



OPEN The efficiency of quantum teleportation with three-qubit entangled state in a noisy environment

Chang-Yue Zhang^{1,2}, Zhu-Jun Zheng^{1,2✉}, Zhao-Bing Fan³ & Hai-Tao Ma³

Quantum teleportation plays a significant role in the field of quantum communication. This paper investigates quantum teleportation through a noisy environment by using GHZ state and non-standard W state as quantum channels. We analyze the efficiency of quantum teleportation by solving analytically a master equation in Lindblad form. Following the quantum teleportation protocol, we obtain the fidelity of quantum teleportation as a function of evolution time. The calculation results show that the teleportation fidelity using non-standard W is higher in comparison to GHZ state at the same evolution time. Moreover, we consider the efficiency of teleportation with weak measurements and reverse quantum measurement under amplitude damping noise. Our analysis suggests that the teleportation fidelity using non-standard W is also more robust to noise than GHZ state in the same conditions. Interestingly, we found that weak measurement and its reverse operation have no positive effect on the efficiency of quantum teleportation by using GHZ and non-standard W state in the amplitude damping noise environment. In addition, we also demonstrate the efficiency of quantum teleportation can be improved by making minor modifications to the protocol.

Quantum entanglement is shown to be an efficient resource for many potential applications that cannot be achieved using classical resources, such as quantum teleportation¹, quantum cryptography², quantum computing³, quantum dense coding⁴, etc. Quantum teleportation (QT) as a typical application of quantum entanglement, which is proposed by Bennett et al.¹ in 1993 and demonstrated experimentally a few years later by Bouwmeester et al.⁵. In the original teleportation scheme¹, Alice transmits an unknown quantum state to Bob by using a maximally entangled state as quantum channel. In 1998, Karlsson et al.⁶ proposed a protocol that teleport a quantum state by using three-particle entanglement, which has a high degree of security. Based on it, Agrawal et al.⁷ investigated a class of W state that can be used for perfect teleportation. And compared to the GHZ state, the W state can still be used as a resource even after particle loss. Thereafter, teleportation using other multi-particle entangled states has also been considered^{8–10}. Because of the properties of quantum entanglement^{11–14}, QT brings some new capabilities that are probably impossible to attain in any classical communication.

In reality, however, quantum entanglement is subjected to decoherence due to the inevitable interaction of entangled qubits with the environment^{15–18}. In QT protocol, the maximally entangled state becomes mixed state due to the inevitable interactions with the environments during entanglement distribution. Hence, the dynamics of quantum entanglement under a noisy environment always affects QT. Naturally, ones need to consider the influence of entanglement evolution in a noisy environment on QT^{19–23}. In 2015, Fortes et al.²¹ investigated the efficiency of QT in the condition that qubits are subjected to noise. Then, the efficiencies of partially entangled states in three-qubit classes under real conditions are analyzed in Re²². Fonseca²³ studied the protocol of qudit teleportation using quantum systems subjected to several kinds of noise for arbitrary dimensionality d . Moreover, the minimum assured fidelity (MASFI) of QT has been studied, which is the best measure of the quality of quantum teleportation protocol when the quantum channel is imperfect^{24–26}.

We noted that little work has focused on the relationship between teleportation efficiency and evolution time in a noisy environment. Nevertheless, lots of studies on the dynamics of quantum entanglement focus on the time evolution of the quantum system^{27–32}. So, we investigate the variation of fidelity by evolution time when

¹Department of Mathematics, South China University of Technology, Guangzhou 510641, People's Republic of China. ²Laboratory of Quantum Science and Engineering, South China University of Technology, Guangzhou 510641, People's Republic of China. ³College of Mathematics Science, Harbin Engineering University, Harbin 15001, People's Republic of China. ✉email: zhengzj@scut.edu.cn

three-qubit entangled state is used as quantum channels for QT under a noisy environment. We focus on the amplitude damping noise, which is a prototype model of a dissipating interaction between the quantum system and its zero-temperature environment¹⁵. Furthermore, some previous works employed the technique of weak measurement (WK) and reverse quantum measurement (RM) in order to enable the preservation of quantum entanglement^{33–38}. The fidelity of quantum teleportation with WK and RM operations is also studied here.

To begin with, we describe the standard quantum teleportation in a noisy environment and give a framework for calculating the fidelity of teleportation. Based on Re⁷, we concern two typical states of three-qubit entangled state as quantum channels, GHZ state and non-standard W state, which are affected by amplitude damping noise. Subsequently, we calculate the fidelities of the final output states by solving Lindblad's master equation in the amplitude damping noise environment. And the relationship among the efficiency of teleportation, decoherence time and initial states is established. We also make a comparison of fidelities of QT between GHZ state and non-standard W under the help of WM and RM. We find that the fidelity of teleportation cannot be increased after using WM and RM to protect GHZ and non-standard W states for the given protocol. Combined with the analysis above, we demonstrate the efficiency of quantum teleportation can be improved by changing the participant who prepares the initial state under amplitude damping, bit flip, bit-phase flip and phase flip noises.

Results

Mathematical description of quantum teleportation in noisy environments. To start, we consider the procedure of QT using an arbitrary three-qubit to transmit quantum information under noisy environments. Due to the limitation of entanglement distribution distance, we describe the protocol in the situation where Dave is honesty and located in the middle of Alice and Bob. To increasing transmission distance, in what follows we give the mathematical description for a general QT protocol that qubits are subjected to noise, which is characterized by Lindblad master equation.

Without loss of generality, suppose that Alice wants to transmit an unknown state $|\varphi\rangle_a$ to Bob, which is

$$|\varphi\rangle_a = \alpha|0\rangle_a + \beta|1\rangle_a, \quad (1)$$

where $|\alpha|^2 + |\beta|^2 = 1$. Its density matrix is

$$\rho_{in} = |\varphi\rangle_a \langle\varphi|_a. \quad (2)$$

Step 1 Consider that Dave prepares a three qubit entangled state $|\phi\rangle_{123}$, whose density matrix formalism is

$$\hat{\rho} = |\phi\rangle_{123} \langle\phi|_{123}. \quad (3)$$

Therefore, the initial total state describing the whole system before any quantum operation is implemented is given by

$$\rho = \rho_{in} \otimes \hat{\rho}. \quad (4)$$

Step 2 Dave sends particles 1 and 2 to Alice, particle 3 to Bob, respectively. In practical, the particles to be sent are affected by the noises.

Here, we adopt the master equation approach that describes quantum noise in continuous time using differential equations. For an open system, which is affected by environmental influences and other factors can be characterized by Lindblad master equation^{15,18},

$$\frac{\partial\rho}{\partial t} = -i[H_s, \rho] + \sum_{i,\alpha} \left(L_{i,\alpha} \rho L_{i,\alpha}^\dagger - \frac{1}{2} \{ L_{i,\alpha}^\dagger L_{i,\alpha}, \rho \} \right), \quad (5)$$

where the first term is the unitary evolution with Hamiltonian H_s , the second term is the nonunitary contribution, $L_{i,\alpha}$ is the Lindblad operator acting on the i th qubit. In order to simplify the calculation, the evolution of $\hat{\rho}$ can be considered first before calculating the result of the whole evolved state ρ (see "Methods" for detail).

Step 3 Alice makes a von Neumann type measurement on her own qubits $a, 1, 2$ by using the proper states $\{|\psi_j\rangle, j = 1, 2, 3, 4\}$. After this measurement, the total state $\rho(t)$ changes to one of the following four states with a certain probability.

$$\rho_j(t) = \frac{M_j \rho(t) M_j^\dagger}{\text{Tr}[M_j^\dagger M_j \rho(t)]}, \quad (6)$$

where Tr is the trace operation and $M_j = |\psi_j\rangle \langle\psi_j|$. In this step, we consider the measurement is perfect, while the particles still suffer from the noise during the measurement.

Step 4 Alice tells the measurement outcome to Bob through classical communication. Now, Bob knows his own state is given by

$$\rho_{Bj}(t) = \text{Tr}_{a12}[\rho_j(t)] = \frac{\text{Tr}_{a12}[M_j \rho(t) M_j^\dagger]}{\text{Tr}[M_j^\dagger M_j \rho(t)]}, \quad (7)$$

where Tr_{a12} is the partial trace over particles $(a, 1, 2)$.

Step 5 Bob implements a unitary operator U_j on his particle 3. Hence, the final state in Bob is

$$\tilde{\rho}_{B_j}(t) = U_j \rho_{B_j}(t) U_j^\dagger = \frac{U_j \text{Tr}_{a_{12}}[M_j \rho(t) M_j^\dagger] U_j^\dagger}{\text{Tr}[M_j^\dagger M_j \rho(t)]}. \quad (8)$$

To describe how much information has been lost in this transmission, we compute the teleportation fidelity, which indicates the degree of overlap between the input and output states. The fidelity of quantum teleportation is

$$F = \sum_{j=1}^4 P_j F_j, \quad (9)$$

where $P_j = \text{Tr}[M_j^\dagger M_j \rho(t)]$ and $F_j = \langle \varphi_a | \tilde{\rho}_{B_j}(t) | \varphi_a \rangle$.

In addition, one way to protect entanglement is based on WM. It was found that the process of performing WM and its reversal on individual qubit is a very useful technique for preserving and enhancing correlations under noisy conditions^{33–36}. Furthermore, WM and RM have also been experimentally demonstrated^{37,38}. So, we further investigate the effect of WM and RM on protocol effectiveness during teleportation. The modified QT under noises with the help of WM and RM is described as follows.

We start with a scenario where Dave prepares a three-qubit pure entangled state $|\phi\rangle_{123}$. The form of the whole initial system is Eq. (4). To reduce the effect of decoherence, suppose that Dave performs WM on each qubit before sending the particle through noisy environments. Here, the weak measurement operator is

$$W_w = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-w} \end{pmatrix}, \quad (10)$$

where w is the strength of WM operations and $0 \leq w < 1$. According to the operators W_w of WM, the initial quantum system is rewritten as

$$\rho^{wk} = \frac{W \rho W^\dagger}{\text{Tr}[W^\dagger W \rho]}. \quad (11)$$

where $W = W_1 W_2 W_3$ and $W_1 = I^{(a)} \otimes W_{w_1}^{(1)} \otimes I^{(2)} \otimes I^{(3)}$, $W_2 = I^{(a)} \otimes I^{(1)} \otimes W_{w_2}^{(2)} \otimes I^{(3)}$, $W_3 = I^{(a)} \otimes I^{(1)} \otimes I^{(2)} \otimes W_{w_3}^{(3)}$, w_1, w_2 and w_3 are the strengths of WM on qubit 1, 2, 3, respectively. $\text{Tr}[W^\dagger W \rho]$ is the success probability of the weak measurement.

Then, Dave sends particles through noise environments. The evolved state $\rho^{wk}(t)$ can be given by solving the master equation Eq. (5) according to the conditions that quantum noise and initial quantum state.

After receiving the qubits, Alice and Bob perform RM on qubit (1, 2) and qubit 3, respectively. The quantum reversal measurement operator is³⁷

$$R_r = \begin{pmatrix} \sqrt{1-r} & 0 \\ 0 & 1 \end{pmatrix}, \quad (12)$$

where r represents the strength of RM operations $0 \leq r < 1$. The quantum system becomes

$$\rho^{rev}(t) = \frac{R \rho^{wk}(t) R^\dagger}{\text{Tr}[R^\dagger R \rho^{wk}(t)]}, \quad (13)$$

where $R = R_1 R_2 R_3$ and $R_1 = I^{(a)} \otimes R_{r_1}^{(1)} \otimes I^{(2)} \otimes I^{(3)}$, $R_2 = I^{(a)} \otimes I^{(1)} \otimes R_{r_2}^{(2)} \otimes I^{(3)}$, $R_3 = I^{(a)} \otimes I^{(1)} \otimes I^{(2)} \otimes R_{r_3}^{(3)}$, r_1, r_2 and r_3 are the strengths of RM respectively performs on qubit 1, 2 and 3. The probability of performing a successful RM operation is $\text{Tr}[R^\dagger R \rho^{wk}(t)]$.

Finally, as in step 3 to step 5, Alice and Bob complete teleportation by using the entangled state $\rho^{rev}(t)$. The fidelity of the quantum teleportation using three-qubit state with the help of WK and RM is

$$F = \sum_{j=1}^4 U_j \text{Tr}_{a_{12}}[M_j \rho^{rev}(t) M_j^\dagger] U_j^\dagger. \quad (14)$$

Quantum teleportation with GHZ state and W_1 state. In general, three-qubit entangled states are classified into two inequivalent classes, namely GHZ class and W class. There is an instance in three-qubit entangled state where the maximally entangled GHZ state can be used for deterministic teleportation, while the standard W state can only be used for probabilistic teleportation. Interestingly, Agrawal et al.⁷ showed that there exists a special class of W states that can be used for perfect teleportation, called W_n . Therefore, we consider GHZ and W_n states in the following.

First, the situation of GHZ state is considered. Suppose Dave prepares a GHZ state $|\phi_g\rangle_{123}$, which is

$$|\phi_g\rangle_{123} = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle)_{123}. \quad (15)$$

The density matrix of $|\phi_g\rangle_{123}$ is denoted by $\hat{\rho}_g(0) = |\phi_g\rangle_{123}\langle\phi_g|$.

Here, we consider the evolution of the state $\hat{\rho}_g(0)$ under amplitude damping noise, which describes the energy dissipation. Equation (46) provides a specific description. By substituting coefficients of $|\phi_g\rangle_{123}$ into Eq. (52), that is, $\lambda_0 = \lambda_1 = \frac{1}{\sqrt{2}}$, one can obtain the evolved state of $|\phi_g\rangle_{123}$, which is the mixed state shared by Alice and Bob.

After Dave sends particles through amplitude damping noise, the whole quantum state has the following form,

$$\begin{aligned} \rho_g(t) &= \rho_{in} \otimes \hat{\rho}_g(t) \\ &= \frac{1}{2} \left[\left(\alpha|0000\rangle + \alpha e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |0111\rangle + \beta|1000\rangle + \beta e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |1111\rangle \right) \right. \\ &\quad \left(\alpha\langle 0000| + \alpha e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 0111| + \beta\langle 1000| + \beta e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 1111| \right) \\ &\quad + e^{-k_1 t} e^{-k_2 t} \bar{e}^{-k_3 t} (\alpha|0110\rangle + \beta|1110\rangle) (\alpha\langle 0110| + \beta\langle 1110|) \\ &\quad + e^{-k_1 t} \bar{e}^{-k_2 t} e^{-k_3 t} (\alpha|0101\rangle + \beta|1101\rangle) (\alpha\langle 0101| + \beta\langle 1101|) \\ &\quad + e^{-k_1 t} \bar{e}^{-k_2 t} \bar{e}^{-k_3 t} (\alpha|0100\rangle + \beta|1100\rangle) (\alpha\langle 0100| + \beta\langle 1100|) \\ &\quad + \bar{e}^{-k_1 t} e^{-k_2 t} e^{-k_3 t} (\alpha|0011\rangle + \beta|1011\rangle) (\alpha\langle 0011| + \beta\langle 1011|) \\ &\quad + \bar{e}^{-k_1 t} e^{-k_2 t} \bar{e}^{-k_3 t} (\alpha|0010\rangle + \beta|1010\rangle) (\alpha\langle 0010| + \beta\langle 1010|) \\ &\quad + \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} e^{-k_3 t} (\alpha|0001\rangle + \beta|1001\rangle) (\alpha\langle 0001| + \beta\langle 1001|) \\ &\quad \left. + \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} \bar{e}^{-k_3 t} (\alpha|0000\rangle + \beta|1000\rangle) (\alpha\langle 0000| + \beta\langle 1000|) \right], \end{aligned} \tag{16}$$

where k_1, k_2 and k_3 are the amplitude damping parameters associated with particles 1, 2 and 3, respectively. And the detailed calculation procedure of $\hat{\rho}_g(0)$ can be found in the “Methods”-“GHZ state”.

Now, Alice makes a von Neumann measurement on particles (a, 1, 2) with a set of orthogonal states given by

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}(|000\rangle + |111\rangle), |\psi_2\rangle = \frac{1}{\sqrt{2}}(|000\rangle - |111\rangle), |\psi_3\rangle = \frac{1}{\sqrt{2}}(|100\rangle + |011\rangle), |\psi_4\rangle = \frac{1}{\sqrt{2}}(|100\rangle - |011\rangle). \tag{17}$$

Subsequently, Alice sends the result of her measurement to Bob by using classical communication with two bits. According to the measurement result, Bob applies a proper unitary operator to recover the state of his particle 3 to that of state $|\varphi\rangle_a$. Hence, one can obtain the output quantum state

$$\begin{aligned} \rho_g^{out}(t) &= \sum_{j=1}^4 \text{Tr}[M_j^\dagger M_j \rho(t)] \frac{U_j \text{Tr}_{a12}[M_j \rho(t) M_j^\dagger] U_j^\dagger}{\text{Tr}[M_j^\dagger M_j \rho(t)]} \\ &= \frac{1}{2} \left[\left(\alpha|0\rangle + \beta e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |1\rangle \right) \left(\alpha\langle 0| + \beta e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 1| \right) \right. \\ &\quad + \left(\alpha e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |0\rangle + \beta |1\rangle \right) \left(\alpha e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 0| + \beta \langle 1| \right) \\ &\quad + \beta^2 e^{-k_1 t} e^{-k_2 t} \bar{e}^{-k_3 t} |0\rangle\langle 0| + \alpha^2 \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} e^{-k_3 t} |1\rangle\langle 1| \\ &\quad + \alpha^2 \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} \bar{e}^{-k_3 t} |0\rangle\langle 0| + \alpha^2 e^{-k_1 t} e^{-k_2 t} \bar{e}^{-k_3 t} |1\rangle\langle 1| \\ &\quad \left. + \beta^2 \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} e^{-k_3 t} |0\rangle\langle 0| + \beta^2 \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} \bar{e}^{-k_3 t} |1\rangle\langle 1| \right]. \end{aligned} \tag{18}$$

where $M_j = |\psi_j\rangle\langle\psi_j|$, $U_j = \sigma_j, j = 1, 2, 3, 4$, and $\sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \sigma_2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \sigma_3 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \sigma_4 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.

Based on Eq. (9), the fidelity of the quantum teleportation using GHZ state as a quantum channel is

$$\begin{aligned} F_g &= \frac{1}{2} \left[(\alpha^4 + \beta^4) \left(1 + e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} + \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} \bar{e}^{-k_3 t} \right) \right. \\ &\quad \left. + \alpha^2 \beta^2 \left(2e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} + e^{-k_1 t} e^{-k_2 t} \bar{e}^{-k_3 t} + \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} e^{-k_3 t} \right) \right]. \end{aligned} \tag{19}$$

We next consider another class of three-qubit entangled state named W state. The standard W state can only be used for probabilistic teleportation. In Re⁷, Agrawal and Pati have shown that there exists a special class of W state can be used for perfect teleportation. We consider an example of this class non-standard W state

$$|W_1\rangle_{123} = \frac{1}{2} \left(|100\rangle + |010\rangle + \sqrt{2}|001\rangle \right)_{123}. \tag{20}$$

Similarly, we assume that Dave prepares an entangled state $|W_1\rangle_{123}$, then transmits particles (1, 2) and 3 to Alice and Bob through the amplitude damping noise environment, respectively. The initial state is

$$\rho_{W_1}(0) = |\varphi\rangle_a \langle\varphi|_a \otimes |W_1\rangle_{123} \langle W_1|_{123}. \tag{21}$$

According to Eq. (59), let $x = \frac{1}{\sqrt{2}}, y = \frac{1}{2}, z = \frac{1}{2}$, one can obtain the expression of the total evolution quantum state, which is

$$\begin{aligned} \rho_{W_1}(t) = & \frac{1}{4} \left(\alpha e^{-\frac{k_1 t}{2}} |0100\rangle + \alpha e^{-\frac{k_2 t}{2}} |0010\rangle + \sqrt{2} \alpha e^{-\frac{k_3 t}{2}} |0001\rangle + \beta e^{-\frac{k_1 t}{2}} |1100\rangle \right. \\ & \left. + \beta e^{-\frac{k_2 t}{2}} |1010\rangle + \sqrt{2} \beta e^{-\frac{k_3 t}{2}} |1001\rangle \right) \left(\alpha e^{-\frac{k_1 t}{2}} \langle 0100| + \alpha e^{-\frac{k_2 t}{2}} \langle 0010| \right. \\ & \left. + \sqrt{2} \alpha e^{-\frac{k_3 t}{2}} \langle 0001| + \beta e^{-\frac{k_1 t}{2}} \langle 1100| + \beta e^{-\frac{k_2 t}{2}} \langle 1010| + \sqrt{2} \beta e^{-\frac{k_3 t}{2}} \langle 1001| \right) \\ & + \left[2\bar{e}^{-k_3 t} + \bar{e}^{-k_2 t} + \bar{e}^{-k_1 t} \right] (\alpha|0000\rangle + \beta|1000\rangle)(\alpha\langle 0000| + \beta\langle 1000|). \end{aligned} \tag{22}$$

Then, Alice makes a von Neumann measurement on qubit $(a, 1, 2)$ using the states $\{|\zeta_j\rangle, j = 1, 2, 3, 4\}$, which is

$$\begin{aligned} |\zeta_1\rangle = & \frac{1}{2} (|010\rangle + |001\rangle + \sqrt{2}|100\rangle), |\zeta_2\rangle = \frac{1}{2} (|010\rangle + |001\rangle - \sqrt{2}|100\rangle), \\ |\zeta_3\rangle = & \frac{1}{2} (|110\rangle + |101\rangle + \sqrt{2}|000\rangle), |\zeta_4\rangle = \frac{1}{2} (|110\rangle + |101\rangle - \sqrt{2}|000\rangle). \end{aligned} \tag{23}$$

After measurement, Alice sends measurement result to Bob by classical communication. Bob performs a unitary operator on his own qubit 3. The final state of Bob is

$$\begin{aligned} \rho_{W_1}^{out}(t) = & \sum_{j=1}^4 V_j \text{Tr}_{a12} [N_j^\dagger N_j \rho_{W_1}(t) N_j] V_j^\dagger \\ = & \frac{1}{8} \left\{ \left[\left(e^{-\frac{k_1 t}{2}} + e^{-\frac{k_2 t}{2}} \right) \alpha |0\rangle + 2e^{-\frac{k_3 t}{2}} \beta |1\rangle \right] \left[\left(e^{-\frac{k_1 t}{2}} + e^{-\frac{k_2 t}{2}} \right) \alpha \langle 0| + 2e^{-\frac{k_3 t}{2}} \beta \langle 1| \right] \right. \\ & + \left[2e^{-\frac{k_3 t}{2}} \alpha |0\rangle + \left(e^{-\frac{k_2 t}{2}} + e^{-\frac{k_1 t}{2}} \right) \beta |1\rangle \right] \left[2e^{-\frac{k_3 t}{2}} \alpha \langle 0| + \left(e^{-\frac{k_2 t}{2}} + e^{-\frac{k_1 t}{2}} \right) \beta \langle 1| \right] \\ & \left. + 2\beta^2 \left(2\bar{e}^{-k_3 t} + \bar{e}^{-k_2 t} + \bar{e}^{-k_1 t} \right) |0\rangle\langle 0| + 2\alpha^2 \left(2\bar{e}^{-k_3 t} + \bar{e}^{-k_2 t} + \bar{e}^{-k_1 t} \right) |1\rangle\langle 1| \right\}, \end{aligned} \tag{24}$$

where $V_j = U_j$ and $N_j = |\zeta_j\rangle\langle\zeta_j|$.

Using fidelity to measure the amount of information loss in the quantum teleportation process, which can be written as

$$F_{W_1} = \frac{1}{8} \left\{ \left[\left(e^{-\frac{k_1 t}{2}} + e^{-\frac{k_2 t}{2}} \right) \alpha^2 + 2e^{-\frac{k_3 t}{2}} \beta^2 \right]^2 + \left[2e^{-\frac{k_3 t}{2}} \alpha^2 + \left(e^{-\frac{k_1 t}{2}} + e^{-\frac{k_2 t}{2}} \right) \beta^2 \right]^2 + 4\alpha^2 \beta^2 \left(2\bar{e}^{-k_3 t} + \bar{e}^{-k_2 t} + \bar{e}^{-k_1 t} \right) \right\}. \tag{25}$$

We consider a situation where the particles to be sent are affected by the same noise parameters, namely, $k_1 = k_2 = k_3 = k$. In order to describe the relationship between the initial state and the fidelity more vividly, we carried out a simulation and plotted the Fig. 1. In reality, the measurement time needs to be considered, but we assume that the measurement time is extremely small in the numerical simulation. Obviously, Dave sends qubits through the perfect channel, that is $kt = 0$, then the fidelities of GHZ and W_1 state are both 1. It also depicts that whether GHZ state or W_1 state is used as a quantum channel in the process of quantum teleportation, the fidelity of the final quantum state obtained by the receiver Bob will decrease with the increase of decoherence time. For GHZ state, as shown in plot (a), the state of the input state has little effect on the fidelity of the final

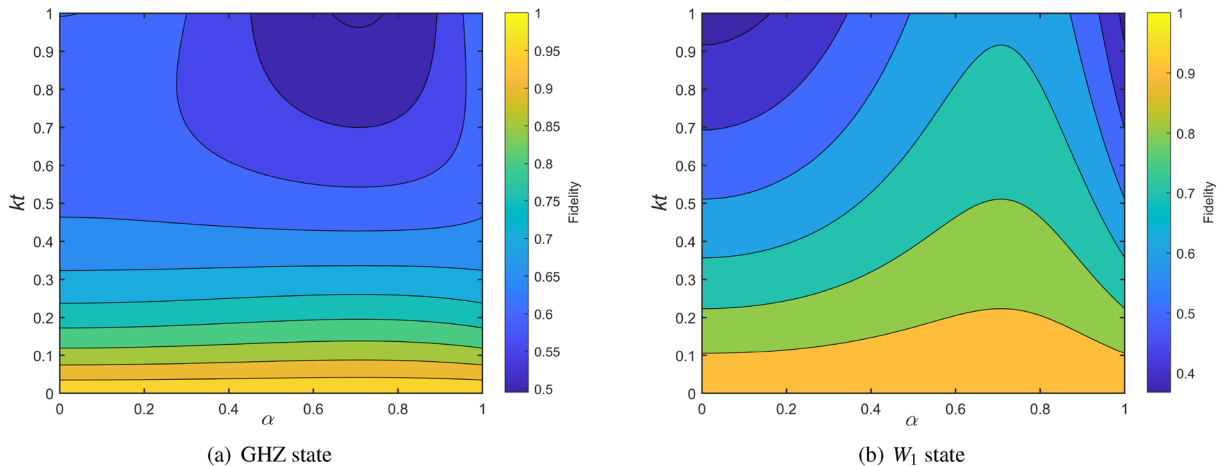


Figure 1. The fidelities of quantum teleportation using GHZ and W_1 state under amplitude damping noise is described by a master equation in the Lindblad form.

output state when $kt \leq 0.5$. If kt continues to increase, the state of the transmission state will affect the fidelity of the transmission. In this case where the smaller the value of α , the higher the fidelity of teleportation. Plot (b) describes the effects of decoherence time and input state on the value of teleportation fidelity of W_1 state. The value of teleportation fidelity of W_1 is always greater than 0.9, if $kt \leq 0.1$. Unlike the former, the fidelity of W_1 is affected not only by decoherence time, but also by the input state when $kt \leq 0.5$. Clearly, the fidelity will be affected by the unknown state as the value of kt increases. When $0.6 < \alpha < 0.8$, the efficiency of protocol performs better. At this time, even kt reaches 0.9, the fidelity $F_{W_1} \geq 0.6$.

Furthermore, we make a comparison between GHZ state and W_1 state in the same condition. The fidelity of GHZ (F_g) subtracted from W_1 (F_{W_1}) is denoted as F . As shown in Fig. 2, we found that the fidelity of W_1 is higher than GHZ, in most cases. The result obtained here confirms that the ability of W_1 to resist amplitude damping noise during the communication process is stronger than GHZ state.

It is worth to discuss the feasibility of the protocol in physical experiments. QT has been demonstrated in different systems such as photonic qubit, continuous variables, nuclear magnetic resonance, atomic ensembles, trapped ions and solid state systems³⁹. Combined with the development trend of quantum communication, transmission distance is one of the key points. Hence, photon is the most suitable carrier of information for the long distance QT. In the numerical simulations, the value of t depends on the value of k . Theoretically, the particles can reach their destinations in a finite time by choosing the appropriate k . In the aspect of experiment, since there are lots of works have been experimentally demonstrated the feasibility of the QT protocol employing photonic qubits^{3,40–42}, it is possible to distribute the entangled particles to the communicating parties even if the decoherence time is very short.

Quantum teleportation with WM and RM. Let us now consider the efficiency of QT protocol with the help of WM and RM operations under noises, by using GHZ state. We assume that Dave prepares a three qubit entangled state $|\phi_g\rangle$. Then, Dave performs WM on qubits 1, 2, 3 before sending them to Alice and Bob. According to the operators W_{wk} of WM, the initial entangled quantum state is became

$$\begin{aligned} \rho_g^{wk}(0) &= W \rho_g(0) W \\ &= \frac{1}{2} \left(|000\rangle + \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} |111\rangle \right) \left(\langle 000| + \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} \langle 111| \right), \end{aligned} \tag{26}$$

where $W = W_1 W_2 W_3$ and $W_1 = W_{w_1 k}^{(1)} \otimes I^{(2)} \otimes I^{(3)}$, $W_2 = I^{(1)} \otimes W_{w_2 k}^{(2)} \otimes I^{(3)}$, $W_3 = I^{(1)} \otimes I^{(2)} \otimes W_{w_3 k}^{(3)}$, $\bar{w}_1 = 1 - w_1$, $\bar{w}_2 = 1 - w_2$, $\bar{w}_3 = 1 - w_3$, w_1 , w_2 and w_3 are the strengths of WM on qubits 1, 2, 3, respectively.

Dave sends particles through amplitude damping noise channel. The evolved state $\hat{\rho}_g^{wk}(t)$ can be given refer to Eq. (52) with the parameter in Eq. (26). It is

$$\begin{aligned} \hat{\rho}_g^{wk}(t) &= \frac{1}{2} \left[\left(|000\rangle + \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |111\rangle \right) \left(\langle 000| + \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 111| \right) \right. \\ &\quad + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} |110\rangle \langle 110| + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} |101\rangle \langle 101| \\ &\quad + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} |100\rangle \langle 100| + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} |011\rangle \langle 011| \\ &\quad + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} |010\rangle \langle 010| + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} |001\rangle \langle 001| \\ &\quad \left. + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} |000\rangle \langle 000| \right]. \end{aligned} \tag{27}$$

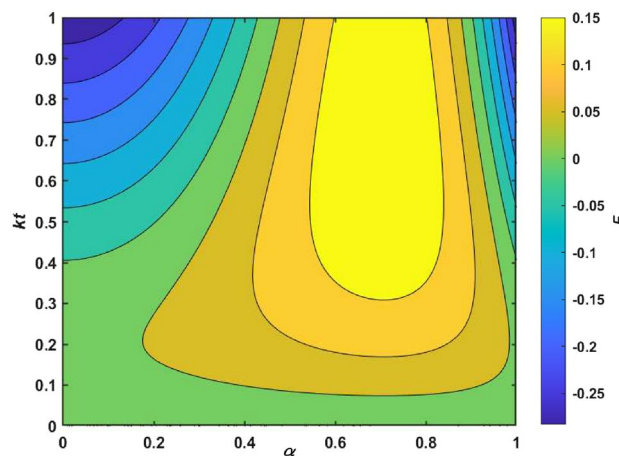


Figure 2. The comparison of fidelities of quantum teleportation between GHZ state and W_1 state. Considering the noise parameters of GHZ and W_1 both are $k_1 = k_2 = k_3 = k$.

Subsequently, Alice and Bob perform RM on qubit (1, 2) and qubit 3, respectively. By Eq. (13), the three-qubit entangled state is changed to

$$\hat{\rho}_g^{rev}(t) = \frac{1}{2} \left[\left(\bar{r}_1^{\frac{1}{2}} \bar{r}_2^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}} |000\rangle + \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |111\rangle \right) \left(\bar{r}_1^{\frac{1}{2}} \bar{r}_2^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}} \langle 000| + \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 111| \right) \right. \\ \left. + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_3 |110\rangle \langle 110| + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_2 |101\rangle \langle 101| \right. \\ \left. + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_2 \bar{r}_3 |100\rangle \langle 100| + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 |011\rangle \langle 011| \right. \\ \left. + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_3 |010\rangle \langle 010| + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_2 |001\rangle \langle 001| \right. \\ \left. + \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_2 \bar{r}_3 |000\rangle \langle 000| \right], \tag{28}$$

where $\bar{r}_1 = 1 - r_1, \bar{r}_2 = 1 - r_2, \bar{r}_3 = 1 - r_3, r_1, r_2$ and r_3 are the strengths of RM respectively performs on qubit 1, 2 and 3.

Now, Alice and Bob share a mixed state $\hat{\rho}_g^{rev}(t)$, which is used to transmit the unknown single-qubit state. Obviously, the total quantum system is

$$\rho_g^{wk}(t) = \rho_{in} \otimes \hat{\rho}_g^{rev}(t) \\ = \left(\alpha \lambda_0 |0000\rangle + \alpha \lambda_1 \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |0111\rangle + \beta \lambda_0 |1000\rangle + \beta \lambda_1 \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |1111\rangle \right) \\ \left(\alpha \lambda_0 \langle 0000| + \alpha \lambda_1 \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 0111| + \beta \lambda_0 \langle 1000| + \beta \lambda_1 \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 1111| \right) \\ + \lambda_1^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} (\alpha |0110\rangle + \beta |1110\rangle) (\alpha \langle 0110| + \beta \langle 1110|) \\ + \lambda_1^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} (\alpha |0101\rangle + \beta |1101\rangle) (\alpha \langle 0101| + \beta \langle 1101|) \\ + \lambda_1^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} (\alpha |0100\rangle + \beta |1100\rangle) (\alpha \langle 0100| + \beta \langle 1100|) \\ + \lambda_1^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} (\alpha |0011\rangle + \beta |1011\rangle) (\alpha \langle 0011| + \beta \langle 1011|) \\ + \lambda_1^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} (\alpha |0010\rangle + \beta |1010\rangle) (\alpha \langle 0010| + \beta \langle 1010|) \\ + \lambda_1^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} (\alpha |0001\rangle + \beta |1001\rangle) (\alpha \langle 0001| + \beta \langle 1001|) \\ + \lambda_1^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} (\alpha |0000\rangle + \beta |1000\rangle) (\alpha \langle 0000| + \beta \langle 1000|). \tag{29}$$

In the subsequent steps, Alice makes a von Neumann type measurement on her own qubits $a, 1, 2$ with the set of orthogonal states given by Eq. (17) and tells the measurement outcome to Bob through classical communication. According to the measurement result, Bob implements a unitary operator U_j on his particle 3. So, the final quantum state is

$$\rho_g^{rev,out}(t) = \sum_{j=1}^4 U_j \text{Tr}_{a12} [M_j \rho_g^{rev}(t) M_j^\dagger] U_j^\dagger \\ = \frac{1}{2} \left[\left(\alpha \bar{r}_1^{\frac{1}{2}} \bar{r}_2^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}} |0\rangle + \beta \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |1\rangle \right) \left(\alpha \bar{r}_1^{\frac{1}{2}} \bar{r}_2^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}} \langle 0| + \beta \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 1| \right) \right. \\ \left. + \alpha^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_2 |1\rangle \langle 1| + \alpha^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_2 \bar{r}_3 |0\rangle \langle 0| \right. \\ \left. + \alpha^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_2 |1\rangle \langle 1| + \alpha^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_2 \bar{r}_3 |0\rangle \langle 0| + \left(\alpha \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |0\rangle + \beta \bar{r}_1^{\frac{1}{2}} \bar{r}_2^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}} |1\rangle \right) \left(\alpha \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 0| + \beta \bar{r}_1^{\frac{1}{2}} \bar{r}_2^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}} \langle 1| \right) \right. \\ \left. + \alpha^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_3 |1\rangle \langle 1| + \beta^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_2 |0\rangle \langle 0| + \beta^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_2 \bar{r}_3 |1\rangle \langle 1| \right]. \tag{30}$$

The fidelity of the quantum teleportation using GHZ state with the help of WK and RM is

$$F_g^{rev} = \frac{1}{2} \left[\left(\alpha^2 \bar{r}_1^{\frac{1}{2}} \bar{r}_2^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}} + \beta^2 \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \right)^2 + \left(\alpha^2 \bar{w}_1^{\frac{1}{2}} \bar{w}_2^{\frac{1}{2}} \bar{w}_3^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} + \beta^2 \bar{r}_1^{\frac{1}{2}} \bar{r}_2^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}} \right)^2 \right. \\ \left. + 2\alpha^2 \beta^2 \bar{w}_1 \bar{w}_2 \bar{w}_3 \left(e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_3 + e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_2 \right) + (\alpha^4 + \beta^4) \bar{w}_1 \bar{w}_2 \bar{w}_3 e^{-k_1 t} e^{-k_2 t} e^{-k_3 t} \bar{r}_1 \bar{r}_2 \bar{r}_3 \right]. \tag{31}$$

For W_1 state, the fidelity of the QT under amplitude damping noise with the help of WM and RM can be calculated by the same method as GHZ state. It is

$$F_{W_1}^{rev} = \frac{1}{8} \left\{ \left[(\bar{w}_1^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} \bar{r}_2^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}} + \bar{w}_2^{\frac{1}{2}} e^{-\frac{k_2 t}{2}} \bar{r}_1^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}}) \alpha^2 + 2\bar{w}_3^{\frac{1}{2}} e^{-\frac{k_3 t}{2}} \bar{r}_1^{\frac{1}{2}} \bar{r}_2^{\frac{1}{2}} \beta^2 \right]^2 + \left[2\bar{w}_3^{\frac{1}{2}} e^{-\frac{k_3 t}{2}} \bar{r}_1^{\frac{1}{2}} \bar{r}_2^{\frac{1}{2}} \alpha^2 + (\bar{w}_1^{\frac{1}{2}} e^{-\frac{k_1 t}{2}} \bar{r}_2^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}} + \bar{w}_2^{\frac{1}{2}} e^{-\frac{k_2 t}{2}} \bar{r}_1^{\frac{1}{2}} \bar{r}_3^{\frac{1}{2}}) \beta^2 \right]^2 + 4\bar{r}_1 \bar{r}_2 \bar{r}_3 \alpha^2 \beta^2 (2\bar{w}_3 e^{-k_3 t} + \bar{w}_2 e^{-k_2 t} + \bar{w}_1 e^{-k_1 t}) \right\}. \tag{32}$$

In previous sections, we considered the fidelities under amplitude damping noise for GHZ and W_1 states, respectively. Now, we concentrate on analyzing the fidelities of these two situations with the help of WM and RM. The comparison between GHZ and W_1 is given in Fig. 3. Here, we consider a case where all the particles to be sent are affected by the same noise, the same strength of WM and RM. That is, $k_i = k, w_i = w, r_i = r, i = 1, 2, 3$. Moreover, we use the optimal reversing condition $r = w + (1 - e^{-kt})(1 - w)^{43,44}$. And we assume that Alice wants to transmit a state $|\varphi\rangle_a = \frac{1}{\sqrt{2}}(|0\rangle_a + |1\rangle_a)$ to Bob.

Our calculations and simulation results are shown in Fig. 3. The fidelities of quantum teleportation are not only related to the quantum noise parameter, but also affected by the strength of WM when the input state is determined. Plot (a) shows that the fidelity of teleportation with WM and RM by using GHZ state as quantum channel under amplitude damping noise. Obviously, the fidelity of teleportation using GHZ state decreases with the increased strength w . It is worth pointing out that the fidelity will tend to 0, when $w > 0.5$ or $kt > 0.5$, namely, the longer the decoherence time and the stronger the WM, the greater the impact on the fidelity. Plot (b) is the result of W_1 state. In this case, the appearance of fidelity is similar to the previous one. Surprisingly, both the teleportation fidelities of GHZ and W_1 state are always smaller if the WM and RM are performed in the communication process. It can be inferred that WM and RM cannot improve the efficiency of QT for the given protocol, although it is able to increase other quantum resources of the entangled state with a certain probability.

In order to further discuss the influence of GHZ and W_1 states as quantum channels on communication efficiency, we subtracted the fidelity of GHZ (F_{GHZ}^{rev}) from the fidelity of W_1 ($F_{W_1}^{rev}$), which is

$$F^{rev} = F_{W_1}^{rev} - F_{GHZ}^{rev}, \tag{33}$$

where $\alpha = \beta = \frac{1}{\sqrt{2}}, k_i = k, w_i = w, r_i = r, i = 1, 2, 3$, and $r = w + (1 - e^{-kt})(1 - w)$. Figure 4 shows the difference between GHZ and W_1 states in their fidelity values under the same condition. When $0 < w < 1$, the value of F^{rev} decreases with the increase of w and it is always greater than or equal to 0, which indicates that W_1 can resist more amplitude damping noise than GHZ state at the same time. The result further suggests the WM and its inverse operation have no positive effect on the efficiency of teleportation with GHZ and W_1 state under amplitude damping noise.

Discussion

Here, we discuss what happens if we make minor modifications to the QT protocol. Different from the description of Results-Quantum teleportation with GHZ state and W_1 state, we assume that Alice prepares the entangled state. In fact, only particle 3 will be sent to Bob by Alice in this situation. Naturally, one can study the fidelity of teleportation by considering particle 3 through amplitude damping noise. For GHZ state, the fidelity can be directly given on the basis of Eq. (19), which is

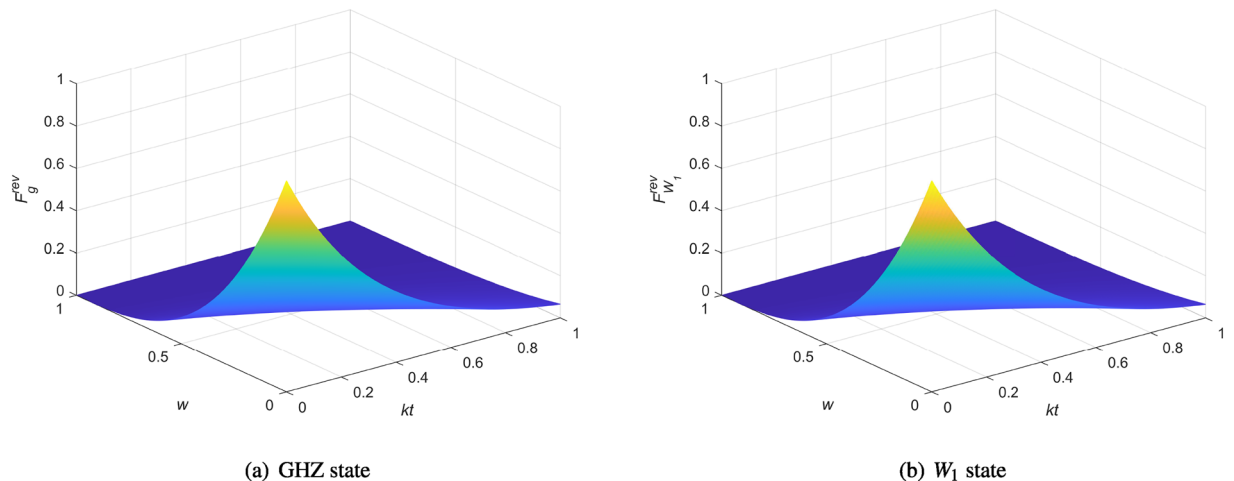


Figure 3. The fidelities of quantum teleportation using three-qubit entangled state with the help of WM and RM under amplitude damping noise.

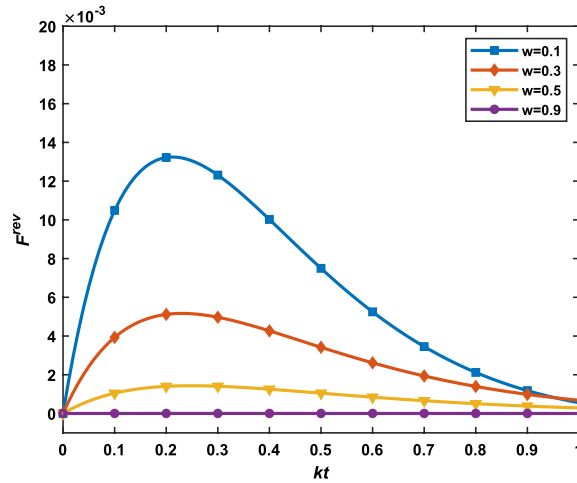


Figure 4. The comparison of fidelities of quantum teleportation between GHZ state and W_1 state with the help of WM and RM.

$$F_g^A = \frac{1}{2} \left[(\alpha^4 + \beta^4) \left(1 + e^{-\frac{k_3 t}{2}} \right) \right] + 2\alpha^2 \beta^2 e^{-\frac{k_3 t}{2}}. \tag{34}$$

For the case of W_1 state, we can obtain the fidelity by Eq. (25), the calculation result is shown as

$$F_{W_1}^A = \frac{1}{8} \left[\left(1 + \alpha^2 e^{-\frac{k_1 t}{2}} + \beta^2 \right)^2 + \left(1 + \alpha^2 + \beta^2 e^{-\frac{k_1 t}{2}} \right)^2 \right]. \tag{35}$$

If we consider that Bob prepares the initial entangled state $|\phi_g\rangle$, then Bob sends qubit-pair (1, 2) to Alice. The fidelities of teleportation also can be obtained by Eqs. (19) and (25) respectively for GHZ state and W_1 state, which are

$$F_g^B = \frac{1}{2} \left[(\alpha^4 + \beta^4) \left(1 + e^{-\frac{k_1 t}{2}} + e^{-\frac{k_2 t}{2}} \right) \right] + 2\alpha^2 \beta^2 e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}}. \tag{36}$$

and

$$F_{W_1}^B = \frac{1}{8} \left[\left(\alpha^2 + \alpha^2 e^{-\frac{k_2 t}{2}} + 2\beta^2 e^{-\frac{k_3 t}{2}} \right)^2 + \left(\beta^2 + 2\alpha^2 e^{-\frac{k_3 t}{2}} + \beta^2 e^{-\frac{k_3 t}{2}} \right)^2 \right]. \tag{37}$$

To compare the fidelities of these two cases with the original case, we performed a simulation of these three cases. Let $k_1 = k_2 = k_3 = k$, $\alpha = \beta = \frac{1}{\sqrt{2}}$, the numerical results are shown in Fig. 5. Plot (a) shows that the

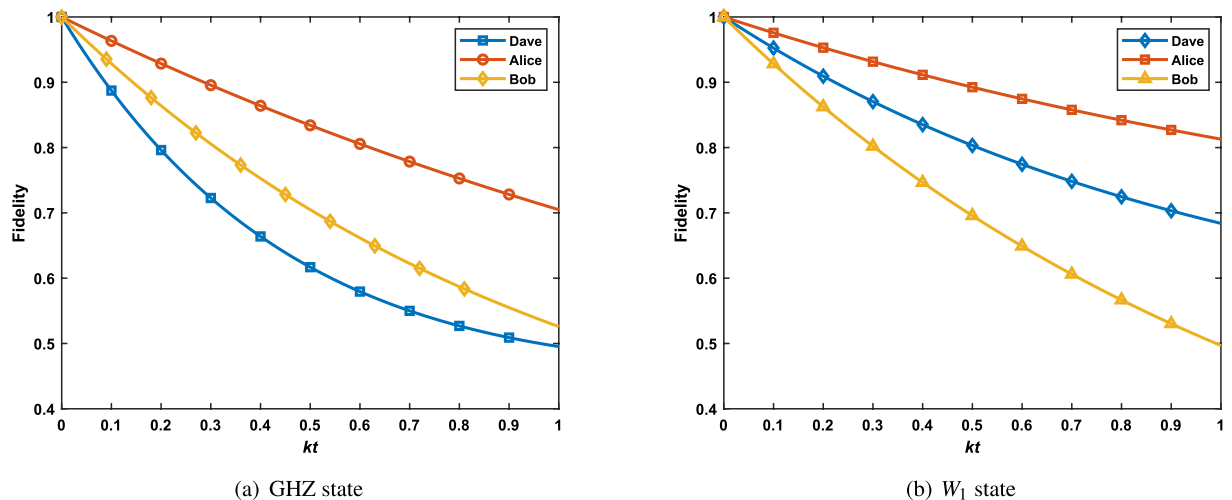


Figure 5. The fidelities of quantum teleportation of the initial states are prepared by Alice, Bob and Dave, respectively.

teleportation fidelity when Alice or Bob prepares GHZ state as the initial state is higher than that when Dave prepares the state. The teleportation fidelities about W_1 state are shown in plot (b). When Alice prepares the W_1 state at her laboratory, the fidelity of QT is more than the situation of preparing state in Dave's laboratory. While Bob prepares the W_1 state, the teleportation fidelity is reduced compared with Dave. Therefore, one can improve teleportation fidelity by selecting the proper preparation laboratory.

In addition, we make comparisons on teleportation fidelities between the modified protocol and original protocol under bit flip, bit-phase flip and phase flip noises, respectively. Similar to the case of amplitude damping noise, one can obtain the fidelities of quantum teleportation under bit flip, bit-phase flip, and phase flip noises by solving the Lindblad master equation, respectively.

Consider the particle to be sent through the noises, which is respectively described by the Lindblad operators $L_{i,\alpha} = \sqrt{k_{i,\alpha}}\sigma_\alpha$ ($\alpha = x, y, z$). Defining $p_{i,\alpha} = \frac{1}{2}(1 + e^{-2k_{i,\alpha}t})$, the evolution described by the Lindblad master equation at time t is equivalent to the Kraus operation represented by

$$E_{i,\alpha}^0 = \sqrt{p_{i,\alpha}}I, \quad E_{i,\alpha}^1 = \sqrt{1-p_{i,\alpha}}\sigma_\alpha, \quad (38)$$

where x, y, z correspond to the bit flip, bit-phase flip and phase flip noises.

Suppose that Dave prepare the initial state $|\phi_g\rangle$ and sends particles to Alice and Bob through bit flip, bit-phase flip and phase flip noises respectively. The fidelities of quantum teleportation are

$$F_{g^x}^D = \frac{1}{8}(D_1^x + D_8^x) + \frac{1}{2}\alpha^2\beta^2(D_2^x + D_7^x), \quad (39)$$

$$F_{g^y}^D = \frac{1}{8}(D_1^y + 4\alpha^2\beta^2D_7^y), \quad (40)$$

and

$$F_{g^z}^D = \frac{1}{8}D_1^z, \quad (41)$$

where

$$\begin{aligned} D_1^\alpha &= (1 + e^{-2k_{1,\alpha}t})(1 + e^{-2k_{2,\alpha}t})(1 + e^{-2k_{3,\alpha}t}), & D_2^\alpha &= (1 + e^{-2k_{1,\alpha}t})(1 + e^{-2k_{2,\alpha}t})(1 - e^{-2k_{3,\alpha}t}), \\ D_3^\alpha &= (1 + e^{-2k_{1,\alpha}t})(1 - e^{-2k_{2,\alpha}t})(1 + e^{-2k_{3,\alpha}t}), & D_4^\alpha &= (1 + e^{-2k_{1,\alpha}t})(1 - e^{-2k_{2,\alpha}t})(1 - e^{-2k_{3,\alpha}t}), \\ D_5^\alpha &= (1 - e^{-2k_{1,\alpha}t})(1 + e^{-2k_{2,\alpha}t})(1 + e^{-2k_{3,\alpha}t}), & D_6^\alpha &= (1 - e^{-2k_{1,\alpha}t})(1 + e^{-2k_{2,\alpha}t})(1 - e^{-2k_{3,\alpha}t}), \\ D_7^\alpha &= (1 - e^{-2k_{1,\alpha}t})(1 - e^{-2k_{2,\alpha}t})(1 + e^{-2k_{3,\alpha}t}), & D_8^\alpha &= (1 - e^{-2k_{1,\alpha}t})(1 - e^{-2k_{2,\alpha}t})(1 - e^{-2k_{3,\alpha}t}). \end{aligned} \quad (42)$$

If Alice or Bob prepares the initial state $|\phi_g\rangle$, one can obtain the corresponding fidelities by setting $k_{3,\alpha} = 0$ or $k_{1,\alpha} = k_{2,\alpha} = 0$.

When Dave prepares the initial state, the fidelities of teleportation using W_1 state under bit flip, bit-phase flip and phase flip noises are

$$F_{W_1^x}^D = \frac{1}{8}(D_1^x + \alpha^2\beta^2D_8^x) + \frac{1}{4}\alpha^2\beta^2(2D_2^x + D_3^x + D_5^x) + \frac{1}{16}[D_4^x + D_6^x + (\alpha^4 + \beta^4)D_7^x], \quad (43)$$

$$F_{W_1^y}^D = \frac{1}{8}D_1^y + \frac{1}{16}[D_4^y + D_6^y + (\alpha^4 + \beta^4)(D_7^y + D_8^y)], \quad (44)$$

and

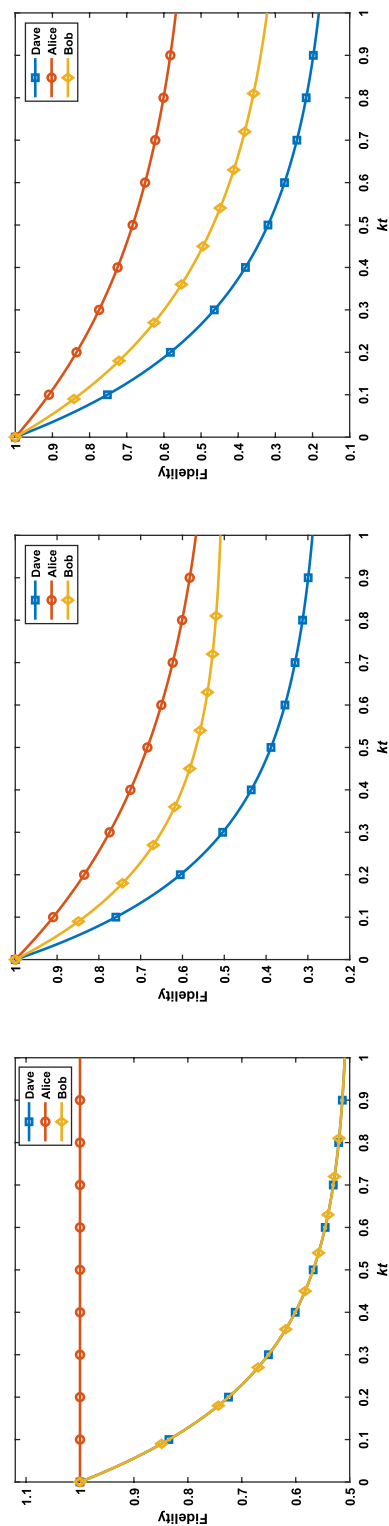
$$F_{W_1^z}^D = \frac{1}{8}(D_1^z + D_8^z) + \frac{1}{16}(\alpha^4 + \beta^4)(D_3^z + D_4^z + D_5^z + D_6^z). \quad (45)$$

Let $k_{3,\alpha} = 0$ or $k_{1,\alpha} = k_{2,\alpha} = 0$, then the fidelities of quantum teleportation of the initial states W_1 are prepared by Alice or Bob are calculated.

To demonstrate the effect of bit flip, bit-phase flip and phase flip noises on the the modified protocol and original protocol, we set all parameters $k_{i,\alpha}$ ($i = 1, 2, 3, \alpha = x, y, z$) be equal to k and $\alpha = \beta = \frac{1}{\sqrt{2}}$. The simulation results for GHZ state and W_1 state are shown in Figs. 6 and 7, respectively. The figures show that when Alice and Bob prepared the initial entangled state, the fidelities of QT are higher than that Dave under all the three types of noises. In particular, the fidelity of QT is not affected by the bit flip noise in the case that the initial state (GHZ state or W_1 state) is prepared by Alice. Therefore, choosing a suitable preparation laboratory can improve the fidelity of QT.

Conclusion

In this work, we study the efficiency of QT with three-qubit entangled state in a noisy environment by solving the Lindblad master equation. The result shows that using W_1 for teleportation can resist more amplitude damping noise than GHZ state during the same time. We further consider the efficiency that quantum teleportation with the help of WM and RM. It shows that W_1 always performs better than GHZ under the same practical condition. Furthermore, we found that for the GHZ and W_1 states used in the given QT protocol, performing WM and its

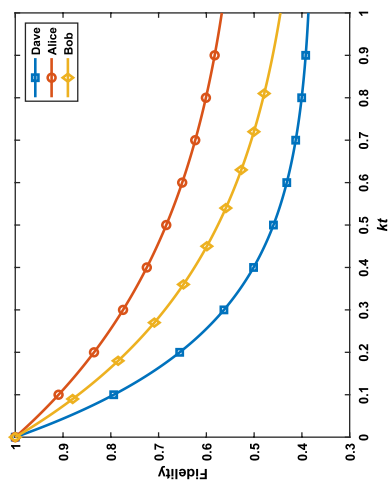


(a) Bit flip

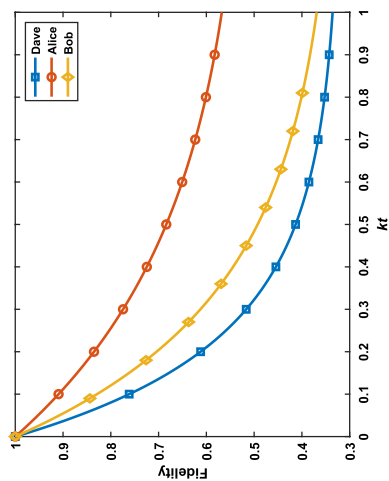
(b) Bit-phase flip

(c) Phase flip

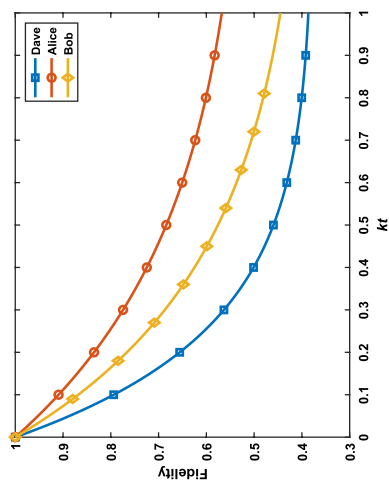
Figure 6. The fidelities of quantum teleportation of GHZ states are prepared by Alice, Bob and Dave, respectively.



(a) Bit flip



(b) Bit-phase flip



(c) Phase flip

Figure 7. The fidelities of quantum teleportation of W_1 states are prepared by Alice, Bob and Dave, respectively.

inverse in amplitude damping noise did not improve fidelity. The research presented here shows that WM and its inverse do not necessarily increase the efficiency of teleportation. Finally, we demonstrate that the fidelities of QT protocol under various types of noises are closely related to the participant who prepares the initial state. It indicates that one can improve the fidelity of QT by modifying the protocol to choose the optimal preparation laboratory.

Methods

Amplitude damping noise. For the given quantum teleportation, Alice and Bob should share a three-qubit entangled state and the three transmitted qubits always undergo quantum noises during the phase of entanglement distribution. Suppose $\hat{\rho}$ is the entangled state before it is sent to Alice and Bob. For what concerns the present work, we restrict our analysis to the amplitude damping noise, which is an important example of open quantum system evolution¹⁵. The master equation describing the dynamics of entangled quantum state under noises is given by Eq. (5).

Consider the particle to be sent through the amplitude damping noise, which is described by the Lindblad operator $L_{i,\alpha} = \sqrt{k_{i,-}}\sigma_{-}^{(i)}, \sigma_{-} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, the constant $k_{i,-}$ (or k_i) is approximately equals to the inverse of decoherence time. Moreover, the first term of Eq. (5) can be removed when we make a change of variables. We consider that H_s is a time independent Hermitian matrix, then one can assume that $H_s = 0$. Accordingly, the equation of motion for $\hat{\rho}$ is easily found to be

$$\begin{aligned} \frac{\partial \hat{\rho}}{\partial t} = & k_1 \left(\sigma_{-}^{(1)} \hat{\rho} \sigma_{+}^{(1)} - \frac{1}{2} \sigma_{+}^{(1)} \sigma_{-}^{(1)} \hat{\rho} - \frac{1}{2} \hat{\rho} \sigma_{+}^{(1)} \sigma_{-}^{(1)} \right) + k_2 \left(\sigma_{-}^{(2)} \hat{\rho} \sigma_{+}^{(2)} - \frac{1}{2} \sigma_{+}^{(2)} \sigma_{-}^{(2)} \hat{\rho} - \frac{1}{2} \hat{\rho} \sigma_{+}^{(2)} \sigma_{-}^{(2)} \right) \\ & + k_3 \left(\sigma_{-}^{(3)} \hat{\rho} \sigma_{+}^{(3)} - \frac{1}{2} \sigma_{+}^{(3)} \sigma_{-}^{(3)} \hat{\rho} - \frac{1}{2} \hat{\rho} \sigma_{+}^{(3)} \sigma_{-}^{(3)} \right), \end{aligned} \tag{46}$$

where $\sigma_{\pm}^{(1)} = \sigma_{\pm} \otimes I \otimes I, \sigma_{\pm}^{(2)} = I \otimes \sigma_{\pm} \otimes I, \sigma_{\pm}^{(3)} = I \otimes I \otimes \sigma_{\pm}$, and $\sigma_{+} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$.

GHZ state. We now investigate the protocol of QT by using a generalized GHZ state under amplitude damping noise. For simplicity, one can directly discuss that the entangled state is affected by the amplitude damping noise, which is shown in Eq. (46). Assume that Alice and Bob use a generalized GHZ state to achieve QT, which has the form of

$$|\phi_g\rangle_{123} = \lambda_0|000\rangle + \lambda_1|111\rangle, \tag{47}$$

where λ_0, λ_1 are real and satisfy $|\lambda_0|^2 + |\lambda_1|^2 = 1$. Its density matrix is

$$\hat{\rho}_g^{(0)} = \begin{pmatrix} \lambda_0^2 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_0\lambda_1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \lambda_0\lambda_1 & 0 & 0 & 0 & 0 & 0 & 0 & \lambda_1^2 \end{pmatrix}. \tag{48}$$

The master equation (46) reduces to 8 diagonal equations and 28 off-diagonal equations. One can easily solve these equations with the initial conditions in Eq. (48). The analytic form of a generalized GHZ state evolution under amplitude damping noise is

$$\hat{\rho}_g(t) = \begin{pmatrix} \rho_{00}^g(t) & \cdots & \rho_{07}^g(t) \\ \vdots & \ddots & \vdots \\ \rho_{70}^g(t) & \cdots & \rho_{77}^g(t) \end{pmatrix}, \tag{49}$$

where the elements are all zero except

$$\left\{ \begin{aligned} \rho_{00}^g(t) &= 1 - \lambda_1^2 \left[e^{-t(k_1+k_2+k_3)} + e^{-tk_1} + e^{-tk_2} + e^{-tk_3} - e^{-t(k_1+k_2)} - e^{-t(k_1+k_3)} - e^{-t(k_2+k_3)} \right], \\ \rho_{11}^g(t) &= \lambda_1^2 \left[e^{-t(k_1+k_2+k_3)} + e^{-tk_3} - e^{-t(k_1+k_2)} - e^{-t(k_2+k_3)} \right], \\ \rho_{22}^g(t) &= \lambda_1^2 \left[e^{-t(k_1+k_2+k_3)} + e^{-tk_2} - e^{-t(k_1+k_2)} - e^{-t(k_2+k_3)} \right], \\ \rho_{33}^g(t) &= \lambda_1^2 \left[-e^{-t(k_1+k_2+k_3)} + e^{-t(k_2+k_3)} \right], \\ \rho_{44}^g(t) &= \lambda_1^2 \left[e^{-t(k_1+k_2+k_3)} + e^{-tk_1} - e^{-t(k_1+k_2)} - e^{-t(k_1+k_3)} \right], \\ \rho_{55}^g(t) &= \lambda_1^2 \left[-e^{-t(k_1+k_2+k_3)} + e^{-t(k_1+k_3)} \right], \\ \rho_{66}^g(t) &= \lambda_1^2 \left[-e^{-t(k_1+k_2+k_3)} + e^{-t(k_1+k_2)} \right], \\ \rho_{77}^g(t) &= \lambda_1^2 \left[e^{-t(k_1+k_2+k_3)} \right], \\ \rho_{07}^g(t)(\rho_{70}^g(t)) &= \lambda_0 \lambda_1 \left[e^{-\frac{t}{2}(k_1+k_2+k_3)} \right]. \end{aligned} \right. \tag{50}$$

On the other hand, defining $p = 1 - e^{-kt}$ at time t , the evolution is equivalent to

$$\hat{\rho}_g(t) = \varepsilon(\hat{\rho}_g(0)) = \sum_{l=0}^1 \sum_{m=0}^1 \sum_{k=0}^1 E_l(1)E_m(2)E_k(3)\hat{\rho}_g(0)E_l^\dagger(1)E_m^\dagger(2)E_k^\dagger(3) \tag{51}$$

where $E_l(1) = E_l \otimes I \otimes I, E_m(2) = I \otimes E_m \otimes I, E_k(3) = I \otimes I \otimes E_k$ and $E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-p} \end{pmatrix}, E_1 = \begin{pmatrix} 0 & 0 \\ 0 & \sqrt{p} \end{pmatrix}$. Therefore, the evolved state is rewritten as

$$\begin{aligned} \hat{\rho}_g(t) &= \left(\lambda_0 |000\rangle + \lambda_1 e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |111\rangle \right) \left(\lambda_0 \langle 000| + \lambda_1 e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 111| \right) + \lambda_1^2 e^{-k_1 t} e^{-k_2 t} \bar{e}^{-k_3 t} |110\rangle \langle 110| \\ &+ \lambda_1^2 e^{-k_1 t} \bar{e}^{-k_2 t} e^{-k_3 t} |101\rangle \langle 101| + \lambda_1^2 e^{-k_1 t} \bar{e}^{-k_2 t} \bar{e}^{-k_3 t} |100\rangle \langle 100| + \lambda_1^2 \bar{e}^{-k_1 t} e^{-k_2 t} e^{-k_3 t} |011\rangle \langle 011| \\ &+ \lambda_1^2 \bar{e}^{-k_1 t} e^{-k_2 t} \bar{e}^{-k_3 t} |010\rangle \langle 010| + \lambda_1^2 \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} e^{-k_3 t} |001\rangle \langle 001| + \lambda_1^2 \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} \bar{e}^{-k_3 t} |000\rangle \langle 000|. \end{aligned} \tag{52}$$

where $\bar{e}^{-k_1 t} = 1 - e^{-k_1 t}, \bar{e}^{-k_2 t} = 1 - e^{-k_2 t}, \bar{e}^{-k_3 t} = 1 - e^{-k_3 t}$. Combined with the analysis above, one can obtain the evolved state $\rho_g(t)$, which describes the whole system under noise. The expression is given by

$$\begin{aligned} \rho_g(t) &= \rho_{in} \otimes \hat{\rho}_g(t). \\ &= \left(\alpha \lambda_0 |0000\rangle + \alpha \lambda_1 e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |0111\rangle + \beta \lambda_0 |1000\rangle + \beta \lambda_1 e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} |1111\rangle \right) \left(\alpha \lambda_0 \langle 0000| + \alpha \lambda_1 e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 0111| \right. \\ &+ \beta \lambda_0 \langle 1000| + \beta \lambda_1 e^{-\frac{k_1 t}{2}} e^{-\frac{k_2 t}{2}} e^{-\frac{k_3 t}{2}} \langle 1111| \left. \right) + \lambda_1^2 e^{-k_1 t} e^{-k_2 t} \bar{e}^{-k_3 t} (\alpha |0110\rangle + \beta |1110\rangle) (\alpha \langle 0110| + \beta \langle 1110|) \\ &+ \lambda_1^2 e^{-k_1 t} \bar{e}^{-k_2 t} e^{-k_3 t} (\alpha |0101\rangle + \beta |1101\rangle) (\alpha \langle 0101| + \beta \langle 1101|) + \lambda_1^2 \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} \bar{e}^{-k_3 t} (\alpha |0100\rangle + \beta |1100\rangle) (\alpha \langle 0100| + \beta \langle 1100|) \\ &+ \lambda_1^2 \bar{e}^{-k_1 t} e^{-k_2 t} e^{-k_3 t} (\alpha |0011\rangle + \beta |1011\rangle) (\alpha \langle 0011| + \beta \langle 1011|) + \lambda_1^2 e^{-k_1 t} e^{-k_2 t} \bar{e}^{-k_3 t} (\alpha |0010\rangle + \beta |1010\rangle) (\alpha \langle 0010| + \beta \langle 1010|) \\ &+ \lambda_1^2 \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} e^{-k_3 t} (\alpha |0001\rangle + \beta |1001\rangle) (\alpha \langle 0001| + \beta \langle 1001|) + \lambda_1^2 \bar{e}^{-k_1 t} \bar{e}^{-k_2 t} \bar{e}^{-k_3 t} (\alpha |0000\rangle + \beta |1000\rangle) (\alpha \langle 0000| + \beta \langle 1000|). \end{aligned} \tag{53}$$

W state. In this part we focus on the evolution of another type of three-qubit state under noise. Considering the W state is

$$|\phi_w\rangle_{123} = x|001\rangle + y|010\rangle + z|100\rangle, \tag{54}$$

where x, y, z are real and $x^2 + y^2 + z^2 = 1$. The initial state prepared by Dave has the form of

$$\hat{\rho}_w(0) = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & x^2 & xy & 0 & 0 & xz & 0 & 0 \\ 0 & xy & y^2 & 0 & 0 & yz & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & xz & yz & 0 & 0 & z^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \tag{55}$$

Suppose that particles 1, 2, 3 are respectively affected by amplitude damping noise, which is also described by Eq. (46). Solving the master equation associate with new boundary conditions of Eq. (55). One can express $\hat{\rho}_w(t)$ analytically in a form

$$\hat{\rho}_w(t) = \begin{pmatrix} \rho_{00}^w(t) & 0 & 0 & 0 & \rho_{04}^w(t) & 0 & 0 & 0 \\ 0 & \rho_{11}^w(t) & \rho_{12}^w(t) & 0 & 0 & \rho_{15}^w(t) & 0 & 0 \\ 0 & \rho_{21}^w(t) & \rho_{22}^w(t) & 0 & 0 & \rho_{25}^w(t) & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \rho_{40}^w(t) & 0 & 0 & 0 & \rho_{44}^w(t) & 0 & 0 & 0 \\ 0 & \rho_{51}^w(t) & \rho_{52}^w(t) & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad (56)$$

where

$$\begin{cases} \rho_{00}^w(t) = 1 - x^2 e^{-tk_3} - y^2 e^{-tk_2} - z^2 e^{-tk_1}, \\ \rho_{11}^w(t) = x^2 e^{-tk_3}, \rho_{22}^w(t) = y^2 e^{-tk_2}, \\ \rho_{44}^w(t) = z^2 e^{-tk_1}, \\ \rho_{04}^w(t)(\rho_{40}^w(t)) = xz \left[e^{-\frac{t}{2}k_1} - e^{-\frac{t}{2}(k_1+2k_3)} \right], \\ \rho_{12}^w(t)(\rho_{21}^w(t)) = xyz e^{-\frac{t}{2}(k_2+k_3)}, \\ \rho_{15}^w(t)(\rho_{51}^w(t)) = xze^{-\frac{t}{2}(k_1+2k_3)}, \\ \rho_{25}^w(t)(\rho_{52}^w(t)) = yze^{-\frac{t}{2}(k_1+k_2+k_3)}. \end{cases} \quad (57)$$

Similar to the situation of GHZ state, the evolved state $\hat{\rho}_w(t)$ can be written as

$$\begin{aligned} \hat{\rho}_g(t) = & (xe^{-\frac{k_3 t}{2}}|001\rangle + ye^{-\frac{k_2 t}{2}}|010\rangle + ze^{-\frac{k_1 t}{2}}|100\rangle)(xe^{-\frac{k_3 t}{2}}\langle 001| + ye^{-\frac{k_2 t}{2}}\langle 010| + ze^{-\frac{k_1 t}{2}}\langle 100|) \\ & + (x^2 e^{-k_3 t} + y^2 e^{-k_2 t} + z^2 e^{-k_1 t})|000\rangle\langle 000|. \end{aligned} \quad (58)$$

Therefore, the whole quantum system under noise becomes

$$\begin{aligned} \rho_w(t) = & (z\alpha e^{-\frac{k_1 t}{2}}|0100\rangle + y\alpha e^{-\frac{k_2 t}{2}}|0010\rangle + x\alpha e^{-\frac{k_3 t}{2}}|0001\rangle + z\beta e^{-\frac{k_1 t}{2}}|1100\rangle \\ & + y\beta e^{-\frac{k_2 t}{2}}|1010\rangle + x\beta e^{-\frac{k_3 t}{2}}|1001\rangle)(z\alpha e^{-\frac{k_1 t}{2}}\langle 0100| + y\alpha e^{-\frac{k_2 t}{2}}\langle 0010| \\ & + x\alpha e^{-\frac{k_3 t}{2}}\langle 0001| + z\beta e^{-\frac{k_1 t}{2}}\langle 1100| + y\beta e^{-\frac{k_2 t}{2}}\langle 1010| + x\beta e^{-\frac{k_3 t}{2}}\langle 1001|) \\ & + (x^2 e^{-k_3 t} + y^2 e^{-k_2 t} + z^2 e^{-k_1 t})(\alpha|0000\rangle + \beta|1000\rangle)(\alpha\langle 0000| + \beta\langle 1000|). \end{aligned} \quad (59)$$

Data availability

The datasets used and analyzed during the current study are available from the corresponding author upon reasonable request.

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Author contributions

Z.J.Z. contributed the idea. C.Y.Z. performed the calculations. C.Y.Z. wrote the main manuscript, Z.B.F, H.T.M, Z.J.Z. and C.Y.Z. made an improvement. All authors contributed to discussion and reviewed the manuscript.

Competing interests

The authors declare no competing interests.

Additional information

Correspondence and requests for materials should be addressed to Z.-J.Z.

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