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Fractal grid-induced turbulence strength characterization via piezoelectric thin-film flapping velocimetry

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The centerline streamwise and cross-sectional ($x/D_h = 0.425$) turbulence characteristics of a 2D planar space-filling square-fractal-grid (SFG) composed of self-similar patterns superimposed at multiple length-scales is experimentally unveiled via piezoelectric thin-film flapping velocimetry (PTFV). The fluid–structure-interaction between a flexible piezoelectric thin-film and SFG-generated turbulent flow at $Re_{Dh} = 4.1 \times 10^4$ is investigated by analysis of the thin-film’s mechanical response. Measurements of the thin-film-tip deflection δ and induced voltage V demonstrate increasing flow fluctuation strength in the turbulence generation region, followed by rapid decay further downstream of the SFG. Interestingly, SFG-induced turbulence enables the generation of maximum centerline thin-film’s response (V_{rms} , δ_{rms}) and millinewton turbulence-forcing (turbulence-induced excitation force acting on the thin-film) F_{rms} which are respectively, 7× and 2× larger than the classical square-regular-grid of similar blockage ratio. The low frequency, large-scale energy-containing eddies at SFG’s central opening plays a critical role in driving the thin-film vibration. Most importantly, the SFG-generated turbulence at ($y/T = 0.106$, $z/T = 0.125$) away from the centerline allows equivalent mechanical characteristics of turbulence generation and decay, with peak of 1.9× nearer from grid. In short, PTFV provides a unique expression of the SFG-generated turbulence, of which, the equivalent turbulence length-scale and induced-forcing deduced could aid in deciphering the flow dynamics for effective turbulence management.

Fractal grid has received an increasing amount of interest recently as a turbulence generator to enhance fluid mixing and convective heat transfer^{1–4}. Fractal is a geometrical structure that repeats itself and diminishes in size, forming a complex pattern of several iterations^{5,6}. A square fractal grid (SFG) restructures the upstream flow to generate more intermittent vorticity field, higher vorticities and higher turbulence intensities when compared to classical regular grid (RG) of similar or smaller blockage ratio⁷. A piezoelectric transducer converts mechanical vibration into AC electrical signal through piezoelectric effect. Despite possessing high mechanical strength, a piezoelectric thin-film has a wide frequency range and is highly responsive to turbulent flow⁸. When immersed in a fractal grid-induced flow domain, the random vortical structures shed from the grid bars will impart a mechanical strain on the piezoelectric to generate electrical charges as a function of fluid fluctuating properties.

Significant work has been carried out lately to investigate the interplays between fluid flow properties and random excitation force against piezoelectric’s structural and electrical responses. Akaydin et al.⁹ theoretically estimated the piezoelectric tip deflection from open-circuit voltage measurement to characterize the interaction between piezoelectric beam and turbulent eddies in the wake of cylinder based on the statistical properties of the voltage and deflection’s local extremes. The piezoelectric tip deflection at various free stream velocities and distances from the test section wall was measured using a strain-gage to study the fluid–structure interaction in turbulent boundary layers¹⁰. Forces measurement and theoretical analysis demonstrated the dependence of turbulence-induced excitation force upon the local mean velocity, turbulence intensity and relative size of the beam length to the integral length scale of turbulence. Experimental work¹¹ showed that the force acting on the piezoelectric beam depends on the velocity and length scales of turbulence. The average power harvested can be modelled as a power-law with respect to the piezoelectric’s distance from the turbulator which closely follows the decay rate of turbulence kinetic energy. A frequency response model¹² was established to compute the drag force applied to a piezoelectric vibration sensor from the experimentally recorded voltage signal. The force

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Figure 1. Schematic of experimental setup: (a) Side-view and (b) top-view.

amplitude was reported to be linearly correlated with the Laser Doppler Anemometry-measured kinetic energy fluctuation intensity rather than the velocity fluctuation¹³.

In spite of the extensive research conducted over the past decade to study the behaviour of piezoelectric thin-film in highly unsteady flow, the majority of them focused on evaluating the piezoelectric performance in turbulent boundary layers^{10,14}, cylinder^{9,14,15} and passive rectangular grid-induced flow^{11,16,17} for energy harvesting purposes. To date, the characterization of fractal grid-induced turbulence based upon the direct interaction of the flow with a piezoelectric thin-film has not been given much consideration in literature. Although recent experimental studies^{18–20} were conducted to investigate the effects of grid-beam distance on the electrical power extracted by a single and two side-by-side piezoelectric beams from fractal grid-induced turbulence, the piezoelectric output profile over the grid's cross-section remains obscure. Moreover, the fractal grid-generated turbulence force acting on a flexible piezoelectric thin-film has yet to be unravelled.

Hence, the current study aims to experimentally unveil the interaction between a flexible piezoelectric thin-film and fractal grid-induced fluid flow by means of analysing the thin-film's physical response towards turbulent flow dynamic. In particular, the expression of local equivalent fluid flow undulation and the corresponding turbulence forcing (turbulence-induced excitation force acting on the thin-film) via an in-house established system, namely, the piezoelectric thin-film flapping velocimetry (PTFV) to comprehend the fundamental turbulence and the strength of grid-generated flow. In the present work, a polyvinylidene fluoride (PVDF) piezoelectric cantilever beam is placed at different lateral positions and centerline streamwise distances leeward of SFG and RG to measure the film's tip deflection and voltage response. A detailed exploration of the various equivalent turbulence statistics determined from the film undulation allows the mechanical characteristics of the grid-induced cross-sectional and centerline streamwise turbulent flow structures to be effectively expressed.

Methods

Experimental setup. A schematic of the experimental setup is presented in Fig. 1. The $T = 0.16 \text{ m}$ wide and 4.41 m long square wind tunnel is constructed from 10 mm thick transparent acrylic sheets with a bellmouth attached to the inlet. Air is drawn into the channel by an axial fan (Kruger MTD200, SG) at a fixed centerline inlet velocity $v_{in} = 4 \text{ m/s}$ with hydraulic diameter-based flow Reynolds number $Re_{Dh} = 4.1 \times 10^4$. Flow straighteners are placed at the inlet right after the bellmouth and in front of the fan at the outlet. A 10 mm thick 2D planar space-filling SFG with fractal iteration $N = 3$ is inserted at 2.02 m from the inlet. The geometry of the fractal insert is the same as that numerically optimized by Hoi et al.⁴ in their numerical study, with thickness ratio $t_r = 9.76$ ($t_r = t_0/t_s$, see Fig. 2a) and blockage ratio (ratio of area covered by grid over channel cross-section): $\sigma = 0.493$. The RG employed in this study has equal thickness and σ as the SFG. Figure 2a,b show the schematic of the inserts with their geometrical parameters listed in Table 1. Note that x_* denotes the wake-interaction length scale representing the location on the centerline where the wakes resulting from the largest grid bars interact, and can be expressed as²¹,

Figure 2. Schematic of (a) ($N=3$, $t_r=9.77$) SFG⁴ and (b) RG. Positions P_0 to P_{22} indicate the individual allocation of piezoelectric thin-film in the lee of (c) SFG and (d) RG.

Parameter	SFG	RG
σ	0.493	0.492
L_0 (mm)	77.0	20.0
L_1 (mm)	45.7	
L_2 (mm)	20.4	
t_0 (mm)	24.4	5.75
t_1 (mm)	5.00	
t_2 (mm)	2.50	
x_* (mm)	243.0	69.6

Table 1. Geometrical parameters of SFG and RG.

$$x_* = \frac{L_0^2}{t_0} \quad (1)$$

where L_0 and t_0 are the length and width of the largest grid bar, respectively.

The unimorph piezoelectric thin-film (TE Connectivity LDT1-028 K, CH) is composed of a PVDF film element with silver ink screen printed electrodes laminated to a Mylar sheet (see Fig. 3). The properties of the thin-film^{8,16,22} are summarized in Table 2. The thin-film is clamped onto a 3D printed holder and placed parallel to the flow with its output connected to a 10 M Ω external electrical load. The thin-film edge is fluorescently labelled and illuminated by an LED light source. The piezoelectric voltage signal is sampled and digitised at a sampling rate of 10³ Hz using a data acquisition device (LabJack U3-HV, US) with a voltage resolution of 4.88 mV/bit. Note that the voltage recorded has a high signal-to-electrical noise ratio of 47.3 even when the thin-film is vibrating at small amplitude. The voltage signal is logged for a period of 60 s to ensure the recording contains large enough samples to achieve stochastic convergence despite the random and unpredictable characteristic of turbulent flow. The lateral fluctuations of the thin-film are simultaneously captured for 10 s using an industrial

Figure 3. Schematic of the SDOF piezoelectric cantilever beam model subjected to a time varying arbitrary force $F(t)$.

Parameter	Symbol	Value
Length (mm)	l	$32^{[m]}$
	l_p	30
Width (mm)	b	$16^{[m]}$
	b_p	12
Thickness (mm)	h	$0.22^{[m]}$
	h_p	0.028
Young's modulus (N/m ²)	E_s	3.79×10^9
	E_p	3×10^9
Density (kg/m ³)	ρ_s	1390
	ρ_p	1780
Second moment of area (mm ⁴)	I	$0.0157^{[c]}$
Strain constant (C/N)	d_{31}	23×10^{-12}
	d_{33}	-33×10^{-12}
Stress constant (Vm/N)	g_{31}	216×10^{-3}
	g_{33}	-330×10^{-3}
Electromechanical coupling factor	k_{31}	12%
	k_t	14%
Permittivity (F/m)	ϵ	1.095×10^{-10}
Capacitance (nF)	C	1.38

Table 2. Dimensions and properties of piezoelectric thin-film^{8,16,22}. Subscript p represents PVDF film element and subscript s the Mylar substrate. Superscript $[c]$ indicates calculated and $[m]$ experimentally measured values.

high-speed camera (fps4000-720, UK) at 10^3 fps. The recorded images of pixel resolution 1280×720 are stored in the camera frame buffer memory before being transferred to the PC for processing. To ensure synchronized measurement of voltage signal and piezoelectric undulation, a simple light-emitting diode (LED) circuit as illustrated in Fig. 1b is employed. Where the start of both recordings is marked by a sharp increase in voltage across the LED just as the LED lights up the moment when circuit is closed. For each set of the experiment, six repeated recordings are performed.

The first series of wind tunnel experiments is conducted to examine the direct fluid–structure interaction of insert-induced turbulent flow and piezoelectric thin-film on the centerline of SFG and RG along different grid-film distances: $5 \leq x \leq 400$ mm i.e. $0.03 \leq x/D_h \leq 2.50$, where $D_h = 160$ mm is the hydraulic diameter of the wind tunnel. Following that, a preliminary study is performed in the turbulence generation region at $x/D_h = 0.43$ to confirm the symmetrical behaviour of the flow field leeward of SFG on positions P_2 , P_{2a} and P_{13} , P_{13a} (see Fig. 2c). In the second series of experiments, the thin-film is positioned in the lower quarter region of SFG and RG (i.e. $-68.5 \leq y \leq 0$ mm and $-67 \leq z \leq 0$ mm corresponding to $-0.43 \leq y/T \leq 0$ and $-0.42 \leq z/T \leq 0$, respectively) at $x/D_h = 0.43$ as indicated in Fig. 2c,d. It is important to note that all the piezoelectric placements are outside

Figure 4. Time history of the piezoelectric voltage V (a) by lightly flicking the thin-film's tip in the absence of air flow, and (b) at $x/D_h = 0.425$ leeward of SFG centerline.

the turbulent boundary layers created by the walls of test section, of which, the boundary layer displacement thickness is estimated to be 3.6 mm at $x/D_h = 0.43$ based on the standard turbulent boundary layer thickness equation in literature²³. This is to ensure the thin-film is completely vibrating in the grid-induced flow dynamic throughout the entire experiment. Lastly, the thin-film is positioned in the lee of SFG on P_5 where turbulence forcing is the maximum, with x/D_h varying from 0.25 to 0.81 to compare its x_{peak} against the one obtained on the centerline (P_0). For each of the positions investigated, the local mean velocities in x , y and z -directions are measured using a hotwire anemometer (Testo 405i, DE) of accuracy ± 0.3 m/s.

Image-detection algorithm. An in-house thin-film fluctuating image-detection algorithm is developed using MATLAB (version 9.7, USA) to identify the turbulence-induced piezoelectric thin-film tip deflection δ . During data post-processing, the camera recorded RGB images are first converted into binary series. Image pixels with luminance greater than certain threshold level are replaced with white pixels, while black pixels are used to substitute the remaining. Noise present in the output images is removed through Area Opening Operation. In addition, boundaries of the thin-film are computed and plotted followed by polynomial curve fitting of the film's immediate edge to justify the pixel length. Once the ratio of the thin-film physical length scale to pixel dimension, i.e. r has been secured, the δ can then be determined using,

$$\delta = \left(\frac{\sum Y_{ins}}{n} - Y_{ins} \right) \times r \quad (2)$$

where Y_{ins} is the instantaneous tip pixel coordinate and n is the total number of images processed. To ensure accurate δ detection, the luminance threshold during RGB to binary image conversion is set based on the following conditions. (1) Before the fan is turned on, the coordinate of the film at clamped end must match the coordinate obtained via zero-crossing edge-detection algorithm, and (2) the difference between the thin-film's pixel length for film fluctuation and initial film position are to be less than $\pm 1\%$. As a consequence of the current camera resolution and lens limitation, the δ detected has an average uncertainty of 0.02 mm.

Expression of turbulence forcing as a function of thin-film flapping. The electromechanical equation describing the δ and voltage V generated by a single degree-of-freedom (SDOF) piezoelectric thin-film cantilever beam subjected to an arbitrary force F (see Fig. 3) has been previously derived by^{24,25}. With the assumptions that the shear forces acting on the beam are very small and that the normal pressure forces are the predominant load^{10,11}, the time-dependent normal flow excitation force acting on the piezoelectric thin-film F_y can be expressed as,

$$m\ddot{\delta} + c\dot{\delta} + k\delta - \vartheta V = F_y \quad (3)$$

where m , c and k are respectively, the mass, damping and stiffness terms for a vibrating piezoelectric beam, while ϑ denotes the electromechanical coupling coefficient. For a forced vibration of a SDOF mass-spring-damper system, k and c are defined as follow,

$$k = (2\pi f_n)^2 m \quad (4)$$

$$c = 2\zeta \sqrt{km} \quad (5)$$

where f_n is the natural frequency and ζ the damping ratio. f_n of the piezoelectric thin-film is experimentally determined by lightly flicking the film tip to obtain the dominant frequency of the measured decaying voltage signal (see Fig. 4a) through Fast Fourier Transform (FFT). ζ on the other hand can be estimated from the FFT of experimentally detected δ using the half-quadratic gain method²⁶, namely,

Parameter	Symbol	Value
Mass (g)	m	0.17 ^[m]
Stiffness (N/m)	k	17.31
Damping (10^{-3} Ns/m)	c	1.30–7.77
Electromechanical coupling coefficient (10^{-6} C/m)	ϑ	7.69
Natural frequency (Hz)	f_n	50.78 ^[m]
Damping ratio (10^{-2})	ζ	1.20–7.16

Table 3. SDOF piezoelectric beam parameters. Superscript [m] denotes experimentally measured values.

$$\zeta = \sqrt{\frac{1}{2} - \sqrt{4 + 4\left(\frac{f_u - f_l}{f_p}\right)^2 - \left(\frac{f_u - f_l}{f_p}\right)^4}}^{-1} \quad (6)$$

where f_p is the frequency of peak amplitude in the FFT spectrum. f_u and f_l are respectively the frequencies obtained at half-quadratic gain level above and below f_p , whereby the amplitudes at half-quadratic gain level are $1/\sqrt{2}$ of the peak amplitude.

It is crucial to mention that the maximum δ recorded for the present piezoelectric thin-film is only 20% of its film length, and that the film length is $145 \times$ larger than the film thickness. Based on the classical Euler–Bernoulli beam theory previously used by^{14,24,25,27} to develop the piezoelectric energy harvester models, the deflection of a cantilevered beam undergoing transverse vibration in y -direction $y(x, t)$ can be related to the bending moment $M(x, t)$ as²⁸,

$$M(x, t) = EI(x) \frac{\partial^2 y(x, t)}{\partial x^2} \quad (7)$$

where x is the position along the beam length L . For a clamped-free beam with first mode vibration, the mode shape $\phi(x)$ at $x=L$ is defined as²⁸,

$$\phi(L) = \cosh \beta_1 L - \cos \beta_1 L - \sigma_1 [\sinh \beta_1 L - \sin \beta_1 L] \quad (8)$$

The weighted frequency $\beta_1 L = 1.875$ ²⁸ while the mode shape coefficients σ_1 is given by,

$$\sigma_1 = \frac{\sinh \beta_1 L - \sin \beta_1 L}{\cosh \beta_1 L + \cos \beta_1 L} = 0.7341 \quad (9)$$

From Elvin and Elvin²⁹, the ϑ in Eq. (3) can be deduced as,

$$\vartheta = d_{31} E_p b_p (2h_a - h_p) \sigma_1 \beta_1 \quad (10)$$

where h_a denotes the distance from the top of the PVDF layer to the neutral axis of composite beam. The calculated SDOF piezoelectric beam parameters are tabulated in Table 3. Once F_y in Eq. (3) are secured, the local turbulence forcing of grid-induced flow at various lateral positions and distances streamwise from the grids can be effectively unravelled.

Expression of local equivalent fluid flow undulation via PTFV. The V generated by the piezoelectric thin-film in the absence and presence of air flow are plotted in Fig. 4a,b, respectively to demonstrate the role of flow fluctuations in piezoelectric beam vibration response. With a light flick of the piezoelectric thin-film's tip in the absence of air flow, the V amplitude in Fig. 4a oscillates about the equilibrium and decays exponentially to zero in merely 0.3 s. This denotes that the thin-film is undergoing an underdamped vibration as a consequence of mechanical damping. Conversely, when air flow is present, the vortical structures shed from the grid bars induce piezoelectric thin-film vibration to generate a continuous V as a function of fluid fluctuating properties (see Fig. 4b). Since piezoelectric transducer is an AC-coupled device where electrical power is virtually generated from the turbulence fluctuations in the flow, and that the average power harvested exhibits trend and behaviour that is similar to that of the turbulence kinetic energy¹¹, we could therefore infer that V and δ are inter-related with the local flow undulation. Hence in the present study, we calculate the local equivalent lateral flow velocity fluctuation v' from the time derivative of experimentally detected δ , i.e., $\frac{d\delta}{dt}$. We stress that the v' calculated represents the velocity fluctuation that is qualitatively, but not quantitatively equivalent to the actual fluid fluctuating velocity obtained using direct hot-wire measurements. The various equivalent turbulence statistics deduced from the present v' will be discussed in the following section to provide insights into the mechanical characteristics of grid-induced turbulent flow.

Results and discussion

Effects of grid-film distance on insert-induced flow characteristics. Figure 5a presents the RMS voltage V_{rms} and RMS tip deflection δ_{rms} of the piezoelectric thin-film recorded at various streamwise distances x/D_h along SFG(P_0 , P_5) and RG centerline, with their corresponding dominant frequencies f_V and f_δ shown in

Figure 5. (a) RMS of experimentally recorded piezoelectric output voltage V_{rms} and tip deflection δ_{rms} , as well as (b) the corresponding dominant frequency of voltage signal f_v and thin-film flapping f_δ at different streamwise distances x/D_h along the leeward of SFG(P_0 , P_5) and RG centerline [note: Symbol denotes piezoelectric measurements and line the camera detection]. Grid-induced equivalent (c) turbulence intensity I_y and (d) lateral integral length scale L_y/T against x/D_h . (e) Interactions between wakes generated by the SFG bars at $y/D_h = -0.106$.

Fig. 5b. The local equivalent turbulence intensity I_y in y -direction, defined as the ratio of RMS v' to the inlet velocity (v'_{rms}/v_{in}) is plotted against x/D_h in Fig. 5c. The autocorrelation function of v' is obtained to determine its internal correlation within a time series by comparing v' at time t with that at time lag k . The first zero-crossing τ_{zc} of the autocorrelation plot denotes the period at which v' decorrelates from the primarily induced turbulence. By integrating the autocorrelation function of v' , the size of the large energy-containing eddies, viz. the equivalent

lent lateral integral length scale L_v is calculated as per Eq. (11). A comparison between the L_v for SFG₀, SFG₅ and RG₀ are depicted in Fig. 5d, where L_v is normalized against the tunnel width T .

$$L_v = v_{in} \int_0^{\tau_{sc}} \frac{\langle v'_t v'_{t+k} \rangle}{\langle v'^2 \rangle} dk \quad (11)$$

Figure 5a,c reveal the existence of two turbulent regions, viz. the turbulence generation and decay regions as described in literature^{7,21,30}. The V_{rms} , δ_{rms} and I_y induced by SFG-generated turbulence along P₀ and P₅ increase, respectively in the generation region to a peak at $x_{peak}/D_h = 0.813$ and 0.425 , which are then decrease exponentially further downstream in the decay region. For RG, the values reach a peak at $x_{peak}/D_h = 0.125$ which are 5 to 7 × lower than SFG₀ before experiencing a power-law decay with respect to x/D_h . The overall 5 × higher I_y flow generated by the SFG along the centerline as compared to RG of similar σ is in line with the previous studies^{7,20}. Essentially, the vorticity of vortical structures (eddies) shed from the grid bars induces fluctuations in velocity perpendicular to the piezoelectric thin-film surface which brings fluid up and down towards the thin-film¹¹. Hence, higher I_y promotes higher conversion of induced fluid dynamic perturbation into vibrational energy, giving rise to a 6 × higher average V_{rms} and δ_{rms} attained downstream of SFG₀ in comparison to RG₀. It is also important to note that although the signal-to-noise ratio of δ for small thin-film undulation is 9.3 × lower than the voltage signal-to-electrical noise ratio, the primary signal, or the large-scale fluctuations of the δ and V responses are rather well fitted to each other as depicted in Fig. 12b. Therefore, the statistical results deduced from the camera recorded δ are deemed reliable as evidenced in Fig. 5a,c where similar profiles are obtained for V_{rms} , δ_{rms} and I_y .

The further peak point observed for SFG₀ as compared to SFG₅ in Fig. 5a,c is due to the wake interactions of different-size bars at different x/D_h . The typical wake width w at a streamwise distance x based on a wake-generating bar of thickness t is approximated as $w \sim \sqrt{xt}$ ²¹. Figure 5e illustrates the possible wake interactions originating from the bars of SFG at $y/D_h = -0.106$. Along position P₅ (shaded in blue), the wakes generated by the largest bar and smallest bar nearest to the midplane interact at a distance $x/D_h \approx 0.4$. This location is considerably closer to the grid as compared to the interaction between the wakes of largest grid bars on the centerline which occurs at $x_*/D_h = 1.5$. On the contrary, the peak location for RG₀ occurs closest to the grid since RG has a smaller x_* compared to SFG. In the present study, $x_{peak}/x_* = 0.54, 0.28$ and 0.29 for SFG₀, SFG₅ and RG₀, respectively. The larger x_{peak}/x_* value attained for SFG₀ as compared to the 0.45 constant determined by Mazellier and Vassilicos²¹ is due to our current SFG having a σ which is almost double than that of their grids. Such high σ is a direct result of the large t_0/L_0 ratio (0.32) which enlarges the size of the recirculation regions immediately leeward the largest grid bars, causing the streamlines of the flow to diverge towards the tunnel walls⁷. This leads to a delay in the wake-interaction from the largest bars and prompts the wakes to meet further downstream.

As seen in Fig. 5b, although f_v and f_δ in the lee of SFG have variations of less than 15% from the f_n of the piezoelectric beam, its vibration frequency is affected to some extent, by the characteristic frequency of the vortices shed from the grid bars. The overall dominant frequencies f of SFG₀ are the lowest, followed by SFG₅ and lastly RG₀. This can be explained by the larger overall L_v/T of SFG₀ evident in Fig. 5d, of which large, slowly rotating eddies possess lower frequency while small eddies experience rapid fluctuations. As a result, the L_v/T profiles are a complete reverse of the f profiles. For the case of SFG₀, the L_v/T calculated are the same order of magnitude as those determined by Hurst and Vassilicos³⁰. The L_v for SFG₀ ranges from 11.5 to 13.7 mm, which on average is 1.1 to 1.3 × larger than SFG₅ and RG₀. In grid-generated turbulence, the size of the largest eddies is set by the characteristic geometric length scale of the mean flow. Since L_0 is smaller for RG, its L_v/T are considerably lower than SFG, whereas the difference in L_v/T for SFG₀ and SFG₅ are due to the multi-scale vortices shed from the multiple-length bars of SFG.

From Fig. 5b,d, we further verify the existence of two distinct flow dynamic regimes downstream of the SFG. The highly irregular f and L_v/T observed near the grid corresponds to the turbulence generation region where the flow is inhomogeneous and anisotropic, while the relative uniform f and L_v/T obtained further downstream signifies the homogeneous and isotropic flow in the decay region. Additionally, it is observed that there is a drop in f when V_{rms} and δ_{rms} are the maximum for SFG₀ and SFG₅. Low f reflects large-scale energy-containing turbulent eddies with significant amplitude of velocity fluctuations, acting as the primary components of the driving mechanism for thin-film flapping. Hence, a peak in film bending and voltage generation is generally accompanied by an increase in L_v/T and a drop in f . This behaviour however, is not observable for the case of RG. Interestingly, even though the eddies for SFG₅ at $x_{peak}/D_h = 0.425$ have a smaller L_v/T and higher f compared to SFG₀ at $x_{peak}/D_h = 0.813$, their V_{rms} and δ_{rms} are almost equivalent, with the I_y of SFG₅ to be 1.1 × higher than SFG₀. The wake-interaction phenomenon illustrated in Fig. 5e could provide a possible explanation, of which, x_{peak} of SFG₅ is in close proximity to the location where the wake from the largest bar encounters the wake from the smallest bar nearest to the midplane. This implies that wake interactions could further intensify the local turbulence strength generated by the SFG which adds to the higher-than-expected peak responses obtained for SFG₅. Conversely, whilst the peak point on the centerline is bounded by the location where the inward spreading wakes from the largest bars interact with each other, its peaks are mainly attributable to the higher energy containing large-scale effective flow structures shed from the grid bars.

To comprehend the Gaussianity of v' , we observe the skewness S_v (Fig. 6a) and flatness F_v (Fig. 6b) of v' against x/D_h for SFG₀, SFG₅ and RG₀, where S_v and F_v are computed as,

$$S_v = \frac{\langle v'^3 \rangle}{\langle v'^2 \rangle^{3/2}} \quad (12)$$

Figure 6. (a) Thin-film undulation skewness S_v , and (b) flapping flatness F_v , against x/D_h .

$$F_v = \frac{\langle v'^4 \rangle}{\langle v'^2 \rangle^2} \quad (13)$$

As shown in Fig. 6a,b, both grids reveal S_v and F_v values correspond to Gaussian distribution ($S_v \approx 0$, $F_v \approx 3$) except for SFG₀ at $x/D_h = 0.43$, where F_v has a relatively larger magnitude ($F_v = 3.5$), suggesting that the v' are generally close to zero yet prone to extreme fluctuations sporadically. Furthermore, SFG demonstrates higher variations in S_v and F_v compared to the rather uniform values obtained for RG at all x/D_h as a consequence of free decay homogenous isotropic turbulence³¹. The variability of S_v and F_v with x/D_h could be an indication of the inhomogeneity in fractal grid-generated turbulence particularly in the turbulence generation region. Nevertheless, the extraordinary large longitudinal velocity flatness F_u and negative velocity skewness S_u observed by^{21,32} along the centerline at $x/x_* \approx 0.2$ are not evident in Fig. 6a,b. We postulate that these intense decelerating flow events either do not occur for a lateral flow or, the inherited strength is rather inferior. This could be due to the complex interaction between the flow and the thin-film structure as compared to direct hot-wire measurements, in which PTFV may have potentially filtered out information relating to the higher order statistics of turbulent flow. However, more research is needed to confirm this hypothesis, such as the use of different geometrical and mechanical properties' piezoelectric thin-film to examine how the inherited strength may be responsible for the strong suppression of the non-Gaussian behaviour of v' .

Figure 7a–c present the streamwise energy spectra E_v for SFG₀, SFG₅ and RG₀ with the left column of figures showing the 3D downstream flow dynamic's energy spectra, and top view along the right. The energy spectra are obtained from the Fourier transform of the autocovariance function, i.e., v' . As shown in Fig. 7, there appears to be multiple peaks in the spectrum for individual streamwise distance, indicating that energy is distributed over a multitude of vortices with a wide range of spatial scales and characteristic frequencies. The major peaks occurring near frequency $f \approx 49$ Hz, 55 Hz and 62 Hz, respectively for SFG₀, SFG₅ and RG₀ are the results of vortex shedding from the grid bars^{33–35}. One can infer that the vortex shedding effect from the SFG bars is more pronounced, with the major peak E_v at x_{peak}/D_h leeward of SFG to be approximately $30 \times$ higher in comparison to RG. Such intense vortex shedding observed for the case of SFG₀ could also have masked the non-Gaussian behaviour of v' in the turbulence generation region³³, causing the corresponding S_v and F_v values in Fig. 6a,b to be close to 0 and 3, respectively. Moreover, more energetic fluctuations ($E_v > 10^{-5}$) are detected for a broader range of f in the turbulence generation region leeward of SFG₀ ($22 \text{ Hz} < f < 98 \text{ Hz}$) and SFG₅ ($29 \text{ Hz} < f < 85 \text{ Hz}$) as compared to RG₀ ($53 \text{ Hz} < f < 69 \text{ Hz}$), suggesting that SFG-induced turbulence contains more effective flow structures contributing to the thin-film undulation.

The f corresponding to the major peaks for all three cases are in excellent agreement with the f_v and f_δ determined earlier in Fig. 5b, of which, they are associated to the first resonant mode of piezoelectric beam amalgamate with their respective vortex shedding frequencies. In addition, the streamwise evolution of the maximum E_v are consistent with the trend previously observed in Fig. 5a,c. One can therefore deduce that more energetic vortex shedding gives rise to higher turbulence intensity flow that yields larger film bending and consequently higher voltage output. On the other hand, minor peaks with relatively smaller E_v tend to appear at higher f close to the second vibration mode of the piezoelectric beam. These peaks are more marked in RG in comparison with SFG, especially in the domain proximate to grid, denoting that higher frequency small-scale eddies possess more significant contribution to RG-induced turbulence. As one proceeds downstream in the decay region, both the mechanically deduced major and minor peaks diminish due to viscous dissipation, with the minor peak intensity of RG attenuating up to $3 \times$ faster than its major peak, since smaller eddies of higher characteristic frequencies decay faster.

Figure 7. Downstream evolution of the energy spectrum (left) on (a) P_0 , (b) P_5 of SFG, and (c) centerline of RG; top view of the respective spectrum (right).

Streamwise evolution of grid-induced turbulence forcing. To demonstrate the turbulence forcing of grid-induced flow at various distances leeward from SFG₀, SFG₅ and RG₀, the RMS of the normal flow excitation force F_{rms} acting on the thin-film was computed as per Eq. (3) and plotted in Fig. 8a. The F_{rms} calculated in the present study are in the range of 2 to 14 mN, which are consonant with the millinewton excitation force dynamically measured by Goushcha et al.¹⁰ in their turbulent boundary layer study with momentum thickness-based Reynolds number: $2.0 \times 10^3 \leq Re_\theta \leq 7.5 \times 10^3$.

As seen in Fig. 8a, the F_{rms} for all three cases display a similar streamwise flow dynamic evolution as the results in Fig. 5a,c. The turbulence generated by SFG₀ generally possesses larger forcing compared to RG₀ except for $x/D_h \leq 0.25$. At x_{peak} , the turbulence forcing of fractal grid-induced flow on the centerline is more than twice of that induced by RG. Perhaps more interestingly may be the observation that the F_{rms} for SFG₅ has a lower peak

Figure 8. Grid-induced (a) RMS excitation force acting on the piezoelectric thin-film F_{rms} and (b) damping ratio ζ against x/D_h . (c) Downstream profile of the normalized acceleration ma_{rms}/ma_{max} , velocity cv'_{rms}/cv'_{max} , deflection $k\delta_{rms}/k\delta_{max}$ and voltage $\theta V_{rms}/\theta V_{max}$ terms along the centerline of SFG (red) and RG (blue).

than SFG₀ despite both having almost the same peak V_{rms} and δ_{rms} (see Fig. 5a). This could be explained by the higher equivalent vortex shedding energy of SFG₀ in comparison to SFG₅ as represented by the wider f range of high energy fluctuations in Fig. 7. The same explanation applies to the case of RG₀ and SFG₀ at $x/D_h = 0.13$, where the former has an equivalent vortex shedding energy distributed among a range of frequencies that is $1.3 \times$ broader than the latter. Moreover, the small-scale eddies of RG₀ possess more energy in exciting the thin-film considering that the E_v of its minor peak is an order of magnitude larger than SFG₀. From these findings, we can deduce that the experimentally recorded V_{rms} and δ_{rms} reflect only the vortex shedding intensity of dominant large eddies. Conversely, the calculated F_{rms} takes into consideration the broad-band random forcing across various eddy sizes including small-scale eddies of larger characteristic frequencies.

Figure 8b shows the damping ratio ζ of the thin-film vibration computed from Eq. (6). The film oscillations for all three cases are underdamped, with $\zeta < 0.1$. From the ζ evolution seen in Fig. 8b, the average ζ for SFG₀ is $1.5 \times$ higher than RG₀, signifying a higher thin-film damping in fractal grid-generated turbulence. This might be due to the complicated multilength-scale flow structures of SFG that flow through the thin-film over time, making the fluid domain around the thin-film becomes rather “crowded” yet highly mechanical effective.

The forced vibration of piezoelectric thin-film is the result of the interplays between fluid excitation force, inertia force, viscous effect and oscillating piezoelectric structure. Here we normalize the RMS of acceleration ma_{rms} , velocity cv'_{rms} , deflection $k\delta_{rms}$ and voltage θV_{rms} terms in Eq. (3) by their respective maximum over the streamwise distances. The downstream profile of the normalized terms (ma_{rms}/ma_{max} , cv'_{rms}/cv'_{max} , $k\delta_{rms}/k\delta_{max}$ and $\theta V_{rms}/\theta V_{max}$) along SFG₀ and RG₀ are depicted in Fig. 8c with their respective maximum terms listed in the side table. Clearly, the downstream evolution of all the four normalized terms for both grids are almost similar to the F_{rms} profiles in Fig. 8a with the exception of the velocity term. Where cv'_{rms}/cv'_{max} peaks at $x/D_h = 1.0$ for SFG₀ and $x/D_h = 0.03$ for RG₀ considering that the ζ at these two locations are the highest, respectively. One can also see that all the four terms are an order of magnitude higher for SFG₀ when compared against RG₀.

From the table in Fig. 8c, one can infer that the force acting on the piezoelectric cantilever beam is generally dominated by the inertia (ma) and bending forces ($k\delta$) followed by the mechanical damping force (cv') of an order of magnitude smaller. The momentum exchange between turbulent fluctuations generated from the

Position	V_{rms} (V)	f_v (Hz)	F_{rms} (mN)	ζ	L_v/T	S_v	F_v	U (m/s)
P ₂	0.8	54.0	4.9	0.050	0.068	-0.023	3.5	2.6
P _{2a}	0.7	54.3	4.2	0.072	0.069	-0.021	3.4	3.2
P ₁₃	1.1	68.5	9.2	0.032	0.055	-0.003	3.1	4.8
P _{13a}	1.1	67.7	9.8	0.034	0.055	-0.004	3.1	4.4

Table 4. Results obtained for SFG at positions P₂ vs. P_{2a} and P₁₃ vs. P_{13a}.

Figure 9. 2D contours of (a) L_v/T , (b) f_v , (c) V_{rms} and (d) local mean velocity U at $x/D_h = 0.425$ leeward of SFG, and (e–h) the corresponding results for RG at positions P₀, P₃, P₈, P₁₀, P₁₉ and P₂₂.

grids contributes predominantly towards the beam forcing in the form of inertia force. The splashing of high momentum fluid transported by the moving vortical structures on the thin-film surface creates a considerable amount of bending force that causes the beam to deflect. At the same time, energy is lost through mechanical damping, viz. viscous air damping and structural damping. The dissipation of the electrical charges generated by piezoelectric through the circuit will also act to dampen out the beam vibration. In view of the low viscosity of air and high elastic compliance of current piezoelectric thin-film employed, mechanical damping force has a relatively minor effect on the beam forcing. Hence, SFG has an overall greater F_{rms} than RG in spite of the higher thin-film damping imposed by fractal grid-generated turbulence. On the other hand, the force due to electromechanical coupling (θV) has an order of magnitude between 10^{-6} and 10^{-5} which is almost negligible, signifying that the electrical damping imposed by the present PVDF film is insignificant.

Cross-sectional turbulence mechanical characteristics in the lee of SFG and RG. In order to have a better comprehension of the grid's cross-sectional turbulence mechanical characteristics in the turbulence generation region, the V and δ of piezoelectric thin-film at $x/D_h = 0.425$ downstream of SFG and RG were recorded by placing the thin-film at the lateral positions indicated in Fig. 2c,d. Table 4 compares the results obtained for SFG(P₂, P_{2a}) and SFG(P₁₃, P_{13a}). All their results are seen to be respectively close to each other, which proves that the flow field leeward of SFG are symmetry and therefore, the present study focuses mainly on the lower quarter grid. Figure 9a–d illustrate the 2D contours of L_v/T , f_v , V_{rms} and local mean velocity U for SFG, of which, U is defined as,

$$U = \sqrt{U_x^2 + U_y^2 + U_z^2} \quad (14)$$

where U_x , U_y and U_z are respectively the local mean velocities in x , y and z -directions measured using hotwire anemometer. The corresponding results obtained for RG at positions P₀, P₃, P₈, P₁₀, P₁₉ and P₂₂ are plotted against SFG in Fig. 9e–h. Note that δ_{rms} and I_y are omitted owing to the similarity with respect to V_{rms} in Fig. 9c,g.

Figure 10. (a) F_{rms} , (b) ζ , (c) S_v and (d) F_v at different lateral positions of SFG and (e–h) RG where $x/D_h = 0.425$.

From Fig. 9a,b, an immediate observation that we can make is the variability of L_v/T and f_v throughout the entire contour owing to the multi-scale nature of SFG as well as the inhomogeneity of fractal grid-generated flow in the turbulence generation region. The L_v of the vortices shed from the different-size bars of SFG ranges from 8.7 to 12.8 mm with an average cross-sectional f_v of 58 Hz. Furthermore, it is apparent that the cross-sectional profile of f_v is a complete reverse of L_v/T , which is consistent with our earlier finding. Large-scale, slow rotating eddies are mostly found at the central opening of the SFG, whereas small-scale eddies with high characteristic frequencies are mainly observed in the wake of the largest grid bar as well as at the bottom left corner of the contour. This observation agrees well with the V_{rms} result in Fig. 9c, of which, large energy-containing eddies induce larger thin-film bending, giving rise to a higher voltage output. On the other hand, energy dissipates faster in smaller size eddies, leading to lower voltage generated. We hypothesize that the small-scale eddies present at bottom left of the contour is the result of vortices shed from smaller size grid bars near the corner of SFG, as well as the breaking up of large-scale eddies into smaller ones when flow strikes the side and bottom walls of test section. This could also explain the slight decrease in L_v/T and the increase in f_v for both grids as one approaches the bottom of the test section (see Fig. 9e,f). At each of the lateral positions investigated, RG has a smaller L_v/T but higher f_v compared to SFG.

In Fig. 9c, high V_{rms} is observed in the region between the largest grid bar and centerline of SFG with P_5 having the greatest V_{rms} . This can be explained by the high velocity jet flowing through the grid opening (see Fig. 9d) as well as the wake-interaction occurring at P_5 as discussed previously (see Fig. 5e). On the contrary, V_{rms} is low on the rear side of the largest grid bar due to the presence of recirculating flow as represented by the low velocity wake region in Fig. 9d. Low V_{rms} is also detected at the bottom left corner of the contour albeit the high U , with P_{22} having the lowest V_{rms} . This could be a consequence of the chaotic flow stirred up by the aforementioned small-scale eddies in the bottom left region of the test section. The force acting on the thin-film provided by the highly disordered eddies would most likely counteract each other, imposing thin-film damping upon fractal grid-generated turbulence as supported by the high ζ in Fig. 10b. On average, the cross-sectional V_{rms} in the lee of SFG are $5\times$ higher than the RG in Fig. 9g. At P_8 , SFG has a lower U but higher V_{rms} than RG (see Fig. 9g,h) owing to the $3\times$ higher localized I_y flow generated by SFG behind the largest grid bar.

The 2D contours of F_{rms} , ζ , S_v and F_v for SFG are presented in Fig. 10a–d. The results are compared against RG(P_0 , P_3 , P_8 , P_{10} , P_{19} , P_{22}) and plotted in Fig. 10e–h. With the exclusion of SFG(P_{12} , P_{13} , P_{19}), the F_{rms} in Fig. 10a generally displays a similar profile as the V_{rms} in Fig. 9c. By taking P_0 as the baseline, one can see that both P_0 and P_{19} have comparable V_{rms} due to the same intensity of vortex shedding, i.e., same maximum E_v (see Fig. 11a). However, due to the unique geometry of SFG, the small-scale eddies on P_{19} possess higher energy than P_0 as depicted by the higher E_v of the minor peak near $f = 240$ Hz, giving rise to a larger F_{rms} upon thin-film flapping. Applying a similar analysis, P_{13} has a lower V_{rms} than P_0 due to the lower shedding intensity of faster rotating yet smaller L_v vortices from the second iteration grid bar. Nevertheless, the additional forcing provided by the higher E_v small eddies on P_{13} gives rise to a larger F_{rms} as compared to P_0 . Moving on to P_{12} , the flow recirculation behind the largest grid bar leads to a less pronounced vortex shedding effect but with no significant impact on the energy content of small-scale eddies, thus the discrepancy in the F_{rms} and V_{rms} on P_{12} . As seen in Fig. 10e, the cross-sectional turbulence forcing of the RG-generated flow is comparatively uniform, which on average is $2\times$ lower than the SFG-induced turbulence. The present findings further justify our speculations made earlier

Figure 11. Lateral variations in energy spectrum (left) along $z/T = -0.419$ and $z/T = 0$ at $x/D_h = 0.425$ leeward of (a) SFG and (b) RG; top view of the respective spectrum (right).

that F_{rms} expresses the force acting on the thin-film contributed by various scales flow structures, including the additional forcing provided by small-scale turbulence with less significant amplitude of velocity fluctuation.

From Fig. 10b,f, we once again observe that the film oscillations at all the lateral positions for both grids are underdamped, with SFG having an average ζ that is $1.5\times$ higher than RG. As discussed previously, the high ζ in the bottom left region leeward of SFG is due to the highly chaotic fluid domain at the corner of the test section, whereas the high ζ behind the largest grid bar is caused by the recirculating flow in the wake region. On the other hand, the jet-like behaviour through the opening is the strongest on the centerline of SFG. When this strong central jet flows over the film surface, it may possibly hinder the upward and downward movement of thin-film, causing a less effective thin-film flapping as supported by the relatively higher ζ on P_0 than P_1 and P_4 , thence lower F_{rms} and V_{rms} .

It is observed in Fig. 10c,d,g,h that the v' distributions for both SFG and RG are Gaussian. This is excepted for SFG(P_0 , P_2 , P_6), where $F_v \approx 3.5$, implying that most of the time v' are near-zero with extreme fluctuations occurring occasionally when larger size eddies of considerable velocity fluctuation amplitude impinge on the thin-film surface. It is also noteworthy to mention that we do not observe any intense accelerating or decelerating events occurring on all the lateral positions investigated for SFG, in contrast to the inhomogeneous S_u and F_u cross-sectional profiles reported by Nagata et al.³⁶ in the turbulence generation region. Given that the displacement of an elastic structure in response to the random turbulence-induced pressures on its surface can be well represented by a Gaussian distribution³⁷, it is reasonable to infer that our S_v and F_v reflect only the Gaussianity of the thin-film's structural response towards turbulent flow. Hence, the present PTFV approach might not be feasible in characterizing the higher order statistics of turbulent velocity fluctuations, instead allows the unique expression of insert-induced turbulence mechanical characteristics.

Figure 11a,b display the lateral evolution of energy spectra along $z/T = -0.419$ and $z/T = 0$ at $x/D_h = 0.425$ in the lee of SFG and RG with the 3D contour presented in the left column, and top view on the right. The immediately notable observation for both grids is that the lateral variations of maximum E_v from the middle to the bottom wall of test section are identical to the V_{rms} trends in Fig. 9c,g. It can also be observed that an increase in the E_v of major peak is generally accompanied by a wider f range of high energy fluctuations as shown in the right side of Fig. 11. Hence, more intense vortex shedding gives rise to more effective flow structures responsible for the higher voltage generated through piezoelectric thin-film flapping. The vortex shedding intensity of SFG is found to be 7 to $37\times$ higher than RG with the exception of P_{22} owing to the lower shedding intensity of high frequency, low energy containing vortices from the local higher iteration grid bars. Moreover, the breaking up of vortical structures by the corner walls as previously discussed could also have possibly weakened the vortex shedding effect. The vortex shedding phenomenon is also observed to be less pronounced on SFG(P_8 , P_{11}) since they are both located in the wake of the largest grid bar where flow recirculation occurs. As one advances towards the bottom wall, the f corresponding to the major peak increases from 48 to 62 Hz for SFG and 61 to 74 Hz for RG, which agrees well with the f_v results in Fig. 9b,f. For all the lateral positions investigated, we once again secure a more prominent minor peak for RG with respect to SFG except for P_{22} , signifying that the contribution of small-scale eddies towards v' is very significant for SFG near the corner walls.

A probabilistic description of piezoelectric voltage response towards grid-induced turbulence. Figure 12a,b show a segment of the recorded time-history for V and δ at $x/D_h = 0.425$ leeward of SFG and RG centerlines. The intermittent behavior observed for both grids are mainly due to the random nature of grid-induced turbulence impingement on the thin-film, of which, the V and δ for SFG are about $6\times$ greater than that of RG. Although at first glance the V and δ responses for both grids seem well fitted, the small-scale fluctuations are not well matched to each other particularly for the RG in Fig. 12b. These small-scale fluctuations are the result of minute eddies impinging on the thin-film which may either intensify or offset the primary film oscillation induced by the large-scale eddies. Nonetheless, if the large eddies contain high enough energy to cause considerable film bending as seen for the case of SFG in Fig. 12a, it will mask the relatively insignificant fluctuations caused by the small eddies. These results are in good agreement with the energy spectra in Fig. 11, where a much prominent minor peak is observed for RG₀ as compared to SFG₀. Furthermore, the small-scale fluctuations are mostly visible in the V response since the output voltage is based on the overall piezoelectric deformation induced by the multilength-scale eddies along the thin-film surface, instead of merely the flow structures revolving around the film tip. This demonstrates that V not only possesses higher signal-to-noise ratio but also comprises of more comprehensive information of the flow dynamic, and is more responsive towards small-scale turbulence in comparison with camera recorded δ .

Considering that V better characterizes grid-induced turbulence than the camera recorded δ , we attempt to examine the probability distribution of a random piezoelectric voltage response towards the turbulent flows generated by SFG and RG. On the left side of Fig. 13, we present the cumulative probability of V^2 at different x/D_h for SFG₀, SFG₅ and RG₀. It can be observed that the variations in the probability distribution with x/D_h for all three cases follow the streamwise profiles in Fig. 5a,c. For SFG₀, at $x/D_h = 0.13$ where V_{rms} , δ_{rms} and I_y are the lowest, the probability in obtaining large V^2 is also the lowest, of which, 99% are less than $3 V^2$. At $x_{peak}/D_h = 0.81$, 95% of V^2 are below $68 V^2$ while the top 1 percentile has $V^2 > 120 V^2$, which is highly similar to the probability distribution at $x_{peak}/D_h = 0.43$ of SFG₅. On the contrary, RG₀ has much smaller V^2 as compared to SFG(P_0 , P_5). For the former, 99% of the V^2 at $x_{peak}/D_h = 0.13$ do not even exceed $3 V^2$ and this is further reduced to $0.2 V^2$ at $x/D_h = 2.50$.

On the right of Fig. 13, we plot the cumulative probability of V^2 at different lateral positions of SFG and RG where $x/D_h = 0.425$. It is immediately apparent that the variations in the distribution plots from middle to the bottom wall of test section for both grids are once again identical to the V_{rms} profile in Fig. 9c,g, with RG having a narrower range of variation than the SFG. For the latter, the probability in obtaining large V^2 is high on P_3 and

Figure 12. Time series plots of V and δ recorded at $x/D_h = 0.425$ downstream of (a) SFG and (b) RG centerlines.

P_4 , where top 5 percentile have $V^2 > 16$ and 47 V^2 while top 1 percentile have $V^2 > 30$ and 88 V^2 , respectively. In the wake of the largest grid bar (P_8 and P_{11}), 99% of V^2 are not more than 6 V^2 on average. V^2 is observed to be the lowest on P_{22} where 99% are below 0.5 V^2 , which is similar to P_{22} of RG. For the rest of the positions, lower V^2 are attained for RG in comparison with SFG. It is also more likely to obtain small V^2 closer to the bottom wall for instance, top 1 percentile of RG(P_0 , P_3) has V^2 above 0.8 V^2 but is narrowed down to 0.3 V^2 on P_{19} .

In short, Fig. 13 not only demonstrates the dependency of voltage response on the grid-film distance and film lateral placement in the lee of grid, but also the unique advantages of SFG-induced turbulence over the turbulence generated by RG. Hence, this crucial finding presents opportunity for future study on correlating the piezoelectric output voltage with the various turbulence statistics and forcing of fractal grid-generated turbulence to unveil the multi-scale turbulence-mechanical interplays of fractal grid.

Furthermore, the overall high voltage response secured from the SFG-induced turbulence presents potential future energy harvesting opportunities. Piezoelectric thin-films could be placed at the regions where local turbulence strength is high to maximize the amount of electrical power harvested, particularly at the four P_5 locations leeward of the SFG. Further research may be undertaken to scale-up the energy harnessing capability of the system, which include but not limited to optimising the geometrical parameters of the SFG and materials properties of the piezoelectric thin-films. Such energy harvesting module which consists of a SFG and an array of piezoelectric cantilever beams could be installed in various sections of the heating, ventilation and air conditioning (HVAC) ducts. The micro to milliwatt-scale electrical output harnessed may be used for self-powering low-power temperature and humidity sensors to monitor, control, and manage the HVAC systems. At the same time, the piezoelectric thin-films could also act as a flow sensing device to characterise the flow in the air ducts which could not be achieved by small wind turbines or solar panels.

Conclusion

The present experimental study was set out to explore the mechanical characteristics of fractal grid-generated turbulence based upon the direct fluid–structure interaction between the flow and a flexible piezoelectric thin-film. The film undulation δ and voltage response V at different grid-film distances uniquely revealed the strength and the corresponding coverage of turbulence generation and decay regimes. Interestingly, the wake-interaction at P_5 not only shifts the peak location x_{peak} upstream but also enhances the local turbulence strength generated using SFG. Centerline results at x_{peak} showed that the V_{rms} and δ_{rms} in the lee of SFG are 7× larger than that of the RG of equivalent blockage ratio σ , with the former having a millinewton turbulence forcing F_{rms} that is more than twice of RG.

The cross-sectional profiles of the thin-film's physical response at $x/D_h = 0.425$ disclosed the inhomogeneity of fractal grid-generated flow in the turbulence generation region as compared to the relatively uniform flow induced by RG. The equivalent lateral integral length scale L_v of the vortices shed from the multi-scale SFG bars ranges from 8.7 to 12.8 mm which on average is 1.3× larger than RG. Low frequency, large-scale energy-containing eddies at the central opening of SFG are primarily responsible for the high voltage generation, of which, the average cross-sectional V_{rms} and F_{rms} are respectively, 5× and 2× higher than RG, with SFG₅ having the highest values. Nevertheless, the recirculating flow in the wake of the largest grid bar, in addition to the highly chaotic

Figure 13. Cumulative probability of V^2 at different streamwise locations along SFG(P_0 , P_5) and RG centreline (left), and at $x/D_h = 0.425$ downstream from the SFG and RG (right).

flow stirred up by the small-scale eddies broken up by the corner walls of test section impose thin-film damping upon fractal grid-generated turbulence, which in turn weakens the local vortex shedding effect.

Our findings demonstrate the unique expression of insert-induced turbulence mechanical characteristics via PTFV, along with the characterization of the large-size flow structures' turbulence length scale desired for effective thermal dissipation and energy harvesting. Such system could potentially be employed in real-world applications for the purpose of evaluating the flow's local turbulence strength to gain insights into the broad-band random forcing across turbulence of various scales.

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Author contributions

T.S.L. conducted the experiments and prepared the manuscript draft. E.H.O., W.S.C. and J.J.F. supervised the project, revised and edited the manuscript. All authors reviewed the manuscript.

Competing interests

The authors declare no competing interests.

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