scientific reports

OPEN

Investigation of sheath properties in a warm plasma with two kappa-distributed electrons and monoenergetic electron beam

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Sheath formation criterion of an electropositive plasma consisting of singly charged positive ions, two kappa-distributed electron species with different effective temperatures and a monoenergetic electron beam is investigated by the Sagdeev potential approach. Using this criterion, effects of electron beam, superthermality of electron species as well as temperature and concentration of positive ions on the sheath properties are studied numerically. It is shown that the temperature of positive ions, concentration and superthermality of electron species and presence of electron beam affect Bohm velocity of positive ions. Also, it is observed that density distribution of the charged particles and sheath thickness increase in the presence of electron beam. In addition, it is found that with increasing the ion temperature, the sheath width and density distribution of the charged particles in the sheath area decrease.

Many observations have confirmed the existence of both cool and hot electron populations with a non-Maxwell distribution function[1–4]. For example, analysing data from Cassini spacecraft by Schippers et al.[4] showed a best fit for both cool and hot electron velocity distributions with kappa distributions with low values of κ. Although the empirical evidence for the kappa distributions function dates back to the 1960s, its statistical justification occurred in the 1980s based on maximizing the q-entropy[5–7]. Kappa distributions may be produced in space and astrophysical plasmas by various mechanisms such as superstatistics[8,9], the effect of shock waves[10], pump acceleration mechanism[11] and etc.

The power law one dimensional kappa velocity distribution is given by:

$$f_\kappa(v) = \left(\frac{n_0 \Gamma(\kappa)}{\Gamma(\kappa - 1/2)(\pi\kappa\theta^2)^{1/2}} \right) \left(1 + \frac{v^2}{\kappa\theta^2} \right)^{-\kappa}, \tag{1}$$

where $f_\kappa(v)$ is distribution function, n_0 is the equilibrium density, Γ is the standard gamma function, κ is the parameter of the superthermal particles and is known as spectral index, $\theta = \left(\frac{2\kappa - 3}{\kappa} \frac{k_B T}{m_p} \right)^{1/2}$ denotes the effective thermal speed or most probable speed of the particles of mass m_p and temperature T and, k_B is the Boltzmann constant. From θ, we note that a well-defined value of effective thermal speed requires $\kappa > 3/2$[12]. Integrating the kappa distribution over velocity space, the electron number density can be written as follows:

$$n_e(\varphi) = n_{0e} \left(1 - \frac{e\varphi}{(\kappa_e - 3/2)k_B T_e} \right)^{-\kappa_e + 1/2}, \tag{2}$$

where n_{0e} and T_e are the equilibrium number density and temperature of electrons, respectively, $\kappa_e > 3/2$ is the spectral index and φ is the electrostatic potential.

During the last decade, kappa distribution has been used by many authors for various research purposes such as occurrence and propagation of electron-acoustic and ion-acoustic waves, dust-ion-acoustic shock waves and electromagnetic cyclotron instabilities in plasma with two kappa-distributed electron species[13–22]. On the other hand, sheath formation in plasmas is one of the fundamental issues in plasma physics. Recently, there has been a lot of interest in the study of plasma sheath formation in the presence of non-Maxwellian electrons[23–31]. As a result of these studies, it has been found that depending on the degree of superthermality of electrons, the

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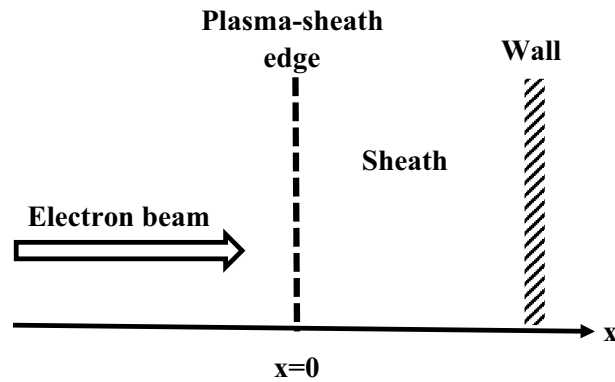
Figure 1. Schematic geometry of the sheath model.

velocity of the ions at the sheath edge could be more or less than their velocity in the corresponding plasma with Maxwellian electrons. Therefore, it can be concluded that the presence of non-Maxwellian electrons makes a significant contribution to the variation of plasma sheath properties.

Since electrons have a significant effect on the plasma sheath layer, the study of this layer in the presence of an electron beam, anisotropic distributed electrons, etc. is very important. For this reason, the effect of presence of an electron beam on plasma sheath properties has been the subject of many studies[33–39]. In 1990, Ingram and Braithwaite have used a quasi-neutral model to examine the nature of the plasma-sheath boundary in the presence of a high-energy monoenergetic electrons and Maxwellian electrons[32]. The behavior of the plasma sheath boundary in the presence of an electron beam with finite temperature has been examined by Bradley and Amemiya[33]. Pal et al.[34] have experimentally investigated the effect of an electron beam on plasma characteristics and boundary sheath in a dc discharge plasma. Demidov et al.[35] have discussed the sheath formation in plasmas with low density nonlocal fast electrons. Sharifian and Shokri[36] have studied the influence of the presence of a fast monoenergetic electron beam on the temporal evolution of the ion-matrix sheath in an electropositive plasma with Maxwellian electrons. Gyergyek et al.[37] have presented a fluid model of the sheath formation in front of a planar electron emitting electrode immersed in a plasma with Maxwellian electrons and a monoenergetic electron beam. Chekour et al.[38] have investigated the Bohm criterion in an electronegative dusty plasmas in the presence of a fast monoenergetic electrons and Maxwellian distributed electrons and negative ions. Ou et al.[39] have presented a sheath model to investigate the plasma-wall interaction in the fusion boundary layer in the presence of a fast monoenergetic electron beam.

In 2012, Badman et al.[40] reported the existence of electron beams at Saturn based on Cassini observations. Also, as mentioned earlier, the analysis of Cassini data revealed the existence of two species of electrons (cool and hot) with a kappa distribution function in Saturn[4]. These facts encourage us to study the structure of plasma sheath in a plasma consisting of fluid positive ions and two kappa-distributed electrons with different effective temperatures in the presence of a monoenergetic electron beam which, to our knowledge, has not yet been investigated. Therefore, the main focus of the current work is to discuss about the effect of presence of an electron beam on the sheath properties of a multi-component electropositive plasma containing ions and two kappa-distributed electron species with different effective temperatures. In order to do this, we first derive the sheath formation criterion for the mentioned plasma system and then we investigate the behavior of the plasma sheath characteristics in the presence of an electron beam. The results of the present work should be useful in understanding the properties of plasma sheath in space plasmas, such as Saturn's magnetosphere[13] and Pulsar magnetosphere[41] where the combined presence of electron beams and excess superthermal electrons may be encountered.

This work is arranged in four sections including the introduction as the first section. In "Model and basic equations" section, the basic equations of the considered plasma are presented. Bohm sheath criterion and the plasma sheath characteristics are investigated in "Results and discussion" section, and finally, our brief conclusion is presented in "Conclusion" section.

Model and basic equations

Let us consider an electropositive plasma containing warm fluid ions and two kappa-distributed electron species with two different effective temperatures in the presence of an electron beam. Assuming the x axis to be normal to the wall and the physical parameters change only along the x (see Fig. 1), the fluid equations for the warm, collisionless ions are described as

$$\frac{d}{dx}(n_i v_i) = 0, \tag{3}$$

$$m_i v_i \frac{dv_i}{dx} = -e\frac{d\varphi}{dx} - \frac{1}{n_i}\frac{dp_i}{dx}, \tag{4}$$

2

where e is the unit electric charge, φ is the electrostatic potential and n_i, v_i, m_i and p_i are density, velocity, mass and pressure of positive ions, respectively. The positive ion pressure p_i is related to the ion density as follows:

$$p_i = k_B T_i n_i^{\gamma_i} / n_{0i}^{\gamma_i - 1}, \tag{5}$$

where T_i is the ion temperature, n_{0i} is the ion density at the sheath edge ($x = 0$) and $\gamma_i = 3, 2, 5/3$ for unidimensional, bidimensional, or tridimensional adiabatic flow, respectively.

The number densities of kappa-distributed electron species are expressed as:

$$n_j = n_{0j} \left(1 - \frac{e\varphi}{k_B T_{ej}(\kappa_j - 3/2)} \right)^{-\kappa_j + 1/2}, \tag{6}$$

where $j = c, h$ refer to cool and hot electron species and T_{ej} and n_{0j} are the temperature and density of electron species j at the sheath edge, and $\kappa_j > 3/2$ is the spectral index of each electron species.

Similar to Ref.[37], we assume that there is an isotropic, monoenergetic electron beam with a unidirectional velocity distribution in the plasma medium. Therefore, in addition to the fact that the velocities of all electrons are the same (called v_b), the directions of their velocities are uniformly distributed in space. As a result, the velocity distribution of the monoenergetic electrons is defined as follows:

$$f_b(v) = \frac{n_{0b}}{4\pi v_b^2} \delta(v - v_b), \tag{7}$$

where n_{0b} is the density of the beam electrons far from the plane wall. Integrating $f_b(v)$ over the velocity space, the density of the beam electrons is written as follows[37]:

$$n_b(x) = \int_v f_b(v) d^3 v = \frac{n_{0b}}{2} \left(1 - \sqrt{-\frac{2e\varphi(x)}{m_e v_b^2}} \right). \tag{8}$$

From relation (8), it is seen that $n_b(x)$ decreases by increasing the electrostatic potential of the sheath layer. Moreover, to avoid complicating the presented fluid model, the effects of the collision of beam electrons in the presheath layer have been neglected.

Finally, the potential in the sheath φ is determined by the Poisson equation:

$$\frac{d^2\varphi}{dx^2} = \frac{e}{\varepsilon_0} (n_c + n_h + n_b - n_i), \tag{9}$$

where ε_0 is the electric permittivity of free space. The quasineutrality condition at the sheath edge of such a multi-component plasma is $n_{0i} = n_{0c} + n_{0h} + n_{0b}$.

We normalize the physical quantities in (3)–(6), (8) and (9) with the following normalized variables:

$$N_i = n_i/n_{0c}, \quad N_j = n_j/n_{0c}, \quad N_b = n_b/n_{0c}, \quad \sigma_j = T_{ej}/T_{ec}, \quad \xi = x/\lambda_D, \quad u_i = v_i/c_s,$$

$$u_e = v_b/c_s, \quad \delta = n_{0h}/n_{0c}, \quad \delta_b = n_{0b}/n_{0c}, \quad \sigma_i = T_i/T_{ec}, \quad \phi = -e\varphi/k_B T_{ec}, \quad \delta_i = n_{0i}/n_{0c}$$

where $j = c, h$, $\lambda_D = (\varepsilon_0 k_B T_{ec}/e^2 n_{0i})^{1/2}$ and $c_s = (k_B T_{ec}/m_i)^{1/2}$.

Using the above mentioned normalized variables for a unidimensional adiabatic ion flow, the basic equations of our fluid model can be written as follows:

$$\frac{d(N_i u_i)}{d\xi} = 0, \tag{10}$$

$$u_i \frac{du_i}{d\xi} = \frac{d\phi}{d\xi} - \frac{3}{2} \left(\frac{\sigma_i}{\delta_i^2} \right) \frac{dN_i^2}{d\xi}, \tag{11}$$

$$\frac{d^2\phi}{d\xi^2} = \frac{1}{\delta_i} \left[N_i - \left(1 + \frac{\phi}{\sigma_c(\kappa_c - \frac{3}{2})} \right)^{-\kappa_c + 1/2} - \delta \left(1 + \frac{\phi}{\sigma_h(\kappa_h - \frac{3}{2})} \right)^{-\kappa_h + 1/2} - \frac{\delta_b}{2} \left(1 - \sqrt{\frac{2\phi}{m u_e^2}} \right) \right], \tag{12}$$

where $m = m_e/m_i$.

Results and discussion

In this section, we are going to analytically derive the sheath formation criterion for an electropositive plasma consisting of singly charged positive ions and two kappa-distributed electron species in the presence of a monoenergetic electron beam. Without knowing this criterion, it will not be possible to use the normalized basic equations to numerically study the sheath structure in the considered plasma.

Multiplying the normalized Poisson equation by $(d\phi/d\xi)$ and integrating, we obtain

$$\left(\frac{d\phi}{d\xi}\right)^2 = \left(\frac{d\phi}{d\xi}\right)^2_{\xi=0} - 2S(\phi, u_{0i}), \tag{13}$$

where

$$S(\phi, u_{0i}) = -u_{0i}\int_{\phi_0}^{\phi}\frac{d\phi}{u_i} - \frac{\sigma_c}{\delta_i}\left(1 + \frac{\phi}{\sigma_c\left(\kappa_c - \frac{3}{2}\right)}\right)^{-\kappa_c+3/2}\Bigg|_{\phi_0}^{\phi}$$
$$- \frac{\sigma_h\delta}{\delta_i}\left(1 + \frac{\phi}{\sigma_h\left(\kappa_h - \frac{3}{2}\right)}\right)^{-\kappa_h+3/2}\Bigg|_{\phi_0}^{\phi} + \left(\frac{\delta_b}{2\delta_i}\phi - \frac{\delta_b}{3\delta_i}\sqrt{\frac{2}{mu_e^2}}\phi^{3/2}\right)\Bigg|_{\phi_0}^{\phi}, \tag{14}$$

is the Sagdeev potential and ϕ_0 and u_{0i} are the normalized electrostatic potential and normalized velocity of the positive ions at the sheath edge ($\xi = 0$), respectively.

From (14), it is found that $S(\phi_0, u_{0i}) = 0$ and $\partial S(\phi_0, u_{0i})/\partial\phi = 0$. Therefore, the sheath edge is an extremum point for the Sagdeev potential. However, it should be mentioned here that the concept of sheath edge resulting from the Bohm criterion is suitable for collisionless plasmas[42]. Maximizing S in the sheath edge, we obtain

$$\frac{\partial^2 S(\phi_0, u_{0i})}{\partial\phi^2} = \frac{1}{u_{0i}^2 - 3\sigma_i} - \frac{\delta_b}{4\delta_i}\sqrt{\frac{2}{mu_e^2\phi_0}} - \frac{1}{\delta_i}(A + B) \le 0, \tag{15}$$

or

$$u_{0i} \ge \left(3\sigma_i + \frac{\delta_i}{A + B + \frac{\delta_b}{4}\sqrt{\frac{2}{mu_e^2\phi_0}}}\right)^{1/2}, \tag{16}$$

where

$$A = -\frac{1}{\sigma_c}\left(\frac{\kappa_c - \frac{1}{2}}{\kappa_c - \frac{3}{2}}\right)\left(1 + \frac{\phi_0}{\sigma_c\left(\kappa_c - \frac{3}{2}\right)}\right)^{-\kappa_c+1/2}, \tag{17}$$

and

$$B = -\frac{\delta}{\sigma_h}\left(\frac{\kappa_h - \frac{1}{2}}{\kappa_h - \frac{3}{2}}\right)\left(1 + \frac{\phi_0}{\sigma_h\left(\kappa_h - \frac{3}{2}\right)}\right)^{-\kappa_h+1/2}. \tag{18}$$

Using the normalized quasineutrality condition at the sheath edge, $(N_c + N_h + N_b = N_i)_{\phi=\phi_0}$, it is easily found that

$$\delta_i = \left(1 + \frac{\phi_0}{\sigma_c(\kappa_c - \frac{3}{2})}\right)^{-\kappa_c+1/2} + \delta\left(1 + \frac{\phi_0}{\sigma_h(\kappa_h - \frac{3}{2})}\right)^{-\kappa_h+1/2} + \frac{\delta_b}{2}\left(1 - \sqrt{\frac{2\phi_0}{mu_e^2}}\right). \tag{19}$$

Relation (16) is the sheath formation criterion for an electropositive plasma consisting of warm positive ions, two species of kappa-distributed electrons and a monoenergetic electron beam. From (16), it is found that the Bohm velocity of positive ions at the sheath edge depends on different plasma parameters such as temperature and initial (unperturbed) density of ions and electron species, spectral index of each electron species, concentration and velocity of the electron beam and electrostatic potential at the sheath edge. Examining Eq. (16) for some specific cases can be useful. For example, ignoring the presence of electron beam and assuming $\phi_0 = 0$, relation (16) leads to the ion velocities at the sheath edge of an electropositive plasma with two species of kappa-distributed electrons[14]. Also, assuming $\delta_b = 0$ in the limit $\kappa_{c,h} \to \infty$, the modified sheath criterion (Eq. (16)) reduces to $u_{0i} \ge (1 + 3\sigma_i)^{1/2}$ for $\phi_0 = 0$ which is the sheath criterion for an ion-electron plasma with warm ions and Maxwellian-distributed electrons[43]. Finally, assuming that the ions are cold and the electrons are in thermal equilibrium ($\kappa_{c,h} \to \infty$), Eq. (16) will lead to the well-known Bohm criterion $u_{0i} \ge 1$ in the absence of the electron beam[44].

Using Eqs. (10)–(12) and (16), one can study the plasma sheath properties of an electropositive plasma consisting of warm fluid positive ions and two species of kappa-distributed electrons with different effective
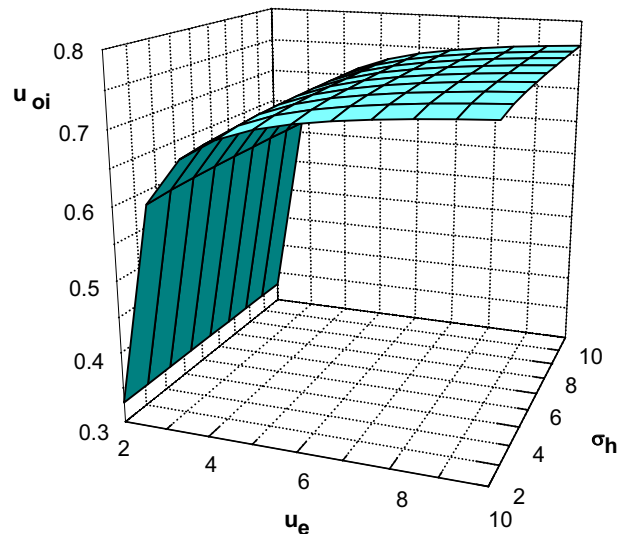
Figure 2. Ion velocities at the sheath edge as a function of u_e and σ_h for $\phi_0 = 0.008$, $\kappa_c = 2.4$, $\kappa_h = 4$, $\sigma_i = 0.02$, $m = 1/1836$, $\delta = 0.2$ and $\delta_b = 0.001$.
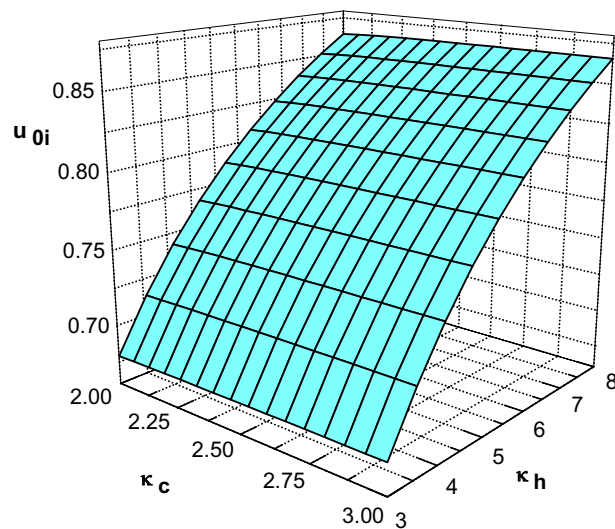

Figure 3. Ion velocities at the sheath edge as a function of κ_c and κ_h for $\phi_0 = 0.008$, $\sigma_h = 10$, $\sigma_i = 0.02$, $m = 1/1836$, $\delta = 0.2$ and $\delta_b = 0.001$.

temperature in the presence of a monoenergetic electron beam. Before doing so, we are going to investigate the behavior of the modified Bohm criterion with different plasma parameters.

Figure 2 shows variation of minimum velocity of the positive ions at the sheath edge u_{0i} with the electron beam velocity u_e and effective temperature ratio of hot-to-cool electron species σ_h for $\phi_0 = 0.008$, $\kappa_c = 2.4$, $\kappa_h = 4$, $\sigma_i = 0.02$, $\delta = 0.2$ and $\delta_b = 0.001$. Hereafter, to prevent the divergence of the second term in (16), we assume that ϕ_0 takes a nonzero but infinitesimal value in our numerical calculation. It is seen that an increase in beam velocity has a negligible effect on the ion velocities at the sheath edge. Also, similar to plasma with two nonextensive electron species[27], it is observed that u_{0i} increases by increasing the effective temperature ratio of hot-to-cool electron species.

Variation of u_{0i} with the spectral index of cool κ_c and hot κ_h electron species is shown in Fig. 3 for $\phi_0 = 0.008$, $\sigma_h = 10$, $\sigma_i = 0.02$, $\delta = 0.2$ and $\delta_b = 0.001$. Considering the typical range of κ_c and κ_h[4], it is seen that u_{0i} increases by increasing the spectral index of each electron species. As we know, increasing $\kappa_{c,h}$ causes the superthermal particles to decrease and therefore an increase in superthermality leads to lower u_{0i}. Similar result has been reported for plasmas without electron beam[14].

Effect of hot-to-cool electron initial densities δ and the beam density δ_b on the modified sheath formation criterion is indicated in Fig. 4 for $\kappa_c = 2.4$, $\kappa_h = 4$ and the other parameters of Fig. 3. One can see that any increase
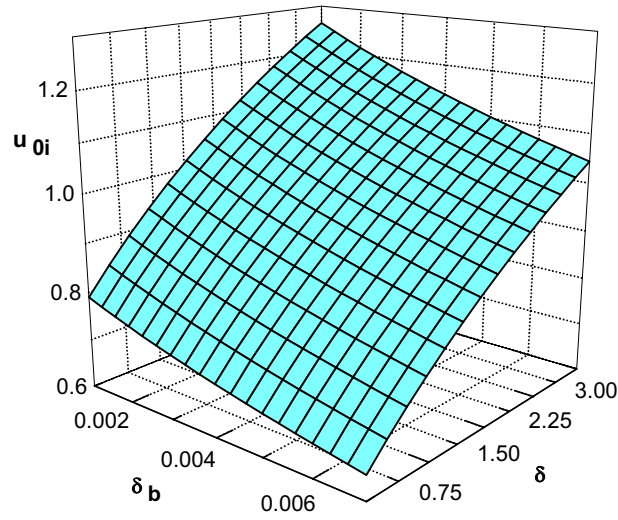
Figure 4. Ion velocities at the sheath edge as a function of δ and δ_b for $\phi_0 = 0.008, \sigma_h = 10, \sigma_i = 0.02$, $m = 1/1836, \kappa_c = 2.4$ and $\kappa_h = 4$.
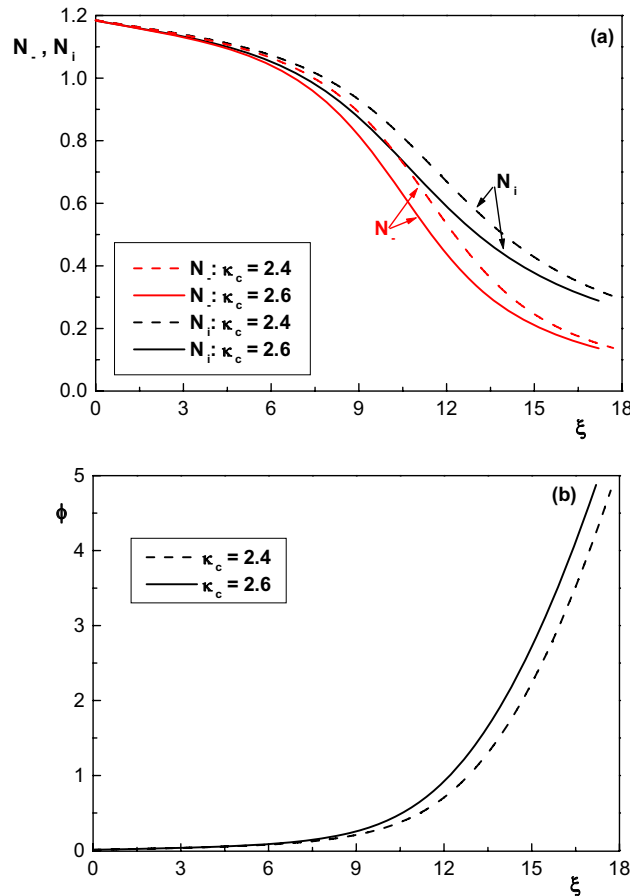

Figure 5. Spatial distribution of (**a**) normalized density of positive and negative particles and (**b**) normalized sheath potential for $\phi_0 = 0.008, \sigma_h = 10, \sigma_i = 0.02, m = 1/1836, \kappa_h = 4, \delta = 0.2, \delta_b = 0.002$ and different values of κ_c.
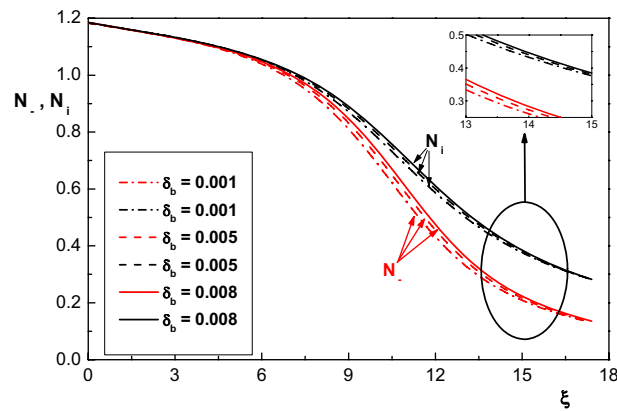
Figure 6. Spatial distribution of the normalized density of positive ions and negatively charged particles for $\kappa_c = 2.4$ and different values of δ_b. The other parameters are the same with Fig. 5.
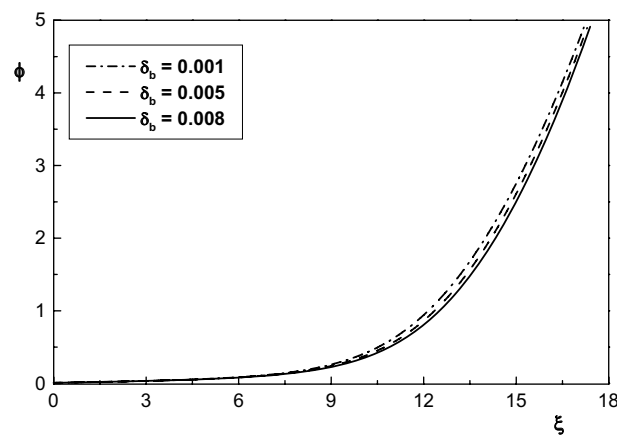

Figure 7. Spatial distribution of the normalized sheath potential for $\kappa_c = 2.4$ and different values of δ_b. The other parameters are the same with Fig. 5.

in density of hot electron species leads to an increase in the Bohm velocity of positive ions but the beam density has a different effect on the behavior of u_{0i} and as can be seen, with increasing δ_b, the Bohm velocity decreases.

Variation of density distribution of the charged particles (N_i and $N_- = N_c + N_h + N_b$) and electrostatic potential of the sheath region with the spectral index of cool electron species (κ_c) is illustrated in Fig. 5a,b, respectively for $\phi_0 = 0.008$, $\sigma_h = 10$, $\sigma_i = 0.02$, $\delta = 0.2$, $\kappa_h = 4$ and $\delta_b = 0.002$. Here, due to the similarity of the results, we have ignored the study of variation in N_i, N_- and ϕ with κ_h. Figure 5a shows density distribution of negative particles N_- decrease with κ_c. Given the fact that the number of fast electrons decreases with increasing the spectral index of electron species (e.g., κ_c), the decrease in negatively charged particle densities is understandable. Normalized density distribution of positive ions also decreases with increasing κ_c. As explained, any increase in κ_c causes a decrease in the population of energetic electrons. The result is a reduction in the number of electrons reaching the wall which in turn reduces the floating potential, and thus the number of ions. Moreover, Fig. 5b shows that the potential of the sheath also increases as κ_c increases which is due to decrease of floating potential with κ_c.

Now, we are going to investigate the effect of presence of monoenergetic electron beam (δ_b) on the properties of the sheath region of an ion-electron plasma with two species of kappa-distributed electrons. Figure 6 indicates variation of density distribution of positive ions with the beam density for the same parameters of Fig. 5. This figure shows that an increase in δ_b leads to an increase in density distribution of positive ions. Also, this figure shows variation of density distribution of negatively charged particles N_- with different values of δ_b which also increases with increasing δ_b. This can be explained by the fact that increasing the beam density increases the number of electrons reaching the cathode and thus increases the cathode potential. Therefore, the density of ions also increases with increasing δ_b. These results are consistent with those reported in Refs.[35,36] for a plasma with Maxwellian electrons and monoenergetic electron beam.

Effect of beam density on the electrostatic potential of the sheath region is depicted in Fig. 7. From this figure one can see that an increase in the beam density causes a decrease in the potential of sheath. Also, it is seen that the sheath thickness increases by increasing δ_b which is in agreement with the result of Refs.[35,36]. The reason
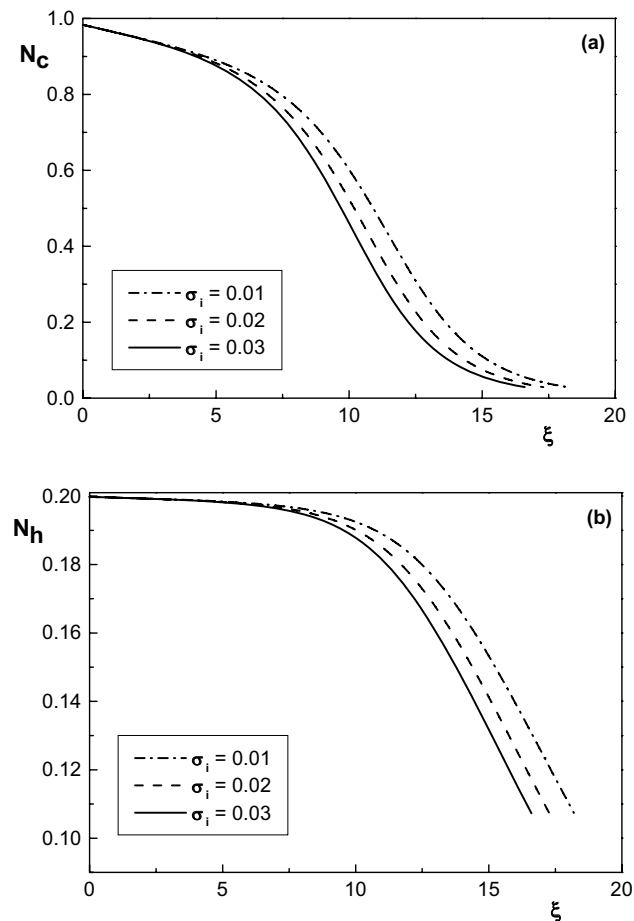
Figure 8. Spatial distribution of the normalized density of (**a**) cool and (**b**) hot electron species for $\kappa_c = 2.4$, $\delta_b = 0.005$ and different values of σ_i. The other parameters are the same with Fig. 5.

behind this is that the potential of the wall increases by increasing δ_b and as a result the sheath width decreases. Therefore, it can be concluded from Figs. 6 and 7 that density distribution of negatively charged particles and positive ions in the sheath increases in the presence of electron beam while the potential of the sheath region decreases.

At the end of this section, we study the effect of temperature of positive ions on the sheath properties of an electropositive plasma containing monoenergetic electron beam, warm positive ions and two kappa-distributed electron species. Figure 8a,b show the effect of positive ion temperatures on the density distribution of cool and hot electron species in the sheath region for $\delta = 0.2, \delta_b = 0.005, \kappa_c = 2.4, \kappa_h = 4, \phi_0 = 0.008, \sigma_h = 10$ and different values of σ_i. It is seen that density of both electron species decreases as the ion temperature increases. In addition, Fig. 9 shows that the behavior of density distribution of the negatively charged particles for different values of σ_i is similar to Fig. 8 and N_- decreases by increasing the positive ion temperatures. It is also seen that the positive ion density N_i is falling, too, which is due to the ion acceleration. A similar result has previously been reported in Refs.[43,45] for a warm Maxwellian plasma.

Finally, the effect of temperature of positive ions on the normalized potential of the sheath region of a plasma with two kappa-distributed electrons and electron beam is depicted in Fig. 10. It is seen that the normalized potential increases by increasing σ_i in the presence of electron beam. Also, in confirming the results of previous works, it is seen that the sheath width decreases with increasing ion temperature[46]. The reason for this is that as the temperature of the ions increases, u_{0i} and consequently the velocity of the ions hits the cathode increases. As a result, the sheath thickness decreases by increasing σ_i. Therefore, it can be concluded that density distribution of the charged particles decreases by increasing σ_i while electrostatic potential of the plasma sheath region increases.

Conclusion

Sheath formation criterion was investigated in a collisionless electropositive plasma with a monoenergetic electron beam by using the hydrodynamic equations. It was assumed that the plasma consisting of singly charged positive ions with finite temperature and two species of kappa-distributed electrons with different effective temperatures. Using Sagdeev potential approach, the modified Bohm criterion was derived and it was shown that the ion velocity at the sheath edge depends on various plasma parameters such as spectral index, concentration and temperature of each electron species, density and velocity of electron beam and concentration and temperature of
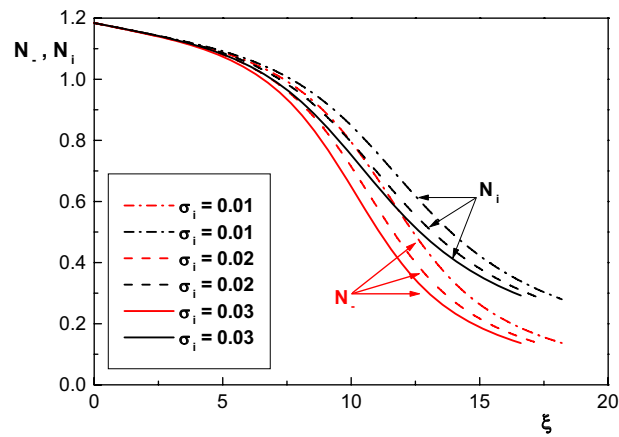
Figure 9. Spatial distribution of the normalized density of positive ions and negatively charged particles for $\kappa_c = 2.4, \delta_b = 0.005$ and different values of σ_i. The other parameters are the same with Fig. 5.
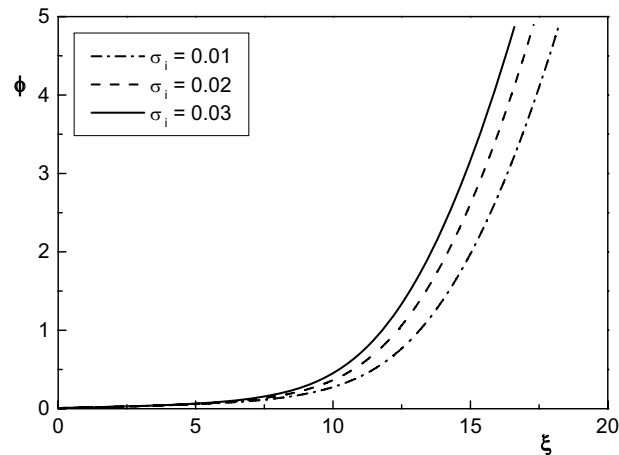

Figure 10. Spatial distribution of the normalized sheath potential for $\kappa_c = 2.4, \delta_b = 0.005$ and different values of σ_i. The other parameters are the same with Fig. 5.

the positive ions. Sheath region structure was also investigated in the presence of electron beam and warm positive ions. It was found that the presence of electron beam causes the density distribution of the charged particles in the sheath region as well as the sheath thickness to increase but the potential of the sheath to decrease. Also, it was shown that an increase in the temperature of positive ions leads to a decrease in the density distribution of ions and both cool and hot electron species while the potential of the sheath region increases. Moreover, it was found that an increase in spectral index of the electron species leads to a decrease (an increases) in density distribution of the charged particles (electric potential of the sheath region).

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Author contributions

M. M. Hatami proposed the concept, performed the analytical calculation as well as numerical simulations and analyzed numerical data.

Competing interests

The author declares no competing interests.

Additional information

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