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Singly-excited resonant open quantum system Tavis-Cummings model with quantum circuit mapping

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Tavis-Cummings (TC) cavity quantum electrodynamical effects, describing the interaction of *N* atoms with an optical resonator, are at the core of atomic, optical and solid state physics. The full numerical simulation of TC dynamics scales exponentially with the number of atoms. By restricting the open quantum system to a single excitation, typical of experimental realizations in quantum optics, we analytically solve the TC model with an arbitrary number of atoms with linear complexity. This solution allows us to devise the Quantum Mapping Algorithm of Resonator Interaction with *N* Atoms (Q-MARINA), an intuitive TC mapping to a quantum circuit with linear space and time scaling, whose *N*+1 qubits represent atoms and a lossy cavity, while the dynamics is encoded through 2*N* entangling gates. Finally, we benchmark the robustness of the algorithm on a quantum simulator and superconducting quantum processors against the quantum master equation solution on a classical computer.

The Tavis-Cummings (TC) model 1 , which describes interaction of N atoms with an optical cavity has been a cornerstone in the studies of quantum optical systems. The collective interactions in this model give an \sqrt{N} increase in the light-matter interaction rate (Fig. 1) and a host of subradiant states with rich phenomenology relevant for the development of quantum networks²⁻⁴, all-photonic quantum simulators⁵, quantum memories^{6,7}, quantum transport⁸, exciton-polarons in semiconductors⁹, superconducting quantum circuits¹⁰, collective interaction of the cavity mode with an ensemble of atoms¹¹⁻¹⁴, and entanglement generation¹⁵⁻²⁰. Rapid progress in experimental development in the field of nanophotonics, renders the impracticality and scarceness of theoretical approaches unsatisfactory, especially in the open quantum system setting where the cavity interacts with the environment. Although recent results demonstrate that generalized TC model is integrable and can be solved using a variant of Quantum Inverse Methods (QIM)^{21,22}, solutions obtained in this way poise difficulties in extracting physical quantities and capturing dynamical correlations in the system. On the other hand, numerical solutions obtained through the quantum master equation²³ are limited by the exponential runtime complexity in Hilbert space size, and have thus far been performed for a single digit number of atoms. Due to the impracticality of analytical approaches based on QIM and exponentially rising cost of numerical solutions of the quantum master equations for such systems, theoretical verifications of experimental results are constrained to low number of atoms. Increasing the size of the Hilbert space has been pursued via approximate methods with polynomial scaling, such as the effective Hamiltonian²⁴, scattering matrix⁴ and quantum trajectories²⁵ approaches. Furthermore, for applications concerned with the singly-excited regime, exact methods can be derived under linear scaling.

The availability of the Noisy Intermediate Scale Quantum (NISQ) devices has attracted interest for simulating open quantum systems. To date, two prevailing directions have emerged, the first using operator sum representation, where Sz.-Nagy theorem is used to relate Kraus operators with unitary dilatation matrices^{26,27} that can then be directly implemented on a quantum circuit. This result has been further generalised and applied to quantum simulate the complex open quantum system, governed by the Fenna-Matthews-Olson Dynamics modelling the quantum theory of electron transfer in biological systems²⁸. An alternative approach is starting directly from the equations of motion in Lindblad and Gorini-Kossakowski-Sudarshan-Lindblad form and mapping the dynamics to a quantum circuit, which has been applied so far to both Markovian and non-Markovian open quantum systems consisting of 1 or 2 qubits²⁹. This approach has recently been verified on a canonical model of

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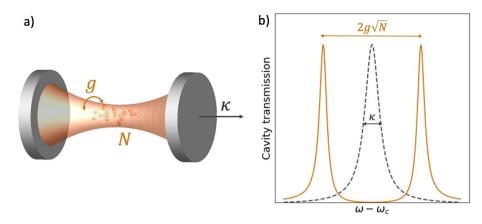


Figure 1. (a) An illustration of an open Tavis-Cummings system consisting of an optical cavity of the loss rate κ with N atoms each coupled at the interaction rate g. (b) The transmission spectrum of an empty cavity (dashed gray line) featuring a lorentzian profile with linewidth κ and a cavity resonantly coupled to N atoms (solid orange line) featuring two polariton peaks separated by $2g\sqrt{N}$.

light matter interaction systems: the Jaynes-Cummings model²⁹. Cavity quantum electrodynamical models that involve multiple emitters, such as the TC model, have not yet been considered. However, this in particular is the area where classical methods quickly saturate numerical resources and quantum devices may be able to expand the Hilbert size of systems studied in quantum communication, memories and simulators. Moreover, studying a quantum system on purely quantum hardware may provide representations that are intuitive in nature, as both emitters and qubits are two-level systems.

In this work, we study a resonant open Tavis-Cummings model with arbitrary number of atoms and first provide an analytical solution for the singly-excited system with linear complexity. We then design the Quantum Mapping Algorithm of Resonator Interaction with N Atoms (Q-MARINA) which maps the open TC system with N atoms to a gate-based quantum circuit with only N+1 qubits. We simulate the system on a superconducting quantum computer available through IBM Quantum program³⁰.

Results

The model

We consider N two-level systems, modeling an ensemble of atoms (or spins), coupling to the environment of discrete bosonic modes. The system and the environment Hamiltonians H_S and H_E are:

$$H_S = \omega_s S_z, \qquad H_E = \sum_k \omega_k b_k^{\dagger} b_k,$$
 (1)

while their interaction is described by the Tavis-Cummings Hamiltonian H_I :

$$H_I = \sum_{k} g_k b_k S^+ + g_k^* b_k^{\dagger} S^-.$$
 (2)

Here, we use the collective system operators $S_z = \sum_{j=1}^N \frac{1}{2} \sigma_z^j$ and $S^{\pm} = \sum_{j=1}^N \sigma_j^{\pm} = \sum_{j=1}^N \frac{1}{2} (\sigma_x^j \pm i \sigma_y^j)$, with commutation relations $[\sigma_j, \sigma_k] = 2i\epsilon_{j,k,l}\sigma_l$ and $[S_z, S^{\pm}] = \pm S^{\pm}$.

To solve the model analytically, we aim to obtain the time dependent Hamiltonian in the interaction picture in the form of:

$$H_I(t) = \sum_k g_k b_k(t) S^+(t) + g_k^* b_k^{\dagger}(t) S^-(t).$$
(3)

Here, the form of $b_k(t) = b_k e^{-i\omega_k t}$ is easily derived, however, finding an elegant expression for $S^{\pm}(t)$ requires closer consideration.

We first note that, in the Hilbert space of the system, the operator $S_z = \frac{1}{2} \sum_{n=1}^{N} \left(\bigotimes^{n-1} I \otimes \sigma_z \otimes^{N-n} I \right) = \operatorname{diag}(\{x_p\})$ is diagonal in terms we will call x_p , $1 \le p \le 2^N$. We find that, x_p is a function of the Hamming weight W(p-1), i.e. the digit sum of the binary representation of the number p-1, as:

$$x_p = \frac{N}{2} - W(p-1).$$

Therefore, the term $e^{iH_St}=\mathrm{diag}(\{e^{i\omega_stx_p}\})$ must too be diagonal, which allows us to obtain a closed form solution:

$$S^{\pm}(t) = S^{\pm} e^{\pm i\omega_S t},\tag{4}$$

thus completing the Eq. (3) for the time-dependent interaction Hamiltonian of the TC model with N identical two-level atoms.

Reduced density matrix

The corresponding reduced density matrix $\rho_S(t)$ of the TC system with N atoms is given by

$$\rho_{S}^{n,n}(t) = \|c_{s_n}(t)\|^2, 1 \le n \le N, \tag{5}$$

$$\rho_{S}^{N+1,N+1}(t) = 1 - \sum_{n=1}^{N} \|c_{s_n}(t)\|^2, \tag{6}$$

where c_{s_n} are the wavefunction coefficients with the following dependence on the cavity loss and cavity-atom interaction parameters:

$$c_{s_n}(t) = c_{s_n}(0) - \frac{1}{N} \sum_{m=1}^{N} c_{s_m}(0) \left[1 - e^{-\frac{\kappa t}{4}} \left(\frac{\kappa}{D} \sinh \frac{Dt}{4} + \cosh \frac{Dt}{4} \right) \right], \tag{7}$$

$$D = \sqrt{-16Ng^2 + \kappa^2}. (8)$$

Real and positive coefficients $c_{s_n}(0)$ are subject to the normalization constraint $\sum_{n=1}^N \|c_{s_n}(0)\|^2 = 1$, and full derivation of the density matrix is given in "Reduced density matrix derivation" section. One of the key considerations to arrive to an exact solution and study the dynamics of the open TC system with N identical two-level atoms is that the total number of excitations in our system is a constant of motion [H,M]=0, where $H=H_I+H_S+H_T$, and $M=S^+S^-+\sum_k\omega_kb_k^\dagger b_k=1$ is the total number of excitations in the system considered here. It has been shown that if one exploits the permutational symmetry originating from the simplification that we are considering N identical emitters, one can gain further insights into closed systems beyond a single excitation manifold both analytically and numerically $^{32-36}$. In the case of the open Tavis-Cummings model studied here, this is seen in the symmetry of our solution for the wavefunction coefficients obtained in Eq. (7), where the identical choice of initial conditions would lead to identical behavior of subradiant states.

In the following, we show that this dynamics can be mapped onto a quantum circuit with N+1 qubit, thus enabling quantum modeling of the Tavis-Cummings open quantum system on a gate-based quantum computer. While the solution derived in this section is a general one for a single-excitation system, for simplicity, we will from now on assume that the first emitter in the system is the one that is initially excited, while others are in the ground state $(c_{S1}(0) = 1, c_{Sm}(0) = 0 \text{ for } m = 2, ..., N)$, and the proposed quantum circuit will reflect that.

Quantum circuit

Here, we devise the Quantum Mapping Algorithm of Resonator Interaction with N Atoms (Q-MARINA), an (N+1)-qubit quantum circuit that evolves an open quantum system of N atoms and a resonant cavity in the single-excitation regime. The quantum circuit consists of N system qubits Q_{Sn} and one environment qubit Q_E . The initial state is the excited state of one of the atoms, here Q_{S1} which is subject to an X-gate. Subsequent application of CU_3 and CNOT gates between Q_{S1} and Q_E entangles the first atom and the environment, and then N-1 sequences of CU_3 and CNOT entangling gates are applied to each of the qubits Q_{S2} ,..., Q_{SN} paired with Q_E , in the opposite direction than for the Q_{S1} . The corresponding quantum circuit is shown in Fig. 2. Here, the parameters of the CU_3 gates, $CU_3(2\theta_n) = CU_3(2\theta_n, 0, 0)$ are selected to implement the Lorentzian density of states of the cavity open to the environment into the circuit:

$$\theta_1 = \arccos\left(c_{s_1}(t)\right),\tag{9}$$

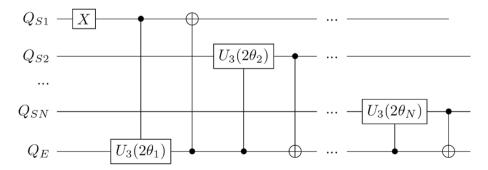


Figure 2. Q-MARINA algorithm that maps an open quantum system of N two-level atoms in a lossy cavity to a quantum circuit with N+1 qubits and 2N entangling gates that encode the interaction of atoms (Q_{Sn}) with the cavity and environment (Q_E).

$$\theta_n = \arcsin\left(\frac{c_{s_n}(t)}{\sin\theta_1 \prod_{m=1}^{n-1} \cos\theta_m}\right),\tag{10}$$

thus resulting in excited state measurement probabilities of the system qubits Q_{Sn} equal to $||c_{s_n}(t)||^2$. Importantly, this quantum circuit maintains the physical connections typical of the TC model where each atom directly interacts only with the cavity, reflected in entangling gates operating solely on system-environment qubit pairs.

Implementation of the Quantum Mapping Algorithm on superconducting circuits

As a testbed for quantum simulations of the lossy TC model, we implement the devised Q-MARINA quantum algorithm on the IBM Q Experience hardware. We first demonstrate agreement of the results for open system dynamics obtained through the implementation of the quantum circuit on the IBM QASM simulator provided via Qiskit library 39 . The comparison of the numerical solution of Quantum Master Equation (QME) for N=7 atoms with the execution of the Q-MARINA quantum circuit in QASM simulator is illustrated in Fig. 3.

We then execute the proposed quantum circuit on the superconducting quantum devices $ibmq_quito$ (Falcon r4T processor) and ibm_oslo (Falcon r5.11H processor), available through the IBM Quantum program. The quantum circuit requires star connectivity as all system qubits Q_{Sn} interact with the environment qubit Q_E , therefore we selected devices that can support that layout in a 3- and 4-qubit circuits within the computers' heavy-hexagon topology. The comparison of our quantum device results with the previously obtained benchmarks on the QASM simulator and numerical QME solutions are shown in Fig. 4. The demonstrated close agreement between the solutions of the QME with Q-MARINA executed on QASM simulator and IBM

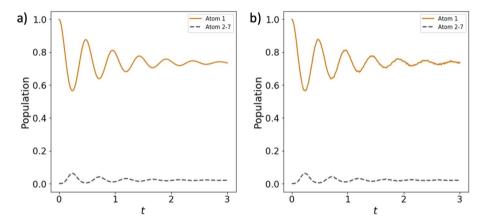


Figure 3. The evolution of the singly excited open quantum system Tavis-Cummings model of $N=7, g=\kappa=5$ calculated using (**a**) quantum master equation in QuTiP software^{37,38} and (**b**) Q-MARINA algorithm in QASM simulator with 40,000 shots per data point.

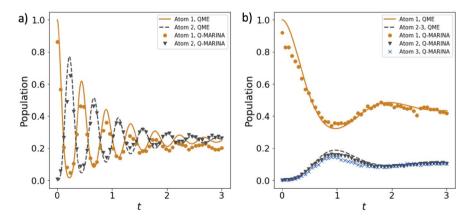


Figure 4. Q-MARINA simulation of the singly excited open TC system for evolving upon excitation of the Atom 1, executed on (a) ibmq_quito quantum computer with 10,000 shots per point for $N=2,g=10,\kappa=5$, and (b) ibm_oslo quantum computer with 10,000 shots per point for $N=3,g=2,\kappa=5$. The exact QME solution is plotted for comparison.

Q quantum devices indicates that NISQ era quantum computers can be used to simulate open quantum system dynamics of highly dimensional models.

Discussion

In this work, we have explored quantum circuit mapping of the dynamics of N two-level atoms in a a lossy optical cavity. By restricting the open quantum system to a single excitation, typical of experimental realizations in quantum optics, we have analytically solved the TC model with an arbitrary number of atoms achieving reduced modeling complexity. This solution enabled us to devise the Quantum Mapping Algorithm of Resonator Interaction with N Atoms (Q-MARINA), an intuitive TC mapping to a quantum circuit with linear space and time scaling. We note here that this work does not aim at quantum advantage, but rather to show that the studied regime of Tavis-Cumming physics in a lossy resonator can be efficiently mapped to N+1 qubit, as opposed to an infinite number of qubits.

It is interesting to note that the execution of the Q-MARINA quantum circuit illustrated in Fig. 2 on the the superconducting quantum devices $ibmq_quito$ and ibm_oslo are in good agreement with the numerical solution of the QME (c.f. Fig. 4), despite the fact that no error mitigation technique has been considered thus far. These results demonstrate that the open quantum system Tavis-Cummings physics can be simulated on the existing quantum hardware with an intuitive mapping between atoms and qubits and a substantial reduction in complexity implemented through the entangling gates with a single environment qubit. That being said, we acknowledge multiple challenges on the hardware side that need to be resolved before achieving e.g. coherence stability of the quantum devices with the number of qubits comparable to the number of atoms where classical solutions of the master equation become intractable. Therefore, a numerical solution of QME³⁸, as well as analytical approaches such as mean-field approximation⁴⁰, or Keldysh's action formalism^{41,42} remain valuable go-to methods for studying the complex dynamics of quantum fluctuations in the TC-like systems.

The devised mapping of the TC system with N identical atoms constitutes a first step toward using superconducting NISQ processors to design new optical quantum devices. The results obtained on existing quantum devices are further limited by the quantum computer size and the corresponding topology which provides the desired star-connectivity to up to 4 qubits. Alternative quantum platforms which provide all-to-all connectivity, such as those based on trapped ions 43,44 or atoms 45 , may provide options to scale the problem size by at least an order of magnitude 46,47 . Once the number of qubits is scaled, the number of entangling gates relative to the qubit coherence time will be the measure of the performance of our algorithm, as the circuit depth scales linearly with the number of atoms.

Methods

Reduced density matrix derivation

The wavefunction of an N-atom Tavis-Cummings system in the low-excitation regime is given by the superposition of the vacuum state $|g0\rangle$, single excitations of the n-th atom $|e_n0\rangle$ and the single excitations of the k-th bosonic mode $|g1_k\rangle$

$$|\Psi_N(t)\rangle = c_0|g_0\rangle + \sum_{n=1}^N c_{s_n}(t)|e_n_0\rangle + \sum_k c_k(t)|g_k\rangle.$$
(11)

The Schrödinger equation with Hamiltonian given in Eq. (3) yields a system of differential equations:

$$\dot{c}_{s_n} = -i \sum_{k} g_k e^{i[(x_{2^N - 2^{N-n}} - x_{2^N})\omega_s - \omega_k]t} c_k(t), \tag{12}$$

$$\dot{c_k} = -ig_k^* \sum_{n=1}^N e^{i[\omega_k + \omega_s(x_{2^N} - x_{2^N - 2^{N-n}})]t} c_{s_n}(t).$$
(13)

We next note that $W(2^N - 1) = N$ and $W(2^N - 2^{N-n} - 1) = N - 1$ therefore $x_{2^N} = -\frac{N}{2}$ and $x_{2^N - 2^{N-n}} = 1 - \frac{N}{2}$. The system of differential equations transforms to

$$\dot{c_{s_n}} = -i \sum_{k} g_k e^{i(\omega_s - \omega_k)t} c_k(t), \tag{14}$$

$$\dot{c_k} = -ig_k^* \sum_{n=1}^N e^{i(\omega_k - \omega_s)t} c_{s_n}(t).$$
 (15)

It follows that the k-th cavity mode and the n-th atom amplitude can be expressed as

$$c_k(t) = -i \int_0^t dt' g_k^* e^{i(\omega_k - \omega_s)t'} \sum_{n=1}^N c_{s_n}(t'),$$
 (16)

$$\dot{c}_{s_n}(t) = -\int d\omega J(\omega) \int_0^t dt' e^{i(\omega_s - \omega)(t - t')} \sum_{m=1}^N c_{s_m}(t'), \tag{17}$$

where we approximate the environment coupling terms with Lorentzian density of states modeling the cavity dynamics $\sum_k \|g_k\|^2 = \int d\omega J(\omega)$. The term

$$J(\omega) = \frac{g^2}{2\pi} \frac{\kappa}{(\omega_s - \omega)^2 + (\kappa/2)^2}$$
 (18)

describes an optical resonator with loss rate κ coupled to an atom at interaction rate g, and represents the channel through which the system interacts with the environment. For a closed system, the cavity would respond to only a singular frequency ($\kappa = 0$). Next, similarly to²⁹, we define

$$f(t - t') = \int d\omega J(\omega) e^{i(\omega_s - \omega)(t - t')}, \tag{19}$$

and the atomic amplitudes simplify to

$$\dot{c}_{s_n}(t) = -\int_0^t dt' f(t - t') \sum_{n=1}^N c_{s_n}(t'). \tag{20}$$

Taking the Laplace transform of l.h.s. and r.h.s. of the previous equation, we obtain:

$$s\tilde{c}_{s_n}(s) - c_{s_n}(0) = -\tilde{f}(s) \sum_{n=1}^N \tilde{c}_{s_n}(s),$$
 (21)

where $\tilde{c}_{s_n}(s)$ and $\tilde{f}(s)$ denote the Laplace transforms of the functions $c_{s_n}(t)$ and f(t-t') defined in Eq. (19). Solving the system of coupled equations given in Eq. (21) for $\tilde{c}_{s_n}(s)$ and performing an inverse Laplace transform gives us wavefunction coefficients which determine the density matrix:

$$c_{s_n}(t) = c_{s_n}(0) - \frac{1}{N} \sum_{m=1}^{N} c_{s_m}(0) \left[1 - e^{-\frac{\kappa t}{4}} \left(\frac{\kappa}{D} \sinh \frac{Dt}{4} + \cosh \frac{Dt}{4} \right) \right], \tag{22}$$

where $D = \sqrt{-16Ng^2 + \kappa^2}$.

To obtain the reduced density matrix $\rho_S(t)$ that describes the state of the system, we remove the environment degrees of freedom through a partial trace:

$$\rho_{S}(t) = \langle 0|\Psi_{N}(t)\rangle\langle\Psi_{N}(t)|0\rangle + \sum_{k}\langle 1_{k}|\Psi_{N}(t)\rangle\langle\Psi_{N}(t)|1_{k}\rangle. \tag{23}$$

From here, we express the diagonal elements of the (N+1)-dimensional density matrix as:

$$\rho_{S}^{n,n}(t) = \|c_{s_n}(t)\|^2, 1 \le n \le N$$
(24)

$$\rho_S^{N+1,N+1}(t) = 1 - \sum_{n=1}^N \|c_{s_n}(t)\|^2, \tag{25}$$

where the first N diagonal elements correspond to the excited state measurement probabilities of the two-level atoms, represented in Fig. 2 by system qubits Q_{S_n} .

Quantum mapping algorithm implementation details

Here, we give further details on the implementation of the devised Q-MARINA quantum algorithm on the IBM Q Experience hardware. The comparison of the results for open system dynamics illustrated in Fig. 3 is obtained by implementing the quantum circuit on the IBM QASM simulator provided via Qiskit³⁹ library and contrasting it with the numerical solution of the Quantum Master Equation (QME) modeled in Quantum Toolbox in Python (QuTiP)^{37,38} on a classical computer. The combination of the system parameters—loss rate κ and coupling constant g—determine whether the light-matter interaction is considered to be in the weak or or the strong coupling regime. Concretely, in our case with N atoms $g\sqrt{N} < \kappa/4$ corresponds to the weak coupling strength, while for $g\sqrt{N} \ge \kappa/4$ we reach the strong coupling regime²⁴, particularly relevant for hybridization of light and matter explored in quantum light generation and extension of coherence in quantum memories. Thus, Fig. 3 compares the QME and the Q-MARINA QASM results for N=7 atoms in the strong coupling regime.

The Q-MARINA implementation on IBM Q hardware shown in Fig. 4 studies 3-qubit and 4-qubit circuits on one-to-all connected subgraphs of <code>ibmq_quito</code> (Falcon r4T processor) and <code>ibm_oslo</code> (Falcon r5.11H processor), respectively, simulates the N = 2 TC system in strong coupling regime and the N = 3 TC system in the borderline regime where an individual atom couples weakly, while the collective coupling is in the strong regime of the cavity QED. The atomic amplitudes follow the exact QME solution closely and leave space for future precision improvement via error mitigation techniques.

Data availibility

The datasets used and/or analysed during the current study available from the corresponding author on reasonable request.

Code availability

The underlying code for this study [and training/validation datasets] is not publicly available but may be made available to qualified researchers on reasonable request from the corresponding author.

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Author contributions

Both authors conceived of the idea, derived the method, implemented quantum simulations, analyzed data and wrote the manuscript.

Competing interests

All authors declare no financial or non-financial competing interests.

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