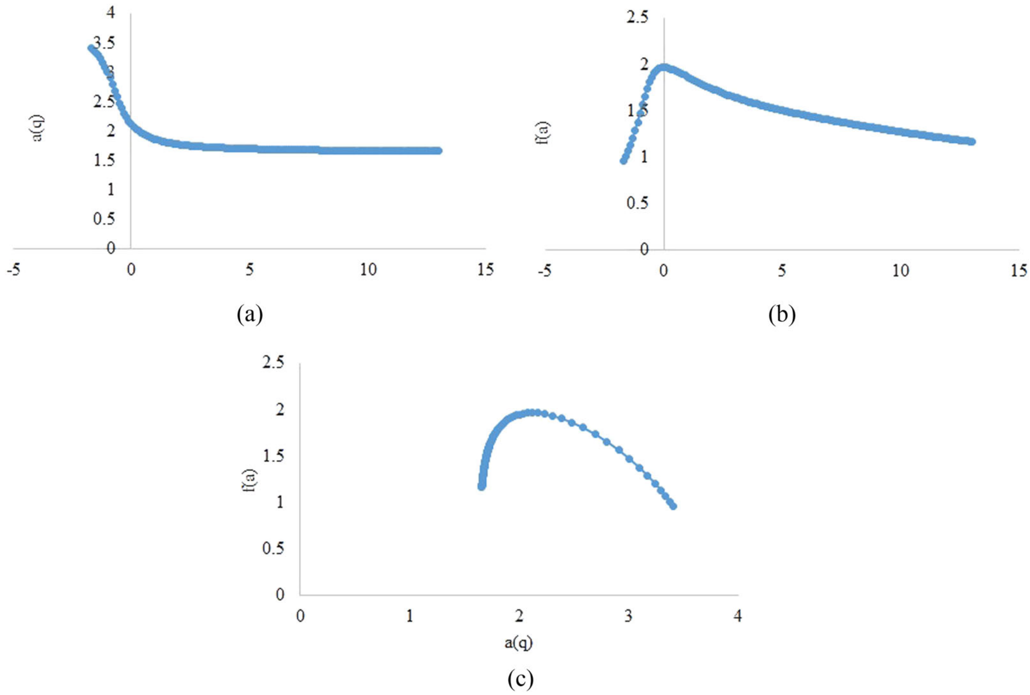

Fig. 4 Local parameter analysis of construction land within Beijing's 6th ring road (1985) at $m = 5$ Level. (**a**) *a*(*q*)–*q*; (**b**) *f*(*a*)–*q*; (**c**) *f*(*a*)–*a*(*q*)

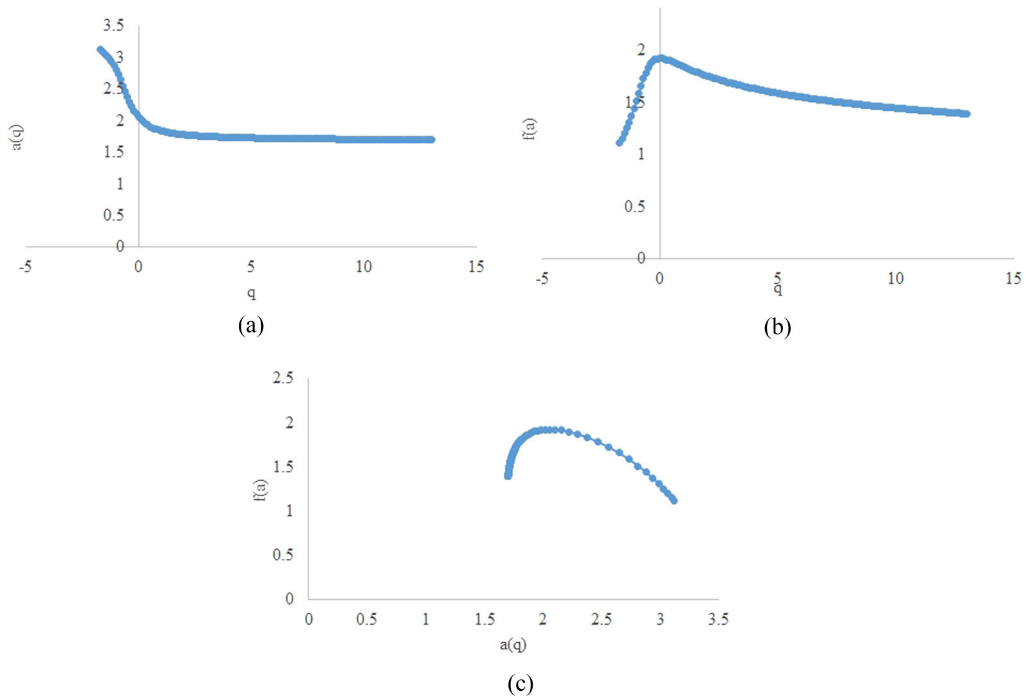

Fig. 5 Local parameter analysis of construction land with Beijing's 6th ring road (1985) at $m = 6$ level. (**a**) *a*(*q*)–*q*; (**b**) *f*(*a*)–*q*; (**c**) *f*(*a*)–*a*(*q*)

conditions of Beijing's construction land on this scale through mathematical analysis. The numerical fitting of various indexes is shown in Fig. 5.

As shown in Fig. 7, comparing the multifractal dimension spectrum at the block level, the road level and the community level, the q values are all between −1.7–13.0, indicating that the form of growth of Beijing's construction land in 1985 is diversified in the micro scale, medium scale, and macro scale. However, from the macroscopic and microscopic perspectives, the left deviation of the multifractal dimension spectrum tends to be more obvious, indicating that the closer it approaches to the micro scale, the stronger the agglomeration of patches of the

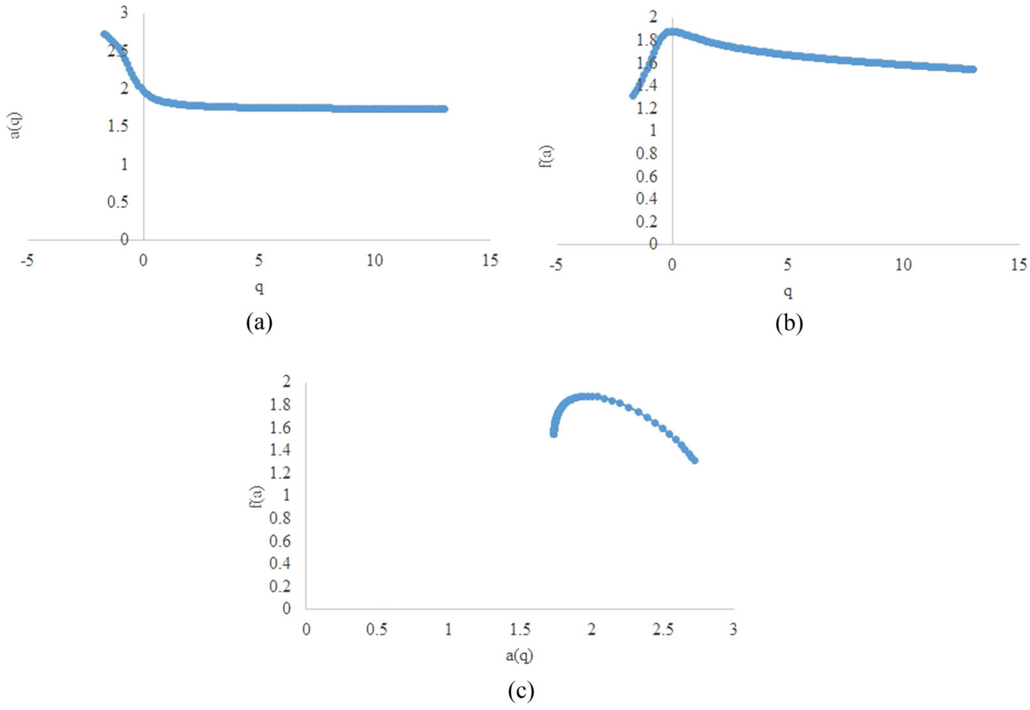

Fig. 6 $m = 7$ Level local parameter analysis of construction land with Beijing's 6th ring road (1985). (**a**) a(q)–q; (**b**) f(a)–q; (**c**) $f(a)$–$a(q)$

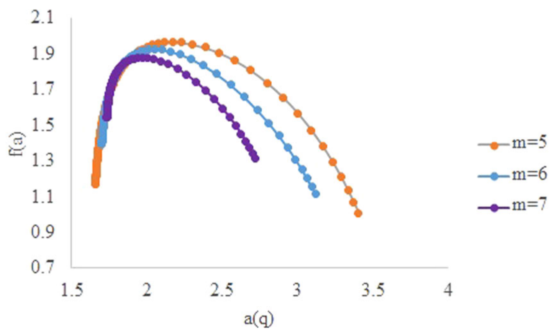

Fig. 7 Comparison of multi-scale multifractal spectrum of construction land within Beijing's 6th ring road (1985)

construction land will become. The spatial complexity tends to be simpler. When it reaches the community level, its spectrum width has been reduced to 0.985. However, from the overall point of view, the basic characteristics of patch growth at the community level basically converges with that at the block level and road level, which jointly constitute a unified integration at the micro-scales, medium-scales, and macro scales.

Brief summary. At the beginning of the reform and opening up, the development of Beijing is wide-ranging (compared with the subsequent three years, the spectrum width of the multifractals of the various spatial scales in 1985 was obviously large) and hierarchically significant (the q value range is large). Thus, it can be considered that the development of the whole area with the 6th ring road at that time in a typical urban spatial expansion and internal filling mode. The macro-spatial, medium-spatial, and micro-spatial patterns of its construction land are all in accordance with the multifractal characteristics. Moreover, from the macro scale to the micro scale, the greater the agglomeration of the construction land, the simpler the spatial complexity. In general, an agglomeration growth of the construction land dominated by filling of the main urban area and radiation on the outskirts has been formed within the 6th ring road. Although the changes at the macro-spatial, medium-spatial, and micro-spatial patterns are different, their basic features are nevertheless convergent.

Local index analysis of 1999. The $f(a)$–$a(q)$ fitting curves of the multifractal characteristics of the construction land within Beijing's 6th ring road of at the block level ($m = 5$), road level ($m = 6$) and community level ($m = 7$) in 1999 are shown in Fig. 8.

Block level ($m = 5$). The q value range of the construction land within Beijing's 6th ring road in 1999 was −0.5–13.0, increased by 1.2 from the lower limit of the q value of the same scale in 1985, indicating that after 14 years of development, the hierarchy of the multifractals of Beijing's construction land at the block level has become somewhat convergent. Moreover, the range of this convergence appears in the negative zone, indicating that the agglomeration phenomenon of newly increased construction land within Beijing's 6th ring road has weakened due to the weakened reliance of the newly increased construction land on the original townships and blocks in the suburbs.

Figure 8 shows that, compared with the convergent growth of the newly increased patches of the construction land in 1985 (Fig. 4), the newly increased construction land in 1999 had a discretization tendency on each scale, and both belonged to different types of multifractal dimension spectrum. As shown in Fig. 1, the spatial pattern of the block level in 1999 had appeared as an enclave and radiant type of compound growth. In addition, the spectral width in 1999 was 0.339, which is much smaller than that of 1985 (1.747), suggesting that the spatial structure of 1999 was much simpler than that of 1985 at the same level. In addition to the characteristic of the very strong space-filling capacity indexed by D_q in the global analysis (see Fig. 3a), it can be considered that the block-level spatial growth, which depended on deviated and

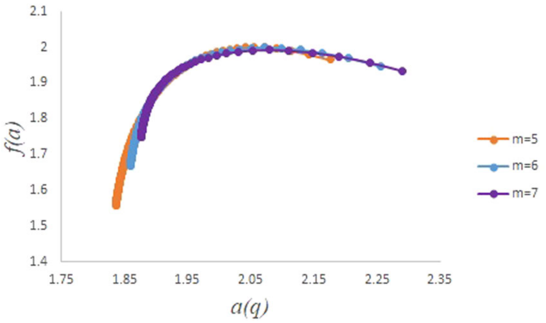

Fig. 8 Comparison of multi-scale multifractal spectrum of construction land within Beijing's 6th ring road

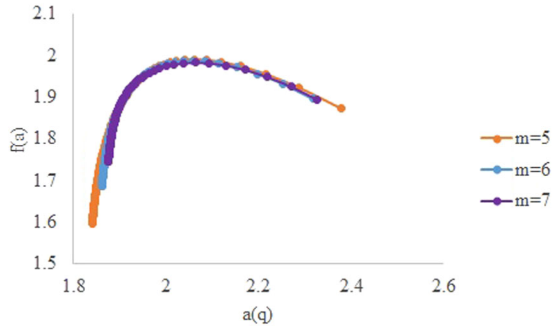

Fig. 9 Comparison of multi-scale multifractal spectrum of construction land within Beijing's 6th ring road

discretized spatial growth points at that time, was not only strong but had a relatively single trend as it was not obviously interfered with by spatial agglomeration. The relatively large difference between the maximum and minimum values of *f(a)* suggests that the overall effect on the whole space by the high-density area (for example, the edge of the main urban area) for outward expansion of the construction land within Beijing's 6th ring road is greater than that of the low-density area, indicating that the enclaves and new development centers play a very significant role in the spatial pattern. Moreover, as the expansion of Beijing's construction land in 1999 happened right in the period of ultra-high-speed development in the 1990s–2000s, the above characteristics also reflect the time characteristics of that time.

Road level ($m = 6$) and community level ($m = 7$). The growth of the construction land within Beijing's 6th ring road at the road level and community level in 1999 also meets the multifractal characteristics that are convergent with that of the block level (Fig. 8). However, the spectrum widths of both were 0.396 and 0.415, respectively. A phenomenon had occurred whereby the spectral width gradually increased from the macro scale to the micro scale and the spatial complexity was greatly weaker than that of 1985, which is an obvious reverse change to that in 1985. This phenomenon was likely caused by the mass construction of illegal buildings along the roads and residential communities by various interested subjects as affected by non-agriculturalization and urbanization at that time. As a whole, driven by the market, many micro scale and medium scale social and economic activities were generally in a state of over-speeding. The trend of "transition urbanization" had already begun.

Brief summary. In 1999, the rapid urbanization and industrialization affected the regional macro scale and medium scale development basis. A large number of rural migrant workers swarmed into the city for work and housing purchase because the non-agriculturalization of the agricultural area had already begun, which greatly changed the development basis within Beijing's 6th ring road.

The comprehensive view shows that the existing state of construction land in 1999 still conforms to the classic mode of urban spatial growth. The scales at which construction land has multifractal features covers the block level (macro) levels, road level (medium), and community level (micro), which is very much in accord with the dynamic evolution system of self-organized criticality (SOC) in a sensitive reaction period. On the whole, as the multiple field evolutions were in the process of coupling, the rapid, deviated, and discretized growth of the space system became the most significant characteristics of that era.

Local index analysis of 2008. Figure 9 shows the *a(q)*-*q* fitting curve of the region in 2008 at the block level ($m = 5$), road level ($m = 6$), and residential district level ($m = 7$), respectively. The analysis of the patches data of Beijing's construction land in 2008 at various space scales shows that the range of q value is between -0.6 and 12.5. From an overall point of view, the outward expansion of construction land brought about by the super-rapid development in 1999 is still continuing. However, the hierarchy of the most grass-roots level of distributed growth has somewhat decreased ($13.0 - 12.5 = 0.5$), and its agglomeration basically remains unchanged $(0.6) - (0.5) = 0.1$).

Block level ($m = 5$). The multifractal dimension spectrum of the regional construction land at the block level shows a moderately left slant, which means that, in 2008, the holistic development of the outskirts and the fringe area of Beijing's main city existed at the same time. Compared with Figs. 7 and 8, the agglomerative expansion in 2008 was weaker than that in 1985, while the distributed expansion was weaker than that in 1999. However, as a whole, the filling of regions with the distributed growth of patches of construction land and the new outward expansion in 1999 still continued in 2008. Nine years later, the trend of agglomeration dominance with a moderate intensity is still maintained within Beijing's 6th ring road. On the whole, the "urban sprawl" trend of spatial spreading in Beijing continues.

According to the shape of the downward opening of the multifractal dimension spectrum, the spectral width is relatively large (0.539), indicating that, as a whole, the spatial growth of construction land within Beijing's 6th ring road demonstrated a relatively more complicated evolution. At the same time, the difference between the maximum and minimum values of *f(a)* is also relatively large, showing that the role played by the high-density area (for instance, fringe of the main urban area) of outward expansion of construction land is greater than that of the low-density area (for instance, the distribution area of rural community centers in suburban areas of Beijing) in the whole spatial expansion. The max q value (12.5) of the spatial system of Beijing's construction land is relatively larger, and the D_q value is also at the high value zone. From the experience point of view, in 2008, the spatial expansion intensity in the suburban and main urban areas caused by social and economic development was very large and had a diverse form. In fact, during this period, because of the Beijing Olympic Games, a comprehensive construction climax at the macro ($m = 5$) level had been achieved in Beijing. Among them, the clearance of illegal buildings may have caused the decrease of the max q value. The construction of related infrastructure may have been the main reason for its relatively larger spectrum width.

Road level ($m=6$) and community level ($m=7$). The multifractal characteristics at the road level and the community level are basically the same as at the block level. After 9 years of changes in rapid development, the regular growth in construction land within Beijing's 6th ring road with multifractal characteristics tends to be consistent at the block level, road level, and community level. Construction land—a carrier of social economic activities conducted along the roads and in community centers—is undergoing an outward expansion with a variety of forms, and in a space-filling process with general complexity (spectrum widths are 0.456, 0.452, respectively).

Brief summary. As shown in Fig. 1, the spatial scale of the construction land of 2008 is much larger than that of 1999. Moreover, this period was also the stage at which the construction land within Beijing's 6th ring road had experienced substantial changes (evolution intensity as represented by the D_q value), and the society and economy had undergone profound transformation. During this period, non-agriculturalization has already become an urgent problem to be solved by the government. "Big city disease" continues. Some production enterprises are moving out. On the whole, the multi-domain coupling in the process of "hyper-speed urbanization" is still not yet satisfactory, and the contradiction between city-rural spatial development is intensifying (He et al., 2006). While the construction land is expanding outward, Beijing is experiencing allometric growth in many fields, while diverse development in a variety of developmental functions, the transfer of urban and rural integration scenarios and spatial structural adjustment have all made corresponding changes. Although this analysis still cannot reflect these connotative changes of the construction land, it has nevertheless found consistency in its macro-spatial, medium-spatial and micro-spatial evolution in this period.

Figures 8 and 9 have morphological similarities. The reason is that the multi-domain hyper-speed development and multi-process complex coupling of Beijing in the 1990s and 2000s were consistent in terms of development characteristics.

Local index analysis in 2015. Figure 10 illustrates the fitting diagram of the macro-local, medium-local, and micro-local indexes of the study area in 2015. In the year 2015, the multifractal characteristics of the construction land within Beijing's 6th ring road are completely different from that of the first three years. First, the q value range was between 1.0 and 13.0. The high density center of the construction land has already disappeared (the range where $q<1$ has already disappeared). Second, the peak value of the multifractal dimension spectrum has completely disappeared at the spatial scale of the block level ($m=5$), the road level ($m=6$), and the community level ($m=7$), presenting a development trend with no dominant q value. This disappearance should be related to Beijing's all-around and "crazy" real estate development in the past ten years. Third, the growth of Beijing's construction land in terms of spatial scales at the district level, road level, and community level, with spectrum widths being 0.115, 0.087, and 0.063, respectively, suggests that Beijing's real estate is not only "crazy" but has a single development trend, which once again proves that "transitional urbanization" is not the subjective assumption of some scholars.

Simply put, the high-density center of new construction land of Beijing's 6th ring road has disappeared, and regional the construction land tends to be homogenized. The shape of a slightly left-deviated multifractal dimension spectrum also shows that the agglomeration of the blocks is no longer strong. All the newly increased patches have a tendency toward spatial discretization, which seems to show that the newly increased patches in 2015 have become a "blossoming" type of spatial existence. The existing studies have indicated that, at present, the multifunction coupling in Beijing did not reach its ideal state. The contradiction of land use in the process of urban and rural development is still in the process of strengthening. Real estate development has already "outshone" others in the process of development of Beijing. However, what is undeniable is that, in this type of spatial pattern of discretization, there is still a certain trend that, as also indicated by the spatial pattern in Fig. 1, the basic pattern within the main urban area and satellite towns has already formed.

Conclusion

From 1985 to 2015, construction land within Beijing's 6th ring road increased year by year. The global and local multifractal indexes suggested that Beijing is a relatively typical urban space system demonstrated by multifractal characteristics in spatial scales of $m=5$ (block level), $m=6$ (road level) and $m=7$ (community level), which generally corresponds to researchers' macro-spatial, medium-spatial, and micro-spatial understanding. That is, under the influence of "three modernizations", the construction land in Beijing has demonstrated a growth pattern of various forms in the last 30 years and presented the following global and local development characteristics.

The overall characteristics of spatial evolution in Beijing in the past three decades. In 1985, the construction land within Beijing's 6th ring road was greatly expanded (the q value range is between -1.7 and 13.0, which means that it is highly hierarchical). However, with the rapid growth, the spatial structural adjustment, and multifunctional development of Beijing, the agglomeration of the newly increased construction land of the study area has weakened (Table 5), which, in the mathematical analysis, is manifested as an increase in the lower limit of the q value (increased from -1.7 to 1.0). It can be concluded from Fig. 1 and Table 5 that the partial decentralized expansion of the construction land in the rural-urban fringe zone of all the years studied has gradually become the principal subject and generally maintained the upper limit of the q value unchanged (13.0 in 1985, 2008, and 2015, and 12.5 in 2008). Corresponding to the reality, on the basis of 1985, the new construction land in the last three years is mainly represented by the expansion of construction land to the suburban area at the micro level and medium level, and the hierarchy of this expansion is strong. This conclusion is also supported by the result of local indicators analysis where the *f(a)–a(q)* fitting curves of the four-year period are all left-slanted and they are mostly located on the left side of the peak.

Moreover, from the D_q value range of each year in Fig. 2 and the D_q–q curve distribution, it is observed that, in the over-thirty years' development, the spatial evolution intensity of the starting year (1985) is the smallest, the mid-term evolution intensity represented by 1999 and 2008 is the biggest, and the evolution intensity of the ending year (2015) is in the middle. Therefore, the development of new construction land in Beijing's 6th ring road can be described as follows: the urban spatial structure gradually evolved from a phase with a strong hierarchy and weak evolution in 1985 to a phase of multi-scale differential change and spatial evolution intensity (1999, 2008). In 2015, the spatial system with the main urban area and satellite towns (enclaves) and slow spatial evolution intensity have basically formed, which can also be verified by the sharp peaks of the multi-dimensional spectra of 1985, 1999, 2008, and 2015 in the analysis of local indicators.

Table 5 Multifractal characteristics of construction land within the 6th ring road of Beijing in 4 years

Year		1985	1999	2008	2015
Global characteristics	q Value range	−1.7–13.0	−0.5–13.0	−0.6–12.5	1.0–13.0
	D_q size	Weak	Strong	Strong	Medium
Block level ($m = 5$)	Spectrum width	1.747	0.339	0.539	0.115
	Degree of left deviation	Strong	Stronger	Stronger	Strong
	Degree of discretization	None	Tendency	Tendency	Yes
Road level ($m = 6$)	Spectrum width	1.417	0.396	0.456	0.087
	Degree of left deviation	Stronger	Medium	Medium	Stronger
	Degree of discretization	None	Tendency	Tendency	Yes
Community level ($m = 7$)	Spectrum width	0.985	0.415	0.452	0.063
	Degree of left deviation	Weak	Weak	Weak	Weak
	Degree of discretization	None	Tendency	Tendency	Yes

Local characteristics of spatial evolution in Beijing in the past three decades. The global indicator reflects the overall characteristics of the changes in Beijing's spatial pattern, while at each scale it must be reflected by specific local indicators. The characteristics of the change in the spatial pattern in Beijing on each scale are reflected by specific local indexes that together form a "telescope" and "microscope" analysis model. After comparative analysis of the multifractal dimension spectra at the block level, road level, and community level (Fig. 11 and Table 4), the following can be found:

The spatial pattern of Beijing tended to be simple. The differences in spectrum width of the spatial scale corresponding to the growth of the construction land each year (m) are very large, and they have been reduced year by year, indicating that the spatial pattern of the construction land within Beijing's 6th ring road has gradually tended to be simple in the past 30 years. Figure 10 shows that the spectral width of each spatial scale was larger in 1985 and then gradually became smaller, which means that in 1985, Beijing's main urban area was small; however, it was fully functional. Therefore, its spatial complexity and hierarchy were relatively strong after the combination development of multiple functions. However, with the enlargement of Beijing's urban area, some functions were relocated to much larger spaces, thereby reducing the degree of the spatial agglomeration and the complexity of its spatial pattern.

The influence of low-density areas of construction land is becoming stronger. Intuitively, the peak of the multi-dimension spectrum corresponds to the development context in which the generalized dimension D_0 is located, with the low-density area distribution area on the left side and the high-density area distribution area on the right side. The structural changes in the peak position and the multi-dimensional spectrum of the four years in Fig. 11 significantly reflect the specific changes in Beijing's space. First, the multi-dimension spectrum of each year and each spatial scale is left-slanted, indicating that Beijing as a whole is a diffuse metropolis. Secondly, this type of left-slant and diffuseness are not the same in each year. In 1985, the overall performance of the scales was moderately left-slanted, and in 1999 and 2008, it had a left-slanted discrepancy, and by 2015 the multi-dimension spectrum of the scales appear to be discretized with a left bias without dominant peaks. In other words, in 1985, the multi-dimensional spread of Beijing still had the distinction between primary and secondary. By 1999 and 2000, the secondary spread had begun to strengthen, and by 2015, it had formed a spread in all directions. In short, with the growth of construction land in Beijing's 6th ring road, the influence of high-density areas (such as the edge of the main urban area) is declining, and the influence of low-density areas (such as the rural residential area in Beijing suburbs) is rising.

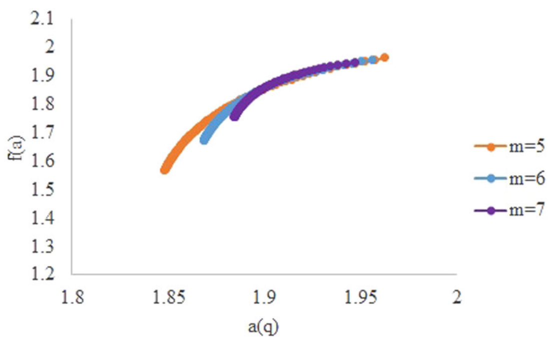

Fig. 10 Comparison of multi-scale multifractal spectrum of construction land within Beijing's 6th ring road

Specifically, the above two evolution trends can also be reflected by the peaks of the multi-dimension spectrum of the four years in Fig. 11. In 1985, 1999, 2008, the $f(a)$–$a(q)$ curves of each m value are covered both sides of the peak (or the large, medium, and small value distribution zones of q value). The large, medium, and small density areas of the construction land display multiple fractal features in some extents. In 2015, the $f(a)$–$a(q)$ curve of each m value appears only on the left side of the peak. That is, the medium and small density area of the construction land (or the large median distribution zone of the q value).

The pedigree evolution shows in all spatial scales. At the block level ($m = 5$), there is little difference in the peak value of dominance of each year. However, at the community level ($m = 7$), the peak value of dominance of 1985 was obviously smaller than that of the other three years, indicating that the evolution intensity of the micro domain after the reform and opening up was greater than that of the initial stage of reform and opening up. However, the change at the road level ($m = 6$) (including spectrum width, shape, and peak value of the multifractal dimension spectrum) is always between the block level ($m = 5$) and the community level ($m = 7$). On the whole, the influences of the spatial system at the community level and road level were increasing day by day.

Moreover, the above local evolutionary characteristics further influence and confluence with each other and produce overall spectrum development characteristics of various spatial scales ($m = 5, 6, 7$), spatial evolutionary breadth (q), and intensity (D_q). Therefore, the specific characteristics of Beijing's four-year periods can be described by fitting curves of $a(q) - q$, $f(a) - q$, *and* $f(a) - a(q)$ on the three spatial scales. The analysis

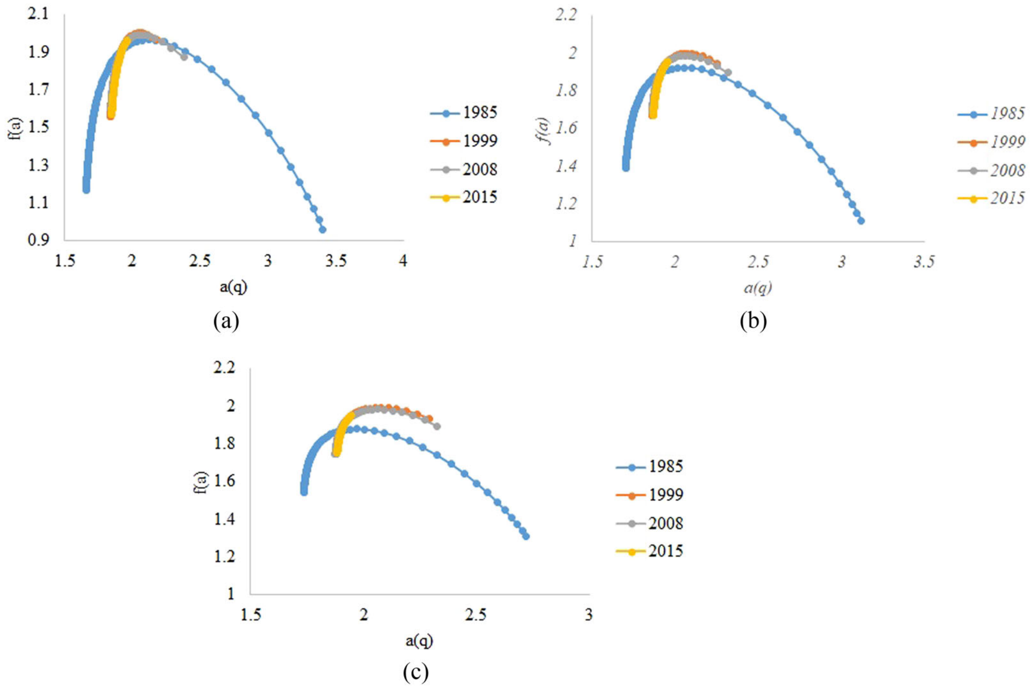

Fig. 11 The four-year multifractal dimension spectrum of each space scale within the 6th ring road of Beijing. (**a**) $m = 5$; (**b**) $m = 6$; (**c**) $m = 7$

conclusions can be found in Section "Local index analysis". Although these conclusions seem cumbersome, they show the specific time-space changes in the space system within Beijing's 6th ring road in detail.

Problems reflected in this analysis. First, the three forms shown by the *f(a)*–*a(q)* fitting curve for each *m* value in Fig. 3 of the 4-year period are actually an in-depth and quantitative understanding of the development status of each period (Fig. 1), which is also conducive to discovering the coupling process and complexity of the process that drives the change of Beijing's spatial structure. The evolution of the spatial structure of construction land reflects that the urbanization and industrialization has not been in an ideal coupling state for a long time.

Moreover, comparing the development status of the various periods and the corresponding developmental schemes and planning measures, it can be observed that there are large differences between them. There have even been objective phenomena such as real estate and excessive urbanization that were not recognized or planned. From above results, this study obtains the following insights: First, the prediction by planners on the evolutionary breadth and strength of the social economic foundation of the various periods in Beijing (especially the late period) is obviously inadequate, resulting in all sorts of development planning that fails to address the growth of the construction land at various spatial scales. Second, understanding the complexity and regularity of this system in the coordinated control of the construction land at the micro level, medium level, and macro level is insufficient. The insights and the use of technical methods in this paper have certain reference value for the in-depth analysis of the complexity of Beijing's space-time development and the effectiveness of government-led planning at all levels.

Discussion

The purpose of this paper is to reveal the complex but regular changes of the construction land within Beijing's 6th ring road in nearly 30 years using global and local multifractal index analysis methods. The course of evolution and specific characteristics of the spatial system of Beijing's construction land (taking the 6th ring road as example) was obtained through an analysis process integrated with global (telescopic) analysis, local (microscope) analysis and historical evolutionary analysis. The research paradigm established by this paper, which, of course, is also the main innovation point of this study, has certain reference value for relevant studies.

Two technical problems in the research paradigm need to be further addressed in future research. First, analysis of a single type of land still has difficulty in accurately and comprehensively reflecting the complex evolutionary characteristics of the study area. Some conclusions of this study could be enriched and supported (or even unsupported) if factors such as cultivated land, traffic, and green spaces were analyzed together. Second, the spatial scale reached level 9 (~100 m) in the global index analysis, but level 7 (~415 m) in the local index analysis, which suggests that the technical route in future studies will require some modifications. For example, the study area can be divided into a number of smaller areas. The analysis conclusion of whole areas then can be verified and supported by the analysis conclusions of the sub-areas.

With regard to the economic geo-spatial system, there have been a great number of multifractal analysis studies using grid-based fractal dimension methods. However, the spatial data obtained were often not ideal. For the purposes of this paper, the main reason for choosing the construction land within Beijing's 6th ring road as the study object is that, after comparing the data of several cities (for instance, cities adjacent to rivers, with spatial divergence), it was found that Beijing's data are the most ideal in that the study area has a square form and its construction land expands basically in a round shape. Therefore, this method is often restricted by irregular spatial data in practical applications, which limits the range of use of this method. However, at present, some scholars have further developed the method, for example, using the spatial angle density and shape index of small areas after selecting the central point of the study area.

Multifractal indexes reflect in the current situation of the spatial pattern of the study area, including various characteristics of the existence and development of geographical objects, such as regularity, tendency, spatial correlation and self-organization. With regard to the existing application studies, multifractal indexes cannot resolve the inherent evolution mechanism, such as the D_q value, spectral deviation and spectral width. At the same time, the specific numerical value of these indexes may also correspond to various development scenarios. Therefore, in the actual analysis, to achieve the aims of removing false information, preserving the truth and discriminating effectively, the results of various supporting studies should be referenced.

Data availability

The datasets generated during and/or analyzed during the current study are not publicly available but are available from the corresponding author on reasonable request.

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Additional information

Competing interests: The authors declare no competing interests.