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Author Correction: Topological quantum criticality in non-Hermitian extended Kitaev chain

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The original version of this Article contained a repeated error, where the sign "*CP*" was incorrectly given as "*Cq*". As a result of this error, in the Results,

"The Hermitian model consists of three critical lines (solid black line) "AB", "BD" and "Cq" distinguishing topological phases W = 0, W = 1 and W = 2."

now reads:

"The Hermitian model consists of three critical lines (solid black line) "AB", "BD" and "CP" distinguishing topological phases W = 0, W = 1 and W = 2."

And, in the Results section, under the subheading 'Zero mode analysis for topological characterization',

"From the Fig. 1, it can be clearly observed that the critical line "Cq" is not present for the non-Hermitian case (critical lines presented in red color)."

now reads:

"From the Fig. 1, it can be clearly observed that the critical line "CP" is not present for the non-Hermitian case (critical lines presented in red color)."

Additionally, the original version of this Article contained errors in the sign in the zero mode solutions in the Method section, where

Zero mode solutions.

The model Hamiltonian can be written as,

$$H_k = \chi_z(k)\sigma_z + \chi_y(k)\sigma_y, \tag{9}$$

where $\chi_z(k) = 2\lambda_1 \cos k + 2\lambda_2 \cos 2k - 2(\mu + i\gamma)$, and $\chi_y(k) = 2\lambda_1 \sin k + 2\lambda_2 \sin 2k$.

Substituting the exponential forms of $\cos k$ and $\sin k$, Eq. (9) becomes,

$$\begin{aligned} H = &\left[2\lambda_1 \frac{1}{2}(e^{-ik} + e^{ik}) + 2\lambda_2 \frac{1}{2}(e^{-2ik} + e^{2ik} + 2(\mu + i\gamma))\right]\sigma_z \\ &+ i\left[2\lambda_1 \frac{1}{2}(e^{ik} - e^{-ik}) + 2\lambda_2 \frac{1}{2}(e^{2ik} - e^{-2ik})\right]\sigma_y. \end{aligned} \tag{10}$$

We replace $e^{-ik} = e^q$, Eq. (10) becomes,

$$H = \left[2\lambda_1 \frac{1}{2}(e^{q} + e^{-q}) + 2\lambda_2 \frac{1}{2}(e^{2q} + e^{-2q}) + 2(\mu + i\gamma)\right]\sigma_z + i\left[2\lambda_1 \frac{1}{2}(e^{-q} - e^{q}) + 2\lambda_2 \frac{1}{2}(e^{-2q} - e^{2q})\right]\sigma_y. \tag{11}$$

To find the zero mode solutions, we make $H^2 = 0$. By solving the Eq. (11), we get,

$$2\lambda_1 \cosh q + 2\lambda_2 \cosh 2q - 2(\mu + i\gamma) = \pm 2\lambda_1 \sinh q + 2\lambda_2 \sinh 2q. \tag{12}$$

Equation 12 shows that there will be more than one solution. Considering the Eq. (11) and squaring both sides with $H = 0$, we get,

$$\big(2\lambda_1 \cosh q + 2\lambda_2 \cosh 2q - 2(\mu + i\gamma)\big) + i\big(2\lambda_1 \sinh q + 2\lambda_2 \sinh 2q\big) = 0. \tag{13}$$

Substituting back the exponential forms to the respective terms, we get,

$$\left[2\lambda_1 \frac{1}{2}(e^{q} + e^{-q}) + 2\lambda_2 \frac{1}{2}(e^{2q} + e^{-2q} + 2(\mu + i\gamma))\right] + \left[2\lambda_1 \frac{1}{2}(e^{-q} - e^{q}) + 2\lambda_2 \frac{1}{2}(e^{-2q} - e^{2q})\right] = 0 \tag{14}$$

Simplifying the Eq. (14), we end up with a quadratic equation,

$$2\lambda_1 \frac{1}{2} e^{q} + 2\lambda_2 \frac{1}{2} e^{2q} + 2(\mu + i\gamma) + 2\lambda_1 \frac{1}{2} e^{q} + 2\lambda_2 \frac{1}{2} e^{2q} = 0. \tag{15}$$

Simplifying the Eq. (15) to a quadratic form and substituting $e^{q} = X$,

$$\lambda_2 X^2 + \lambda_1 X + (\mu + i\gamma) = 0. \tag{16}$$

The roots of this quadratic Equation is given by,

$$X = \frac{-\lambda_1 \pm \sqrt{\lambda_1^2 + 4\lambda_2(\mu + i\gamma)}}{2\lambda_2} \tag{17}$$

The roots Eq. (17) are the solutions of zero modes.

now reads:

Zero mode solutions.

The model Hamiltonian can be written as,

$$H_k = -\big(\chi_z(k)\sigma_z + \chi_y(k)\sigma_y\big), \tag{1}$$

where $\chi_z(k) = 2\lambda_1 \cos k + 2\lambda_2 \cos 2k - 2(\mu + i\gamma)$, and $\chi_y(k) = 2\lambda_1 \sin k + 2\lambda_2 \sin 2k$.

Substituting the exponential forms of $\cos k$ and $\sin k$, Eq. 1 becomes,

$$H_k = 2\lambda_1 \frac{1}{2}(e^{-ik} + e^{ik}) + 2\lambda_2 \frac{1}{2}(e^{-2ik} + e^{2ik} + 2(\mu + i\gamma))\sigma_z + i2\lambda_1 \frac{1}{2}(e^{ik} - e^{-ik}) + 2\lambda_2 \frac{1}{2}(e^{2ik} - e^{-2ik})\sigma_y. \tag{2}$$

We replace $e^{-ik} = e^{q}$, Eq. 2 becomes,

$$H_q = 2\lambda_1 \frac{1}{2}(e^{q} + e^{-q}) + 2\lambda_2 \frac{1}{2}(e^{2q} + e^{-2q}) + 2(\mu + i\gamma)\sigma_z + i2\lambda_1 \frac{1}{2}(e^{-q} - e^{q}) + 2\lambda_2 \frac{1}{2}(e^{-2q} - e^{2q})\sigma_y. \tag{3}$$

We make $H_q^2 = 0$[31], to obtain the zero solutions for certain q where σ_z and σ_y square to 1 or become 0 due to anticommutation.

$$2\lambda_1 \frac{1}{2}(e^{q} + e^{-q}) + 2\lambda_2 \frac{1}{2}(e^{2q} + e^{-2q} + 2(\mu + i\gamma)) + 2\lambda_1 \frac{1}{2}(e^{-q} - e^{q}) + 2\lambda_2 \frac{1}{2}(e^{-2q} - e^{2q}) = 0 \tag{4}$$

Simplifying the Eq. 4, we end up with a quadratic equation,

$$2\lambda_1\frac{1}{2}e^q + 2\lambda_2\frac{1}{2}e^{2q} + 2(\mu + i\gamma) + 2\lambda_1\frac{1}{2}e^q + 2\lambda_2\frac{1}{2}e^{2q} = 0. \quad (5)$$

Simplifying the Eq. 5 to a quadratic form and substituting $e^q = X$,

$$\lambda_2 X^2 + \lambda_1 X + (\mu + i\gamma) = 0. \quad (6)$$

The roots of this quadratic Equation is given by,

$$X = \frac{-\lambda_1 \pm \sqrt{\lambda_1^2 + 4\lambda_2(\mu + i\gamma)}}{2\lambda_2} \quad (7)$$

Finally, the original version of this Article contained an error in the legend of Fig. 2.

"Zero mode solutions plotted with respect to the parameter λ_1 shows both W = 0 to W = 1(λ_2 = 0.5) and W = 1 to W = 2(λ_2 = 2.0) topological phase transitions. The red dot (p1 and p2) represents the transition points. The ZMS, X_+ (red) and X_- (blue) are plotted in y-axis."

now reads:

"Zero mode solutions plotted with respect to the parameter λ_1 shows both W = 0 to W = 1(λ_2 = 0.5) and W = 1 to W = 2(λ_2 = 2.0) topological phase transitions. Blue dots (p1 and p2) represent the transition points. The ZMS, X_+ (red) and X_- (blue) are plotted in y-axis."

The original Article has been corrected.