scientific reports

Check for updates

OPEN

Publisher Correction: A hidden Markov model for lymphatic tumor progression in the head and neck

Roman Ludwig, Bertrand Pouymayou, Panagiotis Balermpas & Jan Unkelbach

Published online: 27 July 2021

Correction to: *Scientific Reports* https://doi.org/10.1038/s41598-021-91544-1, published online 10 June 2021

The original version of this Article contained repeated errors in the Equations.

Equations 1, 4, 15, 16, 21 and 26 were incorrectly aligned.

Equation 2 and 14 contained an incorrect separator (|).

Equation 3 contained a duplication of terms.

The pa(v) in Equations 3, 4 and 5 was incorrectly given in italics.

The product ($\prod$), sum ($\sum$) and fraction operators in Equations 6, 18, 19, 20, 22, 23, 24, 27 and 31 were incorrectly given in a lower-height format.

The numbering indicating Equation 15 was omitted.

The original Equations 1, 2, 3, 4, 5, 6, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 26, 27, 29 and 31 are included below.

$$P_{BN}\left(Z_v^k = z_v^k|X_v = x_v\right) = \left(z_v^k + (-1)^{z_v^k} \cdot s_P^k\right)(1 - x_v) + \left((1 - z_v^k) + (-1)^{1-z_v^k} \cdot s_N^k\right)x_v \tag{1}$$

$$P_{BN}\left(Z_v^k = 0\,|\,X_v = 1\right) = 1 - s_N^k \tag{2}$$

$$P_{BN}\left(X_v = x_v|X_{pa(v)} = x_{pa(v)}, b_v, t_{pa(v)v}\right) = x_v + (-1)^{x_v}(1 - b_v)\left(1 - t_{pa(v)v}\right)^{x_{pa(v)}} = x_v + (-1)^{x_v}(1 - b_v)\left(1 - t_{pa(v)v}\right)^{x_{pa(v)}} \tag{3}$$

$$\begin{aligned} P_{BN}\left(X_v = 0\,|\,X_{pa(v)} = 0\right) &= 1 - b_v \\ P_{BN}\left(X_v = 1\,|\,X_{pa(v)} = 0\right) &= b_v \\ P_{BN}\left(X_v = 0\,|\,X_{pa(v)} = 1\right) &= (1 - b_v)\left(1 - t_{pa(v)v}\right) \\ P_{BN}\left(X_v = 1\,|\,X_{pa(v)} = 1\right) &= 1 - (1 - b_v)\left(1 - t_{pa(v)v}\right) \end{aligned} \tag{4}$$

$$P_{BN}\left(X_v = x_v\,|\left\{X_{pa(v)} = x_{pa(v)}\right\}, \left\{t_{pa(v)v}\right\}, b_v\right) = x_v + (-1)^{x_v}(1 - b_v) \prod_{p \in pa(v)} \left(1 - t_{pv}\right)^{x_p} \tag{5}$$

$$P_{BN}(\mathcal{Z}\,|\theta) = \prod_{n=1}^{N} \sum_{\boldsymbol{x} \in \{0,1\}^V} \prod_{v=1}^{V} \prod_{k \in \mathcal{O}} P_{BN}\left(z_{nv}^k\,|\,x_v\right) P_{BN}\left(x_v\,|\,\left\{x_{\mathrm{pa}(v)}\right\}, \left\{t_{\mathrm{pa}(v)v}\right\}, b_v\right) \tag{6}$$

$$
\begin{aligned}
&P_{HMM}(\boldsymbol{x}[t+1] \mid \boldsymbol{x}[t]) \\
&= \prod_{v \in V} Q(x_v[t+1]; x_v[t]) \Big(P_{BN}\Big(x_v[t+1] | \big\{x_{\mathrm{pa}(v)}[t]\big\}, \big\{\tilde{t}_{\mathrm{pa}(v)v}\big\}, \tilde{b}_v\Big)\Big)^{1-x_v[t]}
\end{aligned} \tag{14}
$$

$$
\begin{aligned}
Q(X_v[t+1] = 0 \mid X_v[t] = 0) &= 1 \\
Q(X_v[t+1] = 0 \mid X_v[t] = 1) &= 0 \\
Q(X_v[t+1] = 1 \mid X_v[t] = 0) &= 1 \\
Q(X_v[t+1] = 1 \mid X_v[t] = 1) &= 1
\end{aligned} \tag{15}
$$

$$
\begin{aligned}
&P_{HMM}\big(\boldsymbol{X}[t+1] = \boldsymbol{\xi}_7 \mid \boldsymbol{X}[t] = \boldsymbol{\xi}_5\big) \\
&= Q(X_1[t+1] = 0 \mid X_1[t] = 0) P_{BN}\Big(X_1[t+1] = 0 \mid \tilde{b}_1\Big)^1 \\
&\quad \cdot\ Q(X_2[t+1] = 1 \mid X_2[t] = 1) P_{BN}\Big(X_2[t+1] = 1 \mid X_1[t] = 0, \tilde{t}_{12}, \tilde{b}_2\Big)^0 \\
&\quad \cdot\ Q(X_3[t+1] = 1 \mid X_3[t] = 0) P_{BN}\Big(X_3[t+1] = 1 \mid X_2[t] = 1, \tilde{t}_{23}, \tilde{b}_3\Big)^1 \\
&\quad \cdot\ Q(X_4[t+1] = 0 \mid X_4[t] = 0) P_{BN}\Big(X_4[t+1] = 0 \mid X_3[t] = 0, \tilde{t}_{34}, \tilde{b}_4\Big)^1 \\
&= \Big(1 - \tilde{b}_1\Big) \cdot 1 \cdot \Big(\tilde{b}_3 + \tilde{t}_{23} - \tilde{b}_3 \tilde{t}_{23}\Big) \cdot \Big(1 - \tilde{b}_4\Big)
\end{aligned} \tag{16}
$$

$$
P\Big(\boldsymbol{z} = \boldsymbol{\zeta}_j\Big) = \sum_{t \in \mathbb{T}} p(t) \cdot P\Big(\boldsymbol{z} = \boldsymbol{\zeta}_j, t\Big) = \left[\sum_{t \in \mathbb{T}} p(t) \cdot \boldsymbol{\pi}^\top \cdot (\boldsymbol{A})^t \cdot \boldsymbol{B}\right]_j \tag{18}
$$

$$
P(\mathcal{Z} | \theta) = \prod_{i=1}^{V \cdot |\mathcal{O}|} P\big(\boldsymbol{\zeta}_i \mid \theta\big)^{f_i} \tag{19}
$$

$$
P(\theta \mid \mathcal{Z}) = \tfrac{P(\mathcal{Z} \mid \theta) P(\theta)}{\int P(\mathcal{Z} \mid \theta') P(\theta') d\theta'} \tag{20}
$$

$$
P(\theta) = \begin{cases} 1 & \text{if } \theta \in \mathcal{S}^{V(V-1)} \\ 0 & \text{otherwise} \end{cases} \tag{21}
$$

$$
\log P(\mathcal{Z} \mid \theta) = \sum_{T=1}^{4} \log \left[\sum_{t \in \mathbb{T}} p_T(t) \cdot \boldsymbol{\pi}^\top \cdot (\boldsymbol{A})^t \cdot \boldsymbol{B}\right] \cdot \boldsymbol{f}_T \tag{22}
$$

$$
R(X_v = 1 \mid \boldsymbol{z}, \theta) = \tfrac{P(\mathbf{Z} = \boldsymbol{z} \mid X_v = 1, \theta) P(X_v = 1 \mid \theta)}{P(\mathbf{Z} = \boldsymbol{z} \mid \theta)} = \tfrac{\sum_{\{i : \xi_{iv} = 1\}} P(\mathbf{Z} = \boldsymbol{z} \mid \boldsymbol{\xi}_i, \theta) P(\boldsymbol{\xi}_i \mid \theta)}{P(\mathbf{Z} = \boldsymbol{z} \mid \theta)} \tag{23}
$$

$$
\mathbb{E}_{\boldsymbol{\theta}}[R(X_v = 1 \mid \boldsymbol{z})] = \tfrac{1}{L} \sum_{k=1}^{L} R(X_v = 1 \mid \boldsymbol{z}, \theta_k) \tag{24}
$$

$$
\mathrm{match}(\boldsymbol{d}, \boldsymbol{z}) := \begin{cases} \text{true} & \text{if } d_v^{\mathcal{O}} = z_v^{\mathcal{O}} \vee d_v^{\mathcal{O}} = \emptyset; \quad \forall v, O \\ \text{false} & \text{else} \end{cases} \tag{26}
$$

$$
R(X_v = 1 \mid \boldsymbol{d}, \theta) = \tfrac{P(\boldsymbol{d} \mid X_v = 1, \theta) P(X_v = 1 \mid \theta)}{P(\boldsymbol{d} \mid \theta)} = \tfrac{\sum_{\{i : \xi_{iv} = 1\}} P(\boldsymbol{d} \mid \boldsymbol{\xi}_i, \theta) P(\boldsymbol{\xi}_i \mid \theta)}{P(\boldsymbol{d} \mid \theta)} \tag{27}
$$

$$
P(\boldsymbol{d} \mid \theta) = \sum_{\left\{j : \mathrm{match}\left(\boldsymbol{d}, \boldsymbol{\zeta}_j\right)\right\}} \left[\sum_{t \in \mathbb{T}} p_T(t) \cdot \boldsymbol{\pi} \cdot (\boldsymbol{A})^t \cdot \boldsymbol{B}\right]_j \tag{29}
$$

$$
P(\boldsymbol{d} \mid \theta) = \sum_{t \in \mathbb{T}} p_T(t) \cdot \boldsymbol{\pi} \cdot (\boldsymbol{A})^t \cdot \boldsymbol{B} \cdot \boldsymbol{c}^{\boldsymbol{d}} \tag{31}
$$

The original Article has been corrected.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

© The Author(s) 2021