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Construction of diverse water wave structures for coupled nonlinear fractional Drinfel'd-Sokolov-Wilson model with Beta derivative and its modulus instability

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This paper aims to analyze the coupled nonlinear fractional Drinfel'd-Sokolov-Wilson (FDSW) model with beta derivative. The nonlinear FDSW equation plays an important role in describing dispersive water wave structures in mathematical physics and engineering, which is used to describe nonlinear surface gravity waves propagating over horizontal sea bed. We have applied the travelling wave transformation that converts the FDSW model to nonlinear ordinary differential equations. After that, we applied the generalized rational exponential function method (GERFM). Diverse types of soliton solution structures in the form of singular bright, periodic, dark, bell-shaped and trigonometric functions are attained via the proposed method. By selecting a suitable parametric value, the 3D, 2D and contour plots for some solutions are also displayed to visualize their nature in a better way. The modulation instability for the model is also discussed. The results show that the presented method is simple and powerful to get a novel soliton solution for nonlinear PDEs.

A solitary wave is a special type of wave that maintains its shape as it propagates through a medium, without changing its speed or amplitude. Solitary waves can arise in various fields, including water waves, metamaterials, engineering, plasma waves, and optical fibers[1–12]. In recent years, there has been increasing interest in the study of solitary waves in nonlinear fractional differential equations (NFDEs), which are differential equations involving fractional derivatives. NFDEs are generalizations of classical differential equations, in which the order of the derivative is not necessarily an integer. Solitary wave solutions of NFDEs have important applications in various fields, including physics, mathematics, engineering, and biology[13–20]. The study of solitary waves in NFDEs is a challenging task, due to the nonlinearity and fractional nature of these equations.

In recent few decades, many efficient methods or techniques have been used to find the analytical solutions for nonlinear models, such as the Ricatti approach[21], the Kudryashov method[22], the Darboux transformation[23], the Jacobi elliptic function approach[24], the sine-cosine approach[25], the direct algebraic technique[26], the extended tanh function method[27–31], sine-Gordon approach[32,33], Fokas technique[34], the Hirota bilinear transformation approach[35,36], the first integral approach[37], the trial solution technique[38], the $\left(\frac{G'}{G}\right)$-expansion approach[39], $\left(\frac{G'}{G^2}\right)$-expansion technique[40], $\left(\frac{G'}{G}, \frac{1}{G}\right)$-expansion technique[41–43], Lie Symmetry method[44], the unified method[45], and so on. The travelling wave solution of DSW was attained by utilizing the auxiliary equation method[46]. By utilizing the modified extended direct algebraic method bell, anti-bell, periodic and dark solitary wave solution of DSW

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has been attained in[47]. The series solution of the DSW model was attained by using the Adomian decomposition method[48].

The coupled (1+1)-dimensional DSW model[49] which read as,

$$\begin{aligned} \Phi_t + a\Psi\Psi_x &= 0 \\ \Psi_t + \gamma_1\Psi\Phi_x + \lambda_1\Phi\Psi_x + \eta_1\Psi_{xxx} &= 0. \end{aligned} \tag{1}$$

We can write the above system in the form of fractional derivative with respect to time is given by,

$$\begin{aligned} D_t^{\alpha}\Phi + a\Psi D_x\Psi &= 0 \\ D_t^{\alpha}\Psi + \gamma_1\Psi D_x\Phi + \lambda_1\Phi D_x\Psi + \eta_1 D_{xxx}\Psi &= 0. \end{aligned} \tag{2}$$

Here, a, γ_1, λ_1 and η_1 are the constant and the α represents the order of fractional derivative with $0 < \alpha \leq 1$. When $\alpha = 1$ Eq. (2) is converted to classical DSW equation, which was first introduced by Drinfel'd and Sokolov[50,51] and studied by Wilson[52]. In this article, we will construct an exact solution for the Drinfel'd-Sokolov-Wilson model using the generalized rational exponential function method approach with the help of well-known Beta derivative. The solutions are attained in the form of singular bright, dark, periodic, bell and lump-type water wave structures. The achieved solutions might be useful to comprehend nonlinear phenomena. It is worth noting that the implemented method for solving NPDEs is efficient, and simple to find further and new-fangled solutions in the area of mathematical physics and coastal engineering. Diverse types of fractional derivatives have been used in the past, such as Caputo fractional[53], Beta derivative[54], Conformable fractional[55], Reimann-Liouville[56] and truncated M-fractional derivative[57] etc. have importance in fractional calculus.

The remaining article is distributed into various sections. Section (2) contain definition from fractional calculus relevant to our study. In Sect. (3) we have discussed the main step of the method. In Sect. (4) solitary wave solutions have been described. Numerical simulations of some attained solutions are given in (5). In Sects. (6) and (7) modulus instability, a conclusion is presented.

Beta derivative

Definition Let $\Pi(t)$ be a function defined for all non-negative t. The function $\Pi(t)$[58] is,

$$D_t^{\alpha}\{\Pi(t)\} = \lim_{\varepsilon\to 0}\frac{\Pi(t+\epsilon(t+\frac{1}{\Gamma(\alpha)})^{1-\alpha}) - \Pi(t)}{\varepsilon}, \tag{3}$$

Theorem *Let Π and g be any two function, $\Pi \neq 0$, and $\alpha \in (0,1]$ then*

1: $D_t^{\alpha}\{b_1\Pi(t) + b_2\Upsilon(t)\} = b_1 D_t^{\alpha}\Pi(t) + b_2 D_t^{\alpha}\Upsilon(t)$,

where $b_1, b_2 \in \Re$

2: $D_t^{\alpha}\{\Pi(t).\Upsilon(t)\} = \Pi(t)D_t^{\alpha}\{\Upsilon(t)\} + \Upsilon(t)D_t^{\alpha}\{\Pi(t)\}$,

3: *For c any constant, the following relation can be easily satisfied* $D_t^{\alpha}c = 0$,

4: $D_t^{\alpha}(\frac{\Pi(t)}{\Upsilon(t)}) = \frac{\Upsilon(t)D_t^{\alpha}\{\Pi(t)\} - \Pi(t)D_t^{\alpha}\{\Upsilon(t)\}}{\Upsilon(t)^2}$,

5: $D_t^{\alpha}\{\Pi(t)\} = (t+\frac{1}{\Gamma(\alpha)})^{1-\alpha}\frac{d\Pi(t)}{dt}$,

Methodology

The GERF method is a quite novel technique for nonlinear partial differential equations (NLPDE)[49]. The main steps are given as:

Step:1

Consider the NLPDE as,

$$H(\Omega, \Omega_x, \Omega_t, \Omega_{xx}, \Omega_{tt} ...) = 0. \tag{4}$$

Suppose the travelling wave transformation,

$$\Omega(x,t) = \Psi(\varpi)e^{\iota\phi(x,t)}. \tag{5}$$

Substituting (5) into (4) then we get ODE given as,

$$F(\Psi, \Psi', \Psi'', \Psi''', ...) = 0. \tag{6}$$

Step:2

Solution of equation of (7) is,

$$\Psi(\varpi) = a_0 + \sum_{n=1}^{N} (a_n \phi(\varpi)^n + b_n \phi(\varpi)^{-n}). \tag{7}$$

Here, a_0, a_n, and b_n are unknown parameters to be found. The function $\phi(\varpi)$ is defined as

$$\phi(\varpi) = \frac{\mu_1 e^{\sigma_1 \varpi} + \mu_2 e^{\sigma_2 \varpi}}{\mu_2 e^{\sigma_2 \varpi} + \mu_3 e^{\sigma_3 \varpi}}. \tag{8}$$

Step:3

We apply the homogeneous balance technique on (7) to attain the value of N.

Step:4 Substituting (7) with equation (8) into (6), then we attain the system of algebraic equations. The system is solved by utilizing Mathematica software, and then the achieved solution of (8) is put into (7) by using (5). Finally, the solution of (4) is attained.

Solitary wave structure

We consider the travelling wave transformation for FDSW (2) as follows,

$$\Phi(x,t) = \Phi(\varpi), \;\; \Psi(x,t) = \Psi(\varpi), \;\; \varpi = \kappa_1 \left(x + \frac{\omega_1}{\alpha} (t + \frac{1}{\Gamma(\alpha)})^{\alpha} \right). \tag{9}$$

Using (9) to (2) and then we get ,

$$a\kappa_1 \Psi \Psi^{'} - \kappa_1 \omega_1 \Phi^{'} = 0. \tag{10}$$

$$\lambda_1 \kappa_1 \Phi \Psi^{'} + \varpi_1 \kappa_1^3 \Psi^{'''} - \kappa_1 \omega_1 \Psi^{'} + \Psi \gamma_1 \kappa_1 \Phi^{'} = 0. \tag{11}$$

From (10), we have

$$\Phi = \frac{a\Psi^2}{2\omega_1}. \tag{12}$$

Putting the value of Φ into (11) and integrating one time then we get,

$$6\varpi_1 \kappa_1^2 \omega_1 \Psi^{''} - 6\omega_1^2 \Psi + a(\lambda_1 + 2\gamma_1)\Psi^3 = 0. \tag{13}$$

Now we have to apply the balancing technique on (13) then we get $N = 1$. Utilizing $N = 1$ in (7) then we get,

$$\Psi(\varpi) = a_0 + a_1 \phi(\varpi) + b_1 \phi(\varpi)^{-1}. \tag{14}$$

where a_0, a_1, and b_1 are unknown constants to be find. The solution of (2) is discussed as,

Case-1 If $[\sigma_1, \sigma_2, \sigma_3, \sigma_4]$=$[1, -1, 1, 1]$ and $[\mu_1, \mu_2, \mu_3, \mu_4]$=$[1, -1, 1, -1]$ then (8) become,

$$\phi(\varpi) = \text{Tanh}(\varpi). \tag{15}$$

When equations (14) and (15) are putting into equation (13), we arrive at a system of algebraic linear equations. By solving these equations simultaneously, we obtain the following set of solitary wave solutions. **set-1**

$$a_0 = 0, b_1 = -a_1, a_1 = a_1, \gamma_1 = \frac{-aa_1^2 \lambda_1 - 48\varpi_1^2 \kappa_1^4}{2aa_1^2}, \omega_1 = 4\varpi_1 \kappa_1^2. \tag{16}$$

Putting (16) into (14) then solution of (2) is,

$$\Psi(\varpi) = a_1(-\text{csch}(\varpi))\text{sech}(\varpi), \;\; \varpi = \kappa_1 \left(x + \frac{4\varpi_1 \kappa_1^2}{\alpha} (t + \frac{1}{\Gamma(\alpha)})^{\alpha} \right). \tag{17}$$

$$\Phi(\varpi) = \frac{aa_1^2 \text{csch}^2(2\varpi)}{2\varpi_1 \kappa_1^2}, \;\; \varpi = \kappa_1 \left(x + \frac{4\varpi_1 \kappa_1^2}{\alpha} (t + \frac{1}{\Gamma(\alpha)})^{\alpha} \right). \tag{18}$$

Set-2

$$a_0 = 0, b_1 = a_1, a_1 = a_1, \gamma_1 = \frac{96\varpi_1^2 \kappa_1^4 - aa_1^2 \lambda_1}{2aa_1^2}, \omega_1 = -8\varpi_1 \kappa_1^2. \tag{19}$$

Substituting (19) into (14) then solution of (2) is,

$$\Psi(\varpi) = a_1(\tanh(\varpi) + \coth(\varpi), \;\; \varpi = \kappa_1 \left(x - \frac{8\varpi_1 \kappa_1^2}{\alpha} (t + \frac{1}{\Gamma(\alpha)})^{\alpha} \right). \tag{20}$$

$$\Phi(\varpi) = -\frac{aa_1^2(\tanh(\varpi)+\coth(\varpi))^2}{16\varpi_1\kappa_1^2}, \quad \varpi = \kappa_1\left(x - \frac{8\varpi_1\kappa_1^2}{\alpha}(t+\frac{1}{\Gamma(\alpha)})^\alpha\right). \tag{21}$$

Set-3

$$a_0 = 0, a_1 = a_1 b_1 = 0, \gamma_1 = \frac{24\varpi_1^2\kappa_1^4 - aa_1^2\lambda_1}{2aa_1^2}, \omega_1 = -2\varpi_1\kappa_1^2. \tag{22}$$

Putting (22) into (14) then solution of (2) is,

$$\Psi(\varpi) = a_1 \mathrm{Tanh}(\varpi), \quad \varpi = \kappa_1\left(\mathrm{x} - \frac{2\varpi_1\kappa_1^2}{\alpha}(\mathrm{t}+\frac{1}{\Gamma(\alpha)})^\alpha\right). \tag{23}$$

$$\Phi(\varpi) = -\frac{aa_1^2\tanh^2(\varpi)}{4\varpi_1\kappa_1^2}, \quad \varpi = \kappa_1\left(x - \frac{2\varpi_1\kappa_1^2}{\alpha}(t+\frac{1}{\Gamma(\alpha)})^\alpha\right). \tag{24}$$

Set-4

$$a_0 = 0, a_1 = 0, b_1 = b_1, \gamma_1 = \frac{24\varpi_1^2\kappa_1^4 - ab_1^2\lambda_1}{2ab_1^2}, \omega_1 = -2\varpi_1\kappa_1^2. \tag{25}$$

Substituting (25) into (14) then solution of (2) is,

$$\Psi(\varpi) = b_1\coth(\varpi), \quad \varpi = \kappa_1\left(x - \frac{2\varpi_1\kappa_1^2}{\alpha}(t+\frac{1}{\Gamma(\alpha)})^\alpha\right). \tag{26}$$

$$\Phi(\varpi) = -\frac{ab_1^2\coth^2(\varpi)}{4\varpi_1\kappa_1^2}, \quad \varpi = \kappa_1\left(x - \frac{2\varpi_1\kappa_1^2}{\alpha}(t+\frac{1}{\Gamma(\alpha)})^\alpha\right). \tag{27}$$

Case-2 If $[\sigma_1,\sigma_2,\sigma_3,\sigma_4] = [\iota,-\iota,1,1]$ and $[\mu_1,\mu_2,\mu_3,\mu_4] = [\iota,-\iota,\iota,-\iota]$ then (8) become,

$$\phi(\varpi) = -\mathrm{Tan}(\varpi). \tag{28}$$

When equations (28) and (15) are putting into equation (13), we arrive at a system of algebraic linear equations. By solving these equations simultaneously, we obtain the following set of solitary wave solutions.

Set-1

$$a_0 = 0, b_1 = -a_1, a_1 = a_1, \gamma_1 = -\frac{2\left(aa_1^2\gamma_1 + 48\varpi_1^2\kappa_1^4\right)}{aa_1^2}, \omega_1 = 8\varpi_1\kappa_1^2. \tag{29}$$

Putting (29) into (14) then solution of (2) is,

$$\Psi(\varpi) = a_1\cos(2\varpi)\csc(\varpi)\sec(\varpi), \quad \varpi = \kappa_1\left(x + \frac{8\varpi_1\kappa_1^2}{\alpha}(t+\frac{1}{\Gamma(\alpha)})^\alpha\right). \tag{30}$$

$$\Phi(\varpi) = \frac{aa_1^2\cot^2(2\varpi)}{4\varpi_1\kappa_1^2}, \quad \varpi = \kappa_1\left(x + \frac{8\varpi_1\kappa_1^2}{\alpha}(t+\frac{1}{\Gamma(\alpha)})^\alpha\right). \tag{31}$$

Set-2

$$a_0 = 0, b_1 = a_1, a_1 = a_1, \gamma_1 = -\frac{2\left(aa_1^2\gamma_1 - 24\varpi_1^2\kappa_1^4\right)}{aa_1^2}, \omega_1 = -4\varpi_1\kappa_1^2. \tag{32}$$

Substituting (32) into (14) then solution of (2) is,

$$\Psi(\varpi) = a_1(-\csc(\varpi))\sec(\varpi), \quad \varpi = \kappa_1\left(x - \frac{4\varpi_1\kappa_1^2}{\alpha}(t+\frac{1}{\Gamma(\alpha)})^\alpha\right). \tag{33}$$

$$\Phi(\varpi) = -\frac{aa_1^2\csc^2(2\varpi)}{2\varpi_1\kappa_1^2}, \quad \varpi = \kappa_1\left(x - \frac{4\varpi_1\kappa_1^2}{\alpha}(t+\frac{1}{\Gamma(\alpha)})^\alpha\right). \tag{34}$$

Set-3

$$a_0 = 0, a_1 = a_1, b_1 = 0, \gamma_1 = -\frac{2\left(aa_1^2\gamma_1 + 12\varpi_1^2\kappa_1^4\right)}{aa_1^2}, \omega_1 = 2\varpi_1\kappa_1^2. \tag{35}$$

Putting Eq. (35) into (14) then solution of (2) is,

$$\Psi(\varpi) = -a_1\mathrm{Tan}(\varpi),\ \ \varpi = \kappa_1\left(\mathrm{x} + \frac{2\varpi_1\kappa_1^2}{\alpha}(\mathrm{t} + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{36}$$

$$\Phi(\varpi) = \frac{a\tan^2(\varpi)}{4\varpi_1\kappa_1^2},\ \ \varpi = \kappa_1\left(x + \frac{2\varpi_1\kappa_1^2}{\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{37}$$

Set-4

$$a_0 = 0, a_1 = 0, b_1 = b_1, \gamma_1 = -\frac{2\left(ab_1^2\gamma_1 + 12\varpi_1^2\kappa_1^4\right)}{ab_1^2}, \omega_1 = 2\varpi_1\kappa_1^2. \tag{38}$$

Substituting (38) into (14) then solution of (2) is,

$$\Psi(\varpi) = b_1(-\cot(\varpi)),\ \ \varpi = \kappa_1\left(x + \frac{2\varpi_1\kappa_1^2}{\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{39}$$

$$\Phi(\varpi) = \frac{ab_1^2\cot^2(\varpi)}{4\varpi_1\kappa_1^2},\ \ \varpi = \kappa_1\left(x + \frac{2\varpi_1\kappa_1^2}{\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{40}$$

Case-3 If $[\sigma_1, \sigma_2, \sigma_3, \sigma_4] = [1 + \imath, 1 - \imath, 1, 1]$ and $[\mu_1, \mu_2, \mu_3, \mu_4] = [\imath, -\imath, \imath, -\imath]$ then (8) become,

$$\phi(\varpi) = 1 - \mathrm{Tan}(\varpi). \tag{41}$$

When equations (41) and (15) are putting into equation (13), we arrive at a system of algebraic linear equations. By solving these equations simultaneously, we obtain the following set of solitary wave solutions.

Set-1

$$a_0 = -a_1, a_1 = a_1, b_1 = 0, \lambda_1 = -\frac{2\left(aa_1^2\gamma_1 + 12\varpi_1^2\kappa_1^4\right)}{aa_1^2}, \omega_1 = 2\varpi_1\kappa_1^2. \tag{42}$$

Putting (42) into (14) then solution of (2) is,

$$\Psi(\varpi) = -a_1\mathrm{Tan}(\varpi),\ \ \varpi = \kappa_1\left(\mathrm{x} + \frac{2\varpi_1\kappa_1^2}{\alpha}(\mathrm{t} + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{43}$$

$$\Phi(\varpi) = \frac{aa_1^2\tan^2(\varpi)}{4\varpi_1\kappa_1^2},\ \ \varpi = \kappa_1\left(x + \frac{2\varpi_1\kappa_1^2}{\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{44}$$

Case-4 If $[\sigma_1, \sigma_2, \sigma_3, \sigma_4] = [2 + \imath, 2 - \imath, 1, 1]$ and $[\mu_1, \mu_2, \mu_3, \mu_4] = [\imath, -\imath, \imath, -\imath]$ then (8) become,

$$\phi(\varpi) = 2 + \mathrm{Tan}(\varpi). \tag{45}$$

When equations (45) and (15) are putting into equation (13), we arrive at a system of algebraic linear equations. By solving these equations simultaneously, we obtain the following set of solitary wave solutions.

Set-1

$$a_0 = -2a_1, a_1 = a_1, b_1 = 0, \lambda_1 = -\frac{2\left(aa_1^2\gamma_1 + 12\varpi_1^2\kappa_1^4\right)}{aa_1^2}, \omega_1 = 2\varpi_1\kappa_1^2. \tag{46}$$

Substituting (46) into (14) then solution of (2) is,

$$\Psi(\varpi) = a_1\mathrm{Tan}(\varpi),\ \ \varpi = \kappa_1\left(\mathrm{x} + \frac{2\varpi_1\kappa_1^2}{\alpha}(\mathrm{t} + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{47}$$

$$\Phi(\varpi) = \frac{aa_1^2\tan^2(\varpi)}{4\varpi_1\kappa_1^2},\ \ \varpi = \kappa_1\left(x + \frac{2\varpi_1\kappa_1^2}{\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{48}$$

Case-5 If $[\sigma_1, \sigma_2, \sigma_3, \sigma_4] = [2, 1, 1, 1]$ and $[\mu_1, \mu_2, \mu_3, \mu_4] = [1, 0, 1, 0]$ then (8) become,

$$\phi(\varpi) = \frac{2e^{\varpi} + 1}{e^{\varpi} + 1}. \tag{49}$$

When equations (49) and (15) are putting into equation (13), we arrive at a system of algebraic linear equations. By solving these equations simultaneously, we obtain the following set of solitary wave solutions.

Set-1

$$a_0 = -\frac{1}{4}(3b_1), a_1 = 0, b_1 = b_1, \lambda_1 = -\frac{2\left(ab_1^2\gamma_1 - 12\varpi_1^2\kappa_1^4\right)}{ab_1^2}, \omega_1 = -\frac{1}{2}\varpi_1\kappa_1^2. \tag{50}$$

Putting (50) into (14) then solution of (2) is,

$$\Psi(\varpi) = \frac{b_1(1-2e^{\varpi})}{8e^{\varpi}+4}, \quad \varpi = \kappa_1\left(x - \frac{\varpi_1\kappa_1^2}{2\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{51}$$

$$\Phi(\varpi) = -\frac{ab_1^2(1-2e^{\varpi})^2}{(8e^{\varpi}+4)^2\varpi_1\kappa_1^2}, \quad \varpi = \kappa_1\left(x - \frac{\varpi_1\kappa_1^2}{2\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{52}$$

Set-2

$$a_0 = -\frac{1}{2}(3a_1), b_1 = 0, a_1 = a_1, \lambda_1 = -\frac{2\left(aa_1^2\gamma_1 - 3\varpi_1^2\kappa_1^4\right)}{aa_1^2}, \omega_1 = -\frac{1}{2}\varpi_1\kappa_1^2. \tag{53}$$

Putting (53) into (14) then solution of (2) is,

$$\Psi(\varpi) = \frac{a_1(e^{\varpi}-1)}{2(e^{\varpi}+1)}, \quad \varpi = \kappa_1\left(x - \frac{\varpi_1\kappa_1^2}{2\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{54}$$

$$\Phi(\varpi) = -\frac{aa_1^2(e^{\varpi}-1)^2}{4(e^{\varpi}+1)^2\varpi_1\kappa_1^2}, \quad \varpi = \kappa_1\left(x - \frac{\varpi_1\kappa_1^2}{2\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{55}$$

Case-6 If $[\sigma_1, \sigma_2, \sigma_3, \sigma_4] = [2, 0, 1, 1]$ and $[\mu_1, \mu_2, \mu_3, \mu_4] = [-1, 0, 1, -1]$ then (8) become,

$$\phi(\varpi) = 1 - \tanh(\varpi). \tag{56}$$

When equations (56) and (15) are putting into equation (13), we arrive at a system of algebraic linear equations. By solving these equations simultaneously, we obtain the following set of solitary wave solutions.

Set-1

$$a_0 = -a_1, a_1 = a_1, b_1 = 0, \lambda_1 = -\frac{2\left(aa_1^2\gamma_1 - 12\varpi_1^2\kappa_1^4\right)}{aa_1^2}, \omega_1 = -2\varpi_1\kappa_1^2. \tag{57}$$

Putting (57) into (14) then solution of (2) is,

$$\Psi(\varpi) = -a_1\mathrm{Tanh}(\varpi), \quad \varpi = \kappa_1\left(\mathrm{x} - \frac{2\varpi_1\kappa_1^2}{\alpha}(\mathrm{t} + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{58}$$

$$\Phi(\varpi) = -\frac{aa_1^2\tanh^2(\varpi)}{4\varpi_1\kappa_1^2}, \quad \varpi = \kappa_1\left(x - \frac{2\varpi_1\kappa_1^2}{\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{59}$$

Case-7 If $[\sigma_1, \sigma_2, \sigma_3, \sigma_4] = [-3, -1, -1, 1]$ and $[\mu_1, \mu_2, \mu_3, \mu_4] = [-1, 1, -1, 1]$ then (8) become,

$$\phi(\varpi) = \tanh(\varpi) - 2. \tag{60}$$

When equations (60) and (15) are putting into equation (13), we arrive at a system of algebraic linear equations. By solving these equations simultaneously, we obtain the following set of solitary wave solutions.

Set-1

$$a_0 = 2a_1, a_1 = a_1, b_1 = 0, \omega_1 = -\frac{\sqrt{a}a_1\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}}, \varpi_1 = \frac{\sqrt{a}a_1\sqrt{2\gamma_1+\lambda_1}}{2\sqrt{6}\kappa_1^2}. \tag{61}$$

Putting (61) into (14) then solution of (2) is,

$$\Psi(\varpi) = a_1\mathrm{Tanh}(\varpi), \quad \varpi = \kappa_1\left(\mathrm{x} - \frac{\sqrt{\mathrm{a}}\mathrm{a}_1\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}\alpha}(\mathrm{t} + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{62}$$

$$\Phi(\varpi) = -\frac{\sqrt{\frac{3}{2}}\sqrt{a}a_1\tanh^2(\varpi)}{\sqrt{2\gamma_1+\lambda_1}}, \quad \varpi = \kappa_1\left(x - \frac{\sqrt{a}a_1\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}\alpha}(t + \frac{1}{\Gamma(\alpha)})^{\alpha}\right). \tag{63}$$

Set-2

$$a_0 = 2a_1, a_1 = a_1, b_1 = 0, \omega_1 = \frac{\sqrt{a}a_1\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}}, \varpi_1 = \frac{\sqrt{a}a_1\sqrt{2\gamma_1+\lambda_1}}{2\sqrt{6}\kappa_1^2}. \quad (64)$$

Putting (64) into (14) then solution of (2) is,

$$\Psi(\varpi) = a_1 \mathrm{Tanh}(\varpi), \quad \varpi = \kappa_1\left(\mathrm{x} + \frac{\sqrt{\mathrm{a}}\mathrm{a}_1\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}\alpha}(\mathrm{t} + \frac{1}{\Gamma(\alpha)})^\alpha\right). \quad (65)$$

$$\Phi(\varpi) = \frac{\sqrt{\frac{3}{2}}\sqrt{a}a_1\tanh^2(\varpi)}{\sqrt{2\gamma_1+\lambda_1}}, \quad \varpi = \kappa_1\left(x + \frac{\sqrt{a}a_1\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}\alpha}(t + \frac{1}{\Gamma(\alpha)})^\alpha\right). \quad (66)$$

Case-8 If $[\sigma_1, \sigma_2, \sigma_3, \sigma_4] = [1, 0, 1, 1]$ and $[\mu_1, \mu_2, \mu_3, \mu_4] = [0, 0, 1, 0]$ then (8) become,

$$\phi(\varpi) = \frac{1}{1+e^{\varpi}}. \quad (67)$$

When equations (67) and (15) are putting into equation (13), we arrive at a system of algebraic linear equations. By solving these equations simultaneously, we obtain the following set of solitary wave solutions.

Set-1

$$a_0 = a_0, a_1 = -2a_0, b_1 = 0, \omega_1 = -\frac{\sqrt{a}a_0\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}}, \varpi_1 = \frac{\sqrt{\frac{2}{3}}\sqrt{a}a_0\sqrt{2\gamma_1+\lambda_1}}{\kappa_1^2}. \quad (68)$$

Putting (68) into (14) then solution of (2) is,

$$\Psi(\varpi) = a_0(1 - \frac{2}{1+e^{\varpi}}), \quad \varpi = \kappa_1\left(x - \frac{\sqrt{a}a_1\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}\alpha}(t + \frac{1}{\Gamma(\alpha)})^\alpha\right). \quad (69)$$

$$\Phi(\varpi) = -\frac{\sqrt{\frac{3}{2}}\sqrt{a}a_0\left(1 - \frac{2}{e^{\varpi}+1}\right)^2}{\sqrt{2\gamma_1+\lambda_1}}, \quad \varpi = \kappa_1\left(x - \frac{\sqrt{a}a_0\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}\alpha}(t + \frac{1}{\Gamma(\alpha)})^\alpha\right). \quad (70)$$

Set-2

$$a_0 = a_0, a_1 = -2a_0, b_1 = 0, \omega_1 = \frac{\sqrt{a}a_0\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}}, \varpi_1 = -\frac{\sqrt{\frac{2}{3}}\sqrt{a}a_0\sqrt{2\gamma_1+\lambda_1}}{\kappa_1^2}. \quad (71)$$

Substituting (71) into (14) then solution of (2) is,

$$\Psi(\varpi) = a_0(1 - \frac{2}{1+e^{\varpi}}), \quad \varpi = \kappa_1\left(x + \frac{\sqrt{a}a_0\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}\alpha}(t + \frac{1}{\Gamma(\alpha)})^\alpha\right). \quad (72)$$

$$\Phi(\varpi) = \frac{\sqrt{\frac{3}{2}}\sqrt{a}a_0\left(1 - \frac{2}{e^{\varpi}+1}\right)^2}{\sqrt{2\gamma_1+\lambda_1}}, \quad \varpi = \kappa_1\left(x + \frac{\sqrt{a}a_0\sqrt{2\gamma_1+\lambda_1}}{\sqrt{6}\alpha}(t + \frac{1}{\Gamma(\alpha)})^\alpha\right). \quad (73)$$

Numerical simulation and discussion

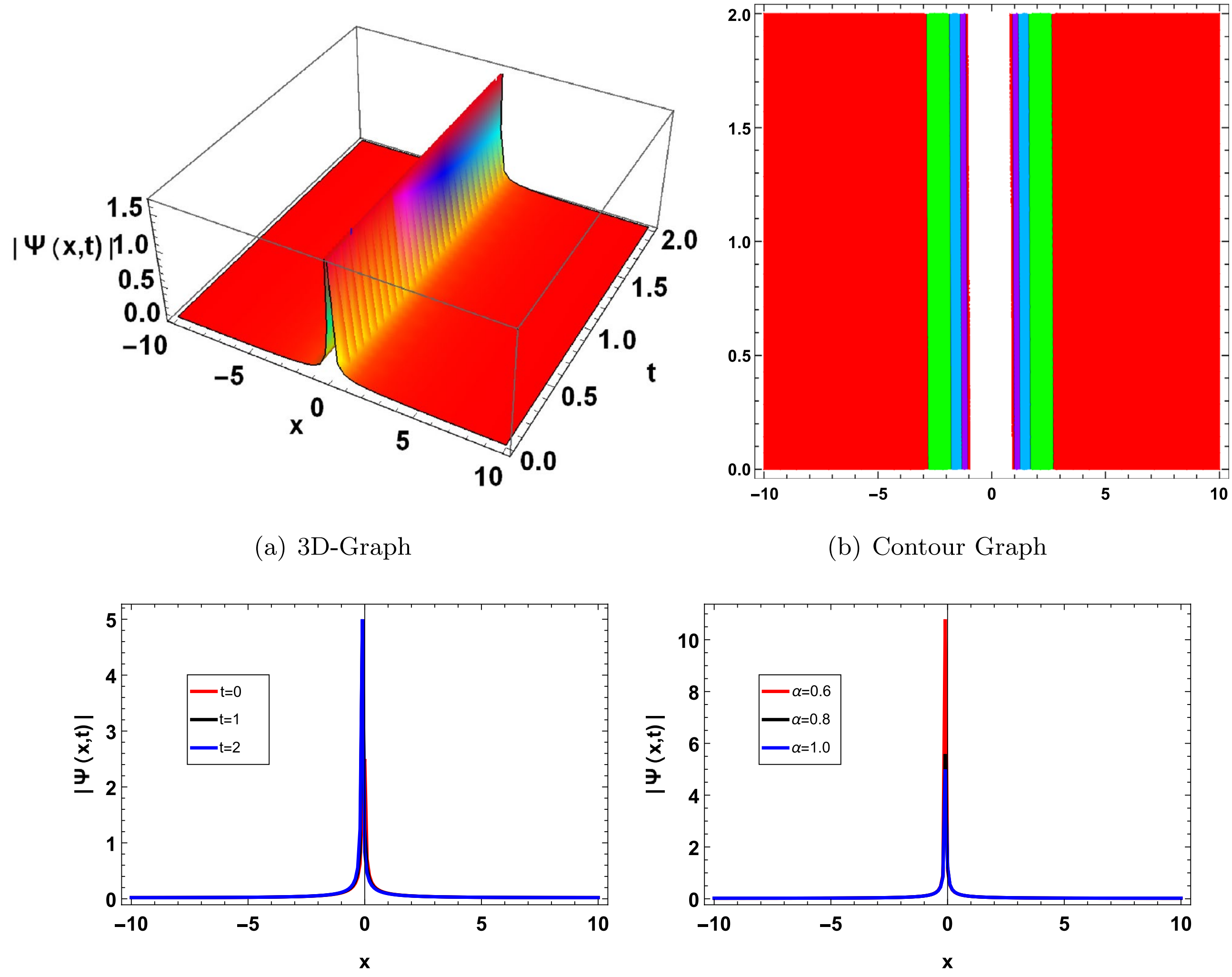
In this section, we have drawn the graph of some attained solutions for the structure solution of solitary waves. The value fractional parameter $\alpha = 1$ is fixed in all 2D graphs. Figs. (1 and 2) shows the singular bright soliton wave structure. Figures 3,4,6, 5, 7 and 8 shows the dark, periodic, bell and lump type soliton wave structure. In[59] authors have attained the bright soliton solutions of the FDSW model by using the homotopy analysis transform method. Similarly in[60] authors have achieved bright type soliton solution with the help of the Laplace Adomian decomposition method. Periodic-type soliton solutions have been attained by using the sine-cosine method[61]. But in this study, we get more generalized soliton solutions such as bright, dark, periodic, bell and lump.

Modulus instability

We have found the modulation instability of the coupled nonlinear DSW model (1) through linear stability. We consider the steady-state solution,

$$\begin{cases} \Phi(x,t) = \sqrt{P} + u(x,t)e^{P\delta\epsilon t} \\ \Psi(x,t) = \sqrt{P} + v(x,t)e^{P\delta\epsilon t}. \end{cases} \quad (74)$$

Substituting (74) into (1) then after linearize we get,



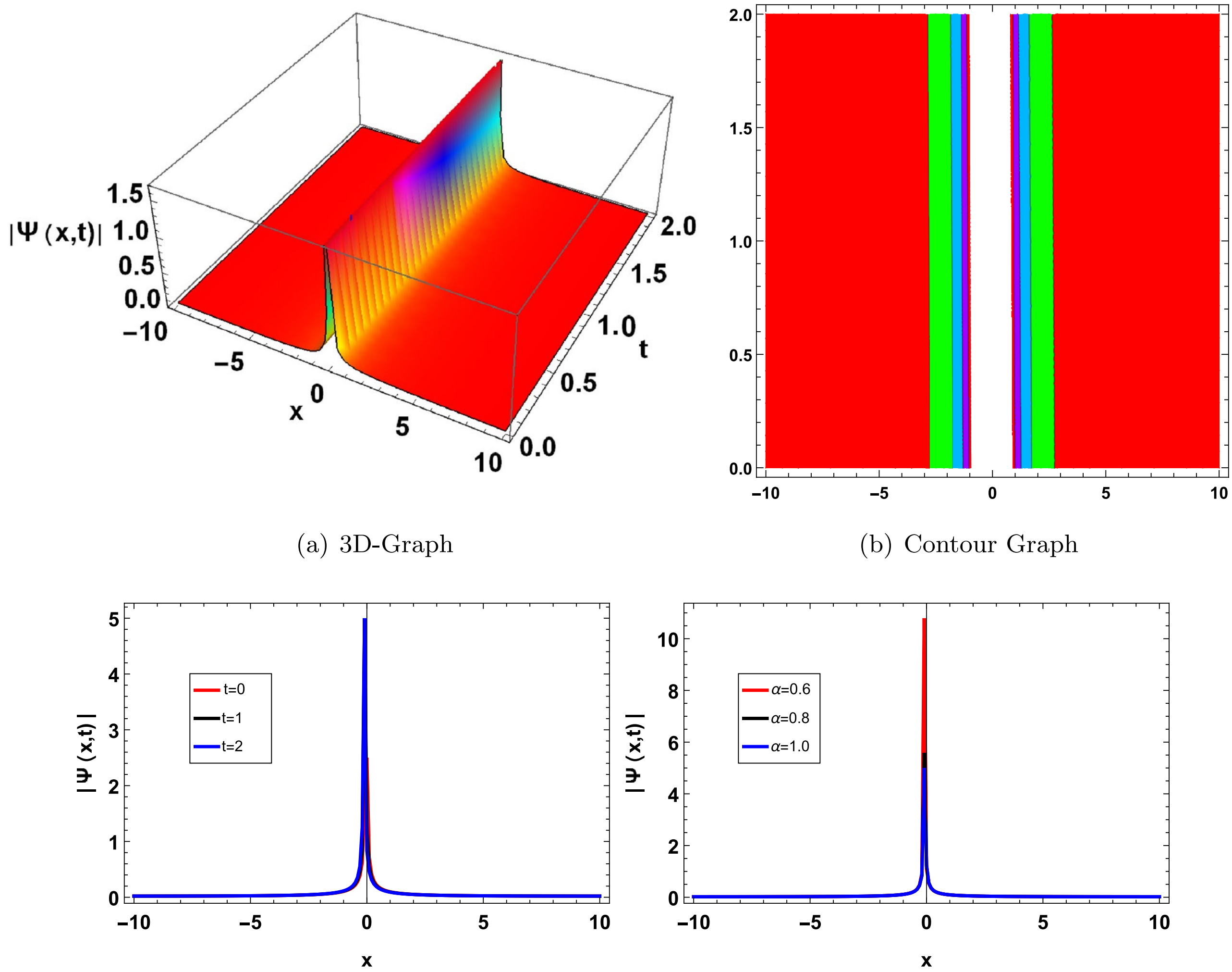
(a) 3D-Graph (b) Contour Graph

(c) 2D-Graph (d) 2D-Graph

Figure 1. Graphical solution of (20) with parameters $\kappa_1 = -0.1, \varpi_1 = -0.5, a_1 = 0.01$.

$$\begin{cases} u_t + P\delta\epsilon u + a\sqrt{P}v_x = 0 \\ v_t + P\delta\epsilon v + \gamma_1\sqrt{P}u_x + \lambda_1\sqrt{P}v_x + \varpi_1 v_{xxx} = 0. \end{cases} \tag{75}$$

It is supposed that the solution of (75) has as,



(a) 3D-Graph (b) Contour Graph

(c) 2D-Graph (d) 2D-Graph

Figure 2. Graphical solution of (21) with parameters $\kappa_1 = 0.2, \varpi_1 = -0.8, a_1 = 0.1, a = 0.5$.

$$\begin{cases} u(x,t) = \rho_1 e^{\kappa x - \omega t} \\ v(x,t) = \rho_2 e^{\kappa x - \omega t}, \end{cases} \tag{76}$$

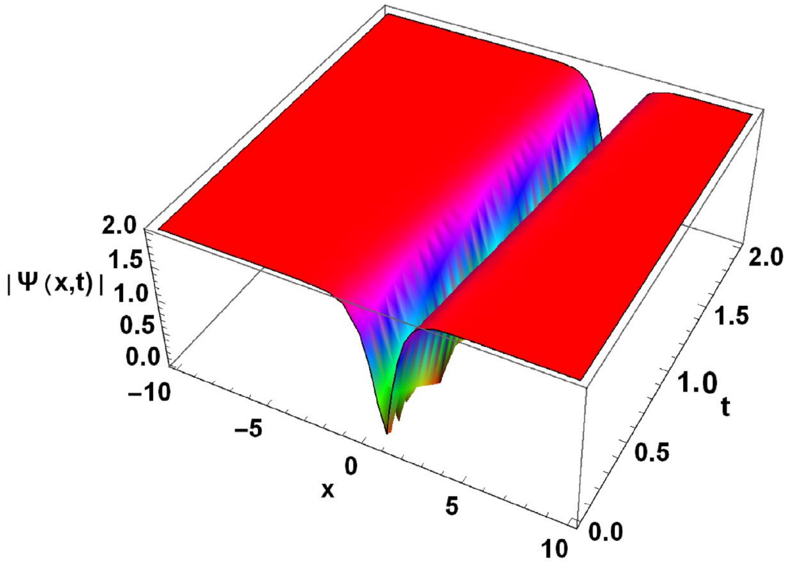
where κ and ω are the wave number and frequency of perturbation. Putting (76) into (75), the dispersion relation (DR) is acquired as

$$\omega = \frac{\rho_2\left(a\kappa\sqrt{P} + \varpi_1\kappa^3 + \lambda_1 + \delta P\epsilon\right) + \rho_1\left(\gamma_1\kappa\sqrt{P} + \delta P\epsilon\right)}{\rho_1 + \rho_2}, \tag{77}$$

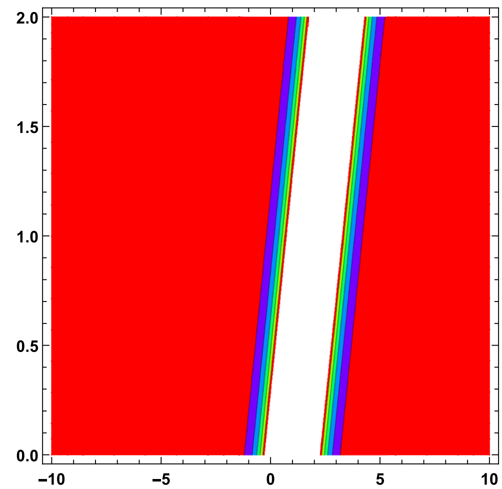
from (77), one can see that the real component is negative for all values of κ then any superposition of the results will appear to decay. So, the dispersion is stable.

Conclusion

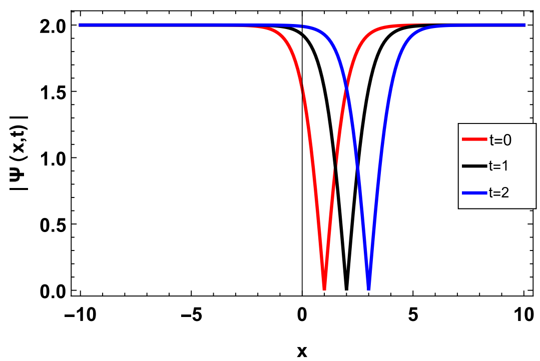
In this work, we have successfully achieved some fresh and further general traveling wave solutions to the nonlinear fractional Drinfel'd-Sokolov-Wilson (FDSW) model with beta derivative. The solutions attained by using the GERF method for the proposed model are competent to examine the scientific model of gravity water waves in shallow water. It is capable of investigating plasma waves in the seaside oceans and breaking down the unidirectional spread of long waves in oceans and harbors. The proposed method is not only more powerful than previous approaches but has also introduced novel solutions that have not been reported before.



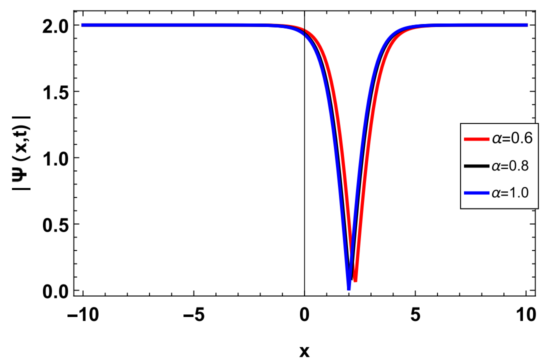
(a) 3D-Graph



(b) Contour Graph



(c) 2D-Graph



(d) 2D-Graph

Figure 3. Graphical solution of (23) with parameters $\kappa_1 = 1, \varpi_1 = 0.5, a_1 = 2$.

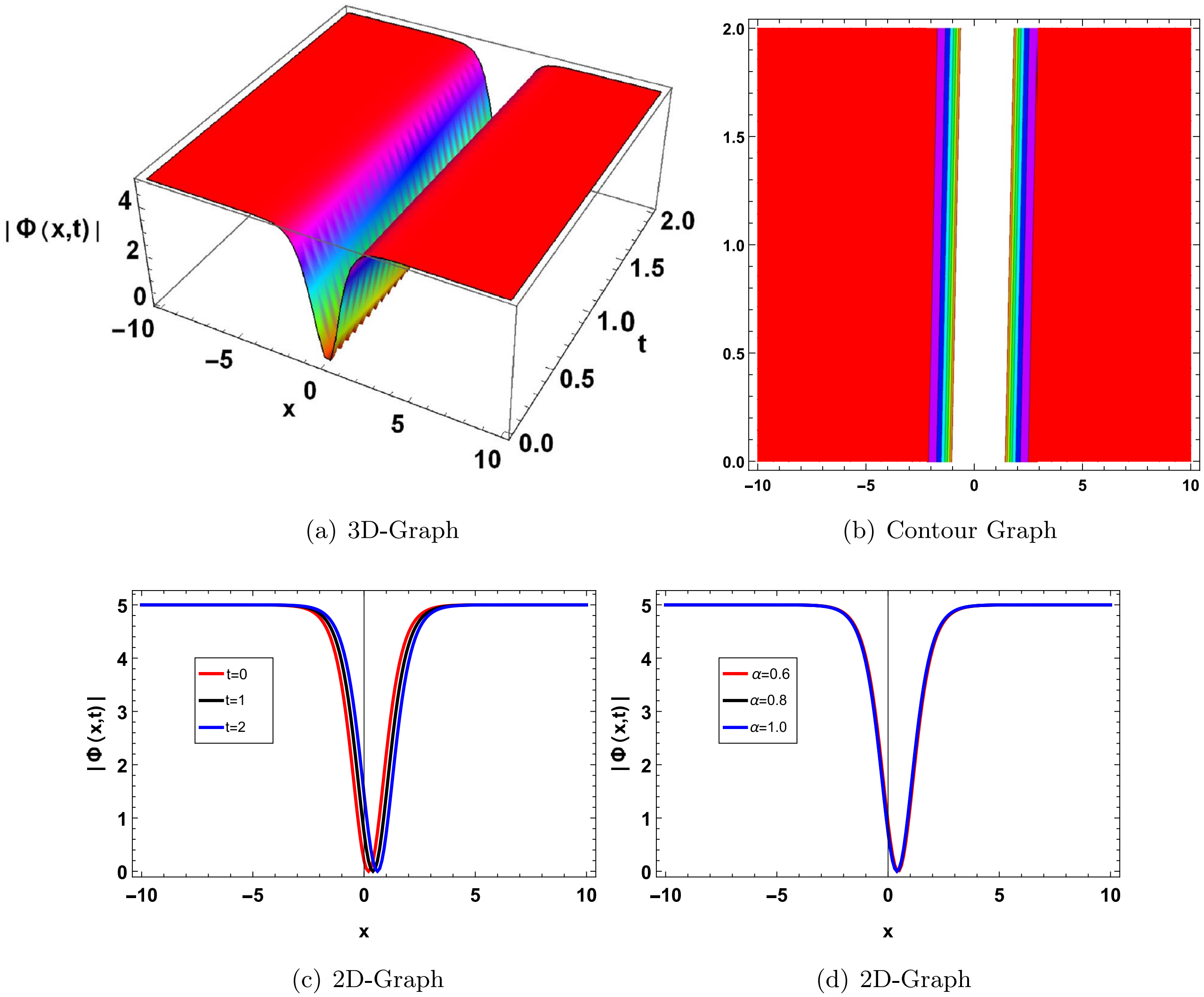
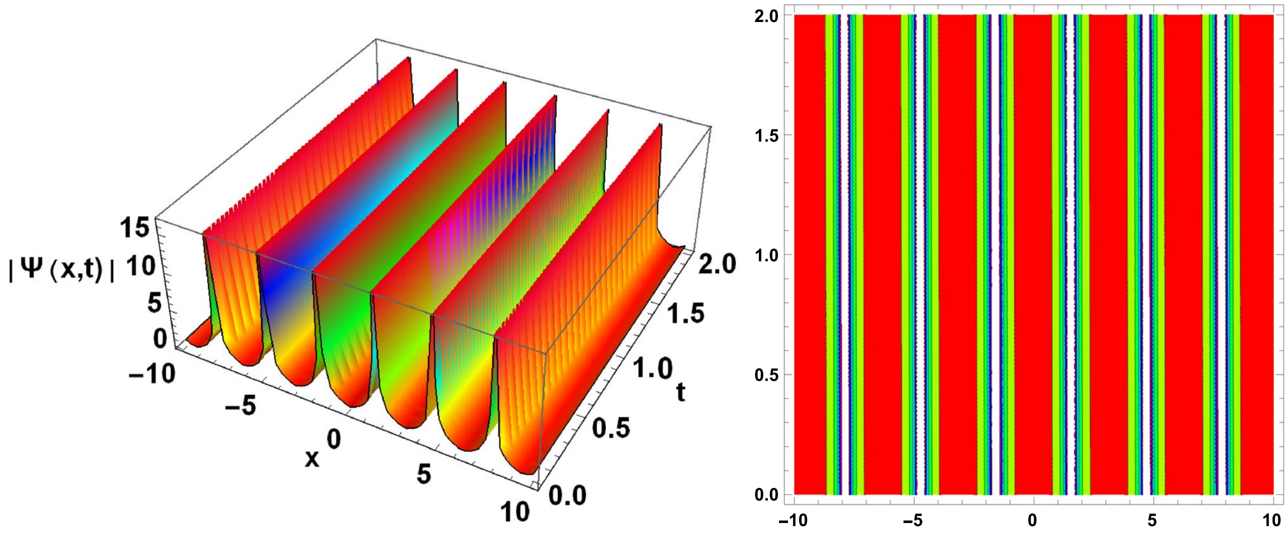
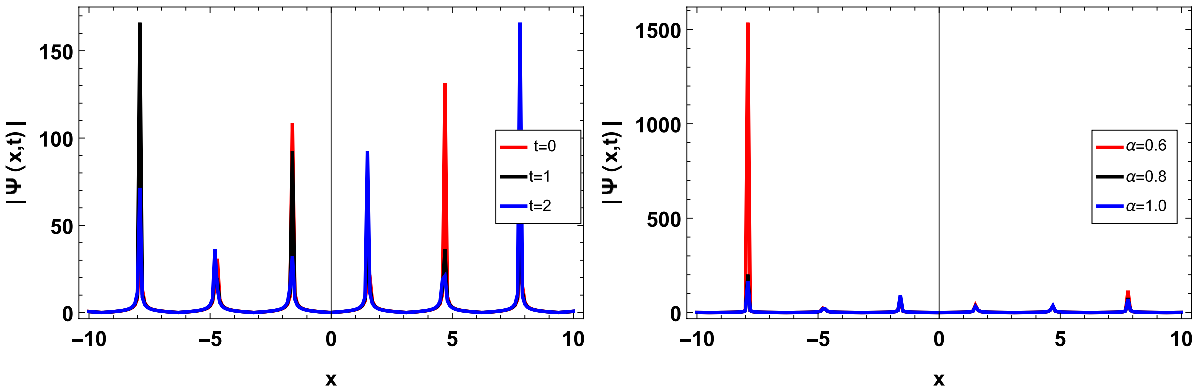


Figure 4. Graphical solution of (24) with parameters $\kappa_1 = 1, \varpi_1 = 0.1, a_1 = 1, a = 2$.

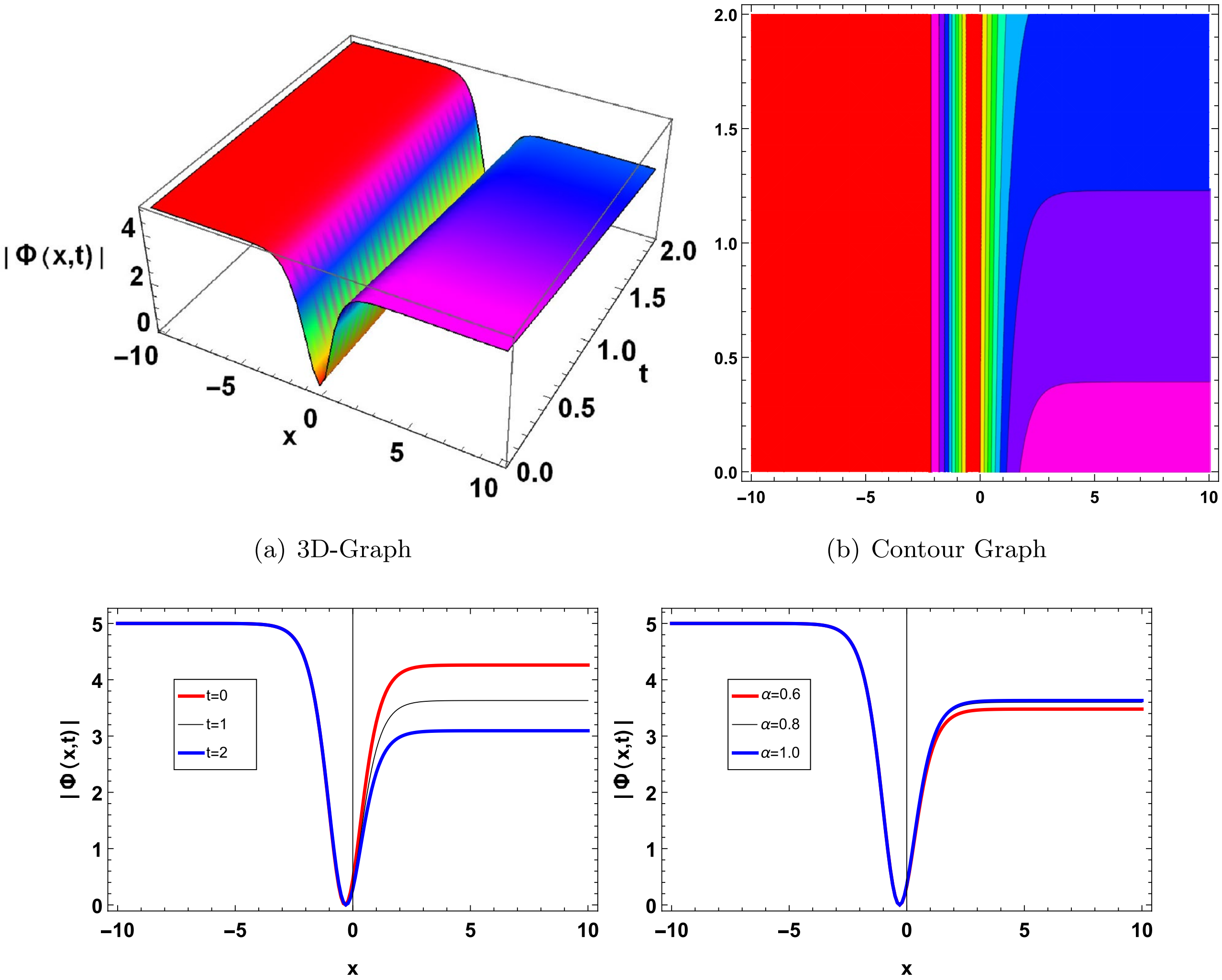


(a) 3D-Graph (b) Contour Graph



(c) 2D-Graph (d) 2D-Graph

Figure 5. Graphical solution of (47) with parameters $\kappa_1 = 1, \varpi_1 = 0.01, a_1 = 1$.



(a) 3D-Graph (b) Contour Graph

(c) 2D-Graph (d) 2D-Graph

Figure 6. Graphical solution of (52) with parameters $\kappa_1 = 2, \varpi_1 = 0.01, b_1 = 2, a = 0.8$.

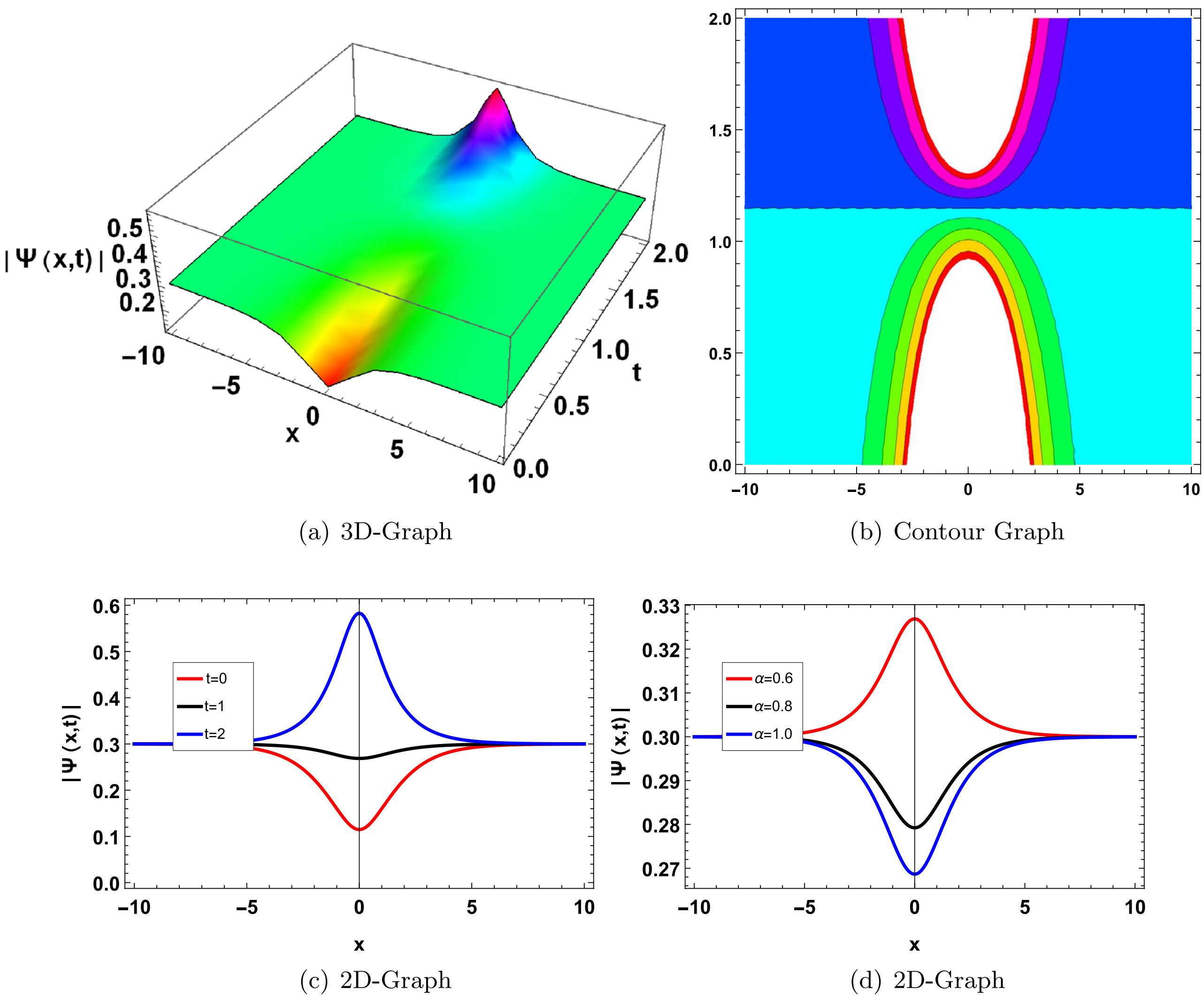
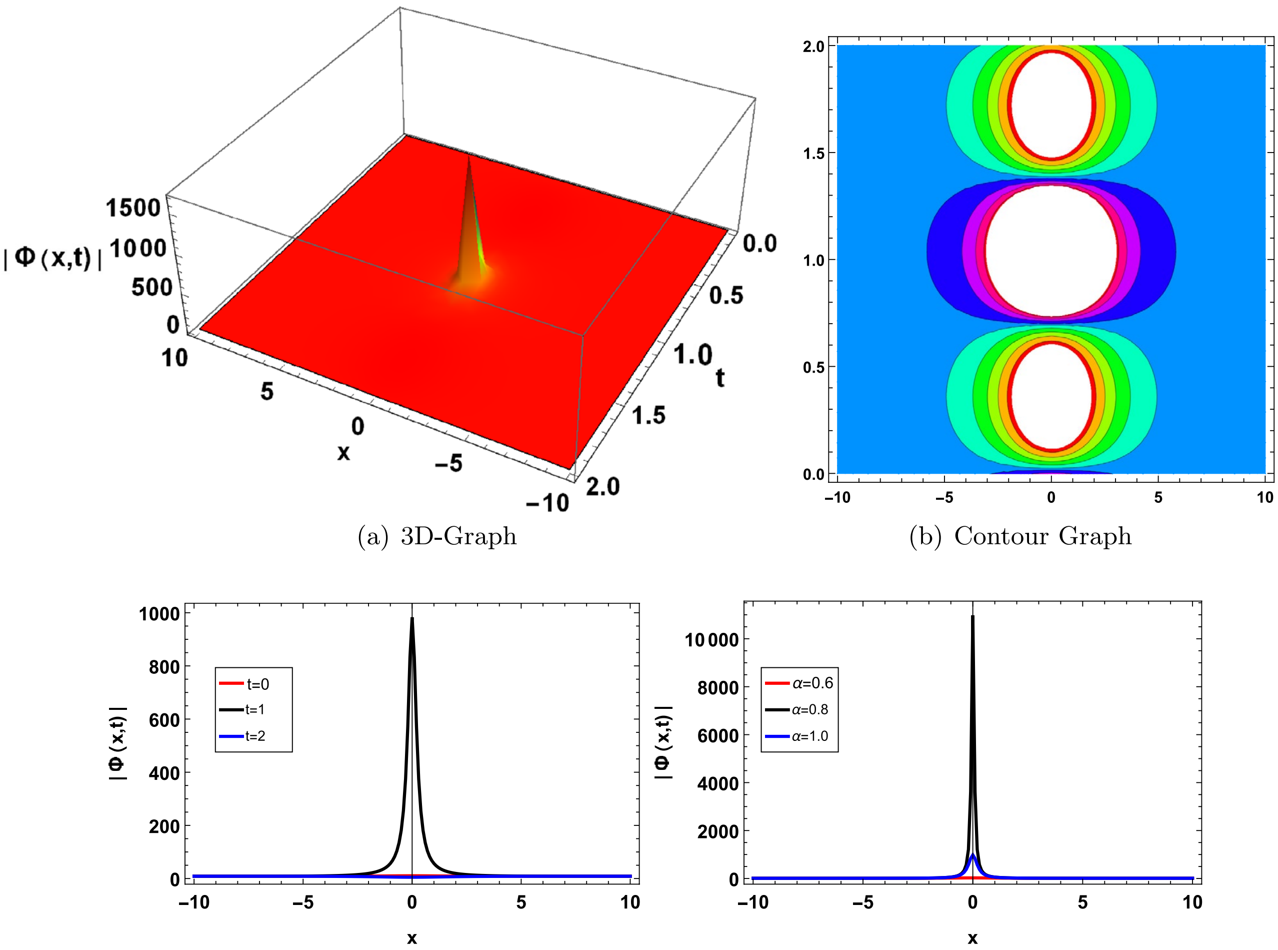


Figure 7. Graphical solution of (69) with parameters $\kappa_1 = -0.8, \gamma_1 = 0.01, \lambda_1 = 0.02, a = -5, a_0 = 0.3, a_1 = -5$.



(a) 3D-Graph (b) Contour Graph

(c) 2D-Graph (d) 2D-Graph

Figure 8. Graphical solution of (70) with parameters $\kappa_1 = -0.8, \gamma_1 = 0.1, \lambda_1 = 0.2, a = -5, a_0 = -2$.

Data availability

All data that support the findings of this study are included in the article.

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All Authors are contributed equally.

Competing interests
The authors declare no competing interests.

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