OPEN Author Correction: Entanglement Availability Differentiation Service for the Quantum Internet

Laszlo Gyongyosi & Sandor Imre

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This Article contains errors.

In Protocol 0, where

Protocol 0 Core Protocol

Step 1. Alice (transmitter node) and Bob (receiver node) agree on a time t , in which they want to establish entanglement between subsystems A and B . Alice generates a separable initial system AB , with no entanglement between A and B as

$$\rho_{AB} = \frac{1}{2} |\psi_+\rangle \langle \psi_+| + \frac{1}{2} |\phi_+\rangle \langle \phi_+|, \quad (1)$$

where $|\psi_+\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$, $|\phi_+\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$. Alice encodes subsystem B , $|\phi_B\rangle = \alpha |0\rangle + \beta |1\rangle$ via an (m, n) redundant quantum parity code as

$$|\delta_B\rangle^{(m,n)} = \alpha |\chi_+\rangle_1^{(m)} \dots |\chi_+\rangle_n^{(m)} + \beta |\chi_-\rangle_1^{(m)} \dots |\chi_-\rangle_n^{(m)}, \quad (2)$$

where $|\chi_\pm\rangle^{(m)} = |0\rangle^{\otimes m} \pm |1\rangle^{\otimes m}$, and sends subsystem B to Bob through the network of n intermediate transfer nodes. Each intermediate transfer nodes $\mathcal{A}'_{1\dots n}$ receives and retransmits $|\delta_B\rangle^{(m,n)}$. Bob receives $|\delta_B\rangle^{(m,n)}$ and decodes it.

Step 2. Alice prepares a system C , denoted by density $\rho_C = \frac{1}{2} (I + s\sigma^x)$ which is completely uncorrelated from ρ_{AB} , where I is the identity operator, σ^x is the Pauli X matrix, while s is a constant. Alice applies a unitary U'_{AC} on A and C , which produces the initial system ABC with no entanglement between A and B as

$$\rho_{ABC} = U'_{AC} \rho_{AB} \rho_C (U'_{AC})^\dagger = \frac{1}{2} |\psi_+\rangle \langle \psi_+| |+\rangle \langle +| + \frac{1}{2} |\phi_+\rangle \langle \phi_+| |-\rangle \langle -|, \quad (3)$$

where $|\pm\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle)$.

Step 3. Alice applies the unitary U_{AC} on subsystem AC for a time t , as

$$U_{AC} = \exp(-iH_{AC}t) = \cos(t)I - i\sin(t)\sigma_A^x\sigma_C^x, \quad (4)$$

where

$$H_{AC} = \sigma_A^x\sigma_C^x \quad (5)$$

is the Hamiltonian with energy E_{AC}

$$E_{AC} = \frac{1}{2} \hbar 2\pi \left(\frac{1}{4t} \right), \quad (6)$$

where $\hbar$ is the reduced Planck constant, which results in the maximally entangled AB system with probability $p = 1$ as

$$\sigma_{AB} = \frac{1}{2} (|\psi_+\rangle - i|\phi_+\rangle) (\langle \psi_+| + i\langle \phi_+|). \quad (7)$$

should read:

Protocol 0 Core Protocol

Step 1. Alice (transmitter node) and Bob (receiver node) agree on a time t , in which they want to establish entanglement between separable subsystems A and B . Alice and Bob generate a tripartite system ABC with a separable initial system AB [12], as

$$\rho_{ABC} = \frac{1}{2} |\psi_+\rangle \langle \psi_+| |+\rangle \langle +| + \frac{1}{2} |\phi_+\rangle \langle \phi_+| |-\rangle \langle -|, \quad (1)$$

where $|\psi_+\rangle = \frac{1}{\sqrt{2}} (|01\rangle + |10\rangle)$, $|\phi_+\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle)$, and $|\pm\rangle = \frac{1}{\sqrt{2}} (|0\rangle \pm |1\rangle)$.

Step 2. Alice keeps subsystems A and C .

Step 3. Alice applies the unitary U_{AC} on subsystem AC for a time t , as

$$U_{AC} = \exp(-iH_{AC}t) = \cos(t) I - i \sin(t) \sigma_A^x \sigma_C^x, \quad (2)$$

where I is the identity matrix, σ^x is the Pauli X matrix, and

$$H_{AC} = \sigma_A^x \sigma_C^x \quad (3)$$

is the Hamiltonian with energy E_{AC}

$$E_{AC} = \frac{1}{2} \hbar 2\pi \left(\frac{1}{4t} \right), \quad (4)$$

where $\hbar$ is the reduced Planck constant, which results in the maximally entangled AB system at $t = \pi/4$, with probability $p = 1$ as

$$\sigma_{AB} = \frac{1}{2} (|\psi_+\rangle - i|\phi_+\rangle) (\langle \psi_+| + i\langle \phi_+|). \quad (5)$$

In addition, in Equation 13 and 14

$$\begin{aligned} \sigma_{ABC} &= |\varphi(t)\rangle \langle \varphi(t)|_{ABC} = U \rho_0 U^\dagger \\ &= \frac{1}{2} (U_{AC} |\psi_+\rangle \langle \psi_+| |+\rangle \langle +| U_{AC}^\dagger) + \frac{1}{2} (U_{AC} |\phi_+\rangle \langle \phi_+| |-\rangle \langle -| U_{AC}^\dagger), \end{aligned}$$

where $|\varphi(t)\rangle_{ABC}$ at time t is evaluated as

$$\begin{aligned} |\varphi(t)\rangle_{ABC} &= \frac{1}{\sqrt{2}} (\cos(t)(|\psi_+\rangle |+\rangle) - i \sin(t)(|\phi_+\rangle |+\rangle)) \\ &\quad + \frac{1}{\sqrt{2}} (\cos(t)(|\phi_+\rangle |-\rangle) + i \sin(t)(|\psi_+\rangle |-\rangle)) \\ &= \frac{1}{\sqrt{2}} (\cos(t)(|\psi_+\rangle) - i \sin(t)(|\phi_+\rangle)) |+\rangle \\ &\quad + \frac{1}{\sqrt{2}} (\cos(t)(|\phi_+\rangle) + i \sin(t)(|\psi_+\rangle)) |-\rangle \end{aligned}$$

should read:

$$\sigma_{ABC} = \frac{1}{2} \left(U_{AC}(t) |\psi_+\rangle \langle \psi_+| |+\rangle \langle +| U_{AC}^\dagger(t) \right) + \frac{1}{2} \left(U_{AC}(t) |\phi_+\rangle \langle \phi_+| |-\rangle \langle -| U_{AC}^\dagger(t) \right),$$

while $|\varphi(t)\rangle_{ABC}$ at time t is defined as

$$\begin{aligned} |\varphi(t)\rangle_{ABC} &= \frac{1}{\sqrt{2}} (U_{AC}(t)(|\psi_+\rangle |+\rangle) + U_{AC}(t)(|\phi_+\rangle |-\rangle)) \\ &= \frac{1}{\sqrt{2}} (\cos(t)(|\psi_+\rangle |+\rangle) - i \sin(t)(|\phi_+\rangle |+\rangle)) + \frac{1}{\sqrt{2}} (\cos(t)(|\phi_+\rangle |-\rangle) + i \sin(t)(|\psi_+\rangle |-\rangle)) \\ &= \frac{1}{\sqrt{2}} (\cos(t)(|\psi_+\rangle) - i \sin(t)(|\phi_+\rangle)) |+\rangle + \frac{1}{\sqrt{2}} (\cos(t)(|\phi_+\rangle) + i \sin(t)(|\psi_+\rangle)) |-\rangle \end{aligned}$$

Furthermore, in Equation 22 and 23

$$\begin{aligned}\sigma_{ABC} &= |\varphi(t)\rangle\langle\varphi(t)|_{ABC} = U\rho_0U^\dagger \\ &= \frac{1}{2}(U_{AC}|\psi_+\rangle\langle\psi_+||+\rangle\langle+|U_{AC}^\dagger) + \frac{1}{2}(U_{AC}|\phi_+\rangle\langle\phi_+||-\rangle\langle-|U_{AC}^\dagger),\end{aligned}$$

where $|\varphi(t)\rangle_{ABC}$ at t is evaluated as

$$\begin{aligned}|\varphi(t)\rangle_{ABC} &= \frac{1}{\sqrt{2}}(U_{AC}(|\psi_+\rangle|+\rangle) + U_{AC}(|\phi_+\rangle|-\rangle)) \\ &= \frac{1}{\sqrt{2}}\left(\cos(t)\left(\frac{1}{2}(|010\rangle + |011\rangle + |100\rangle + |101\rangle)\right) + \cos(t)\left(\frac{1}{2}(|000\rangle - |001\rangle + |110\rangle - |111\rangle)\right) \right. \\ &\quad \left. - i\sin(t)\left(\frac{1}{2}(|111\rangle + |110\rangle + |001\rangle + |000\rangle)\right) \right. \\ &\quad \left. + i\sin(t)\left(\frac{1}{2}(|101\rangle - |100\rangle + |011\rangle - |010\rangle)\right)\right) \\ &= \frac{1}{\sqrt{2}}(\cos(t)(|\psi_+\rangle|+\rangle + |\phi_+\rangle|-\rangle) - i\sin(t)(|\phi_+\rangle|+\rangle - |\psi_+\rangle|-\rangle)) \\ &= \frac{1}{\sqrt{2}}((\cos(t)(|\psi_+\rangle) - i\sin(t)(|\phi_+\rangle))|+\rangle + (\cos(t)(|\phi_+\rangle) + i\sin(t)(|\psi_+\rangle))|-\rangle),\end{aligned}$$

should read:

$$\sigma_{ABC} = \frac{1}{2}\left(U_{AC}(t)|\psi_+\rangle\langle\psi_+||+\rangle\langle+|U_{AC}^\dagger(t)\right) + \frac{1}{2}\left(U_{AC}(t)|\phi_+\rangle\langle\phi_+||-\rangle\langle-|U_{AC}^\dagger(t)\right),$$

and $|\varphi(t)\rangle_{ABC}$ at t is defined as

$$\begin{aligned}|\varphi(t)\rangle_{ABC} &= \frac{1}{\sqrt{2}}(U_{AC}(t)(|\psi_+\rangle|+\rangle) + U_{AC}(t)(|\phi_+\rangle|-\rangle)) \\ &= \frac{1}{\sqrt{2}}\left(\cos(t)\left(\frac{1}{2}(|010\rangle + |011\rangle + |100\rangle + |101\rangle)\right) + \cos(t)\left(\frac{1}{2}(|000\rangle - |001\rangle + |110\rangle - |111\rangle)\right) \right. \\ &\quad \left. - i\sin(t)\left(\frac{1}{2}(|111\rangle + |110\rangle + |001\rangle + |000\rangle)\right) + i\sin(t)\left(\frac{1}{2}(|101\rangle - |100\rangle + |011\rangle - |010\rangle)\right)\right) \\ &= \frac{1}{\sqrt{2}}(\cos(t)(|\psi_+\rangle|+\rangle + |\phi_+\rangle|-\rangle) - i\sin(t)(|\phi_+\rangle|+\rangle - |\psi_+\rangle|-\rangle)) \\ &= \frac{1}{\sqrt{2}}((\cos(t)(|\psi_+\rangle) - i\sin(t)(|\phi_+\rangle))|+\rangle + (\cos(t)(|\phi_+\rangle) + i\sin(t)(|\psi_+\rangle))|-\rangle),\end{aligned}$$

Finally, in equation A.3 and A.4 in the Supplementary Information file,

$$\begin{aligned}\sigma_{ABC} &= |\varphi(t)\rangle\langle\varphi(t)|_{ABC} = U\rho_0U^\dagger \\ &= \frac{1}{2}\left(U_{AC}|\psi_+\rangle\langle\psi_+||+\rangle\langle+|U_{AC}^\dagger\right) + \frac{1}{2}\left(U_{AC}|\phi_+\rangle\langle\phi_+||-\rangle\langle-|U_{AC}^\dagger\right),\end{aligned}$$

where $|\varphi(t)\rangle_{ABC}$ at t is evaluated as

$$\begin{aligned}|\varphi(t)\rangle_{ABC} &= \frac{1}{\sqrt{2}}(U_{AC}(|\psi_+\rangle|+\rangle) + U_{AC}(|\phi_+\rangle|-\rangle)) \\ &= \frac{1}{\sqrt{2}}\left(\cos(t)\left(\frac{1}{2}(|010\rangle + |011\rangle + |100\rangle + |101\rangle)\right) + \cos(t)\left(\frac{1}{2}(|000\rangle - |001\rangle + |110\rangle - |111\rangle)\right) \right. \\ &\quad \left. - i\sin(t)\left(\frac{1}{2}(|111\rangle + |110\rangle + |001\rangle + |000\rangle)\right) + i\sin(t)\left(\frac{1}{2}(|101\rangle - |100\rangle + |011\rangle - |010\rangle)\right)\right),\end{aligned}$$

should read:

$$\sigma_{ABC} = \frac{1}{2} \left(U_{AC}(t) |\psi_+\rangle \langle \psi_+| |+\rangle \langle +| U_{AC}^\dagger(t) \right) + \frac{1}{2} \left(U_{AC}(t) |\phi_+\rangle \langle \phi_+| |-\rangle \langle -| U_{AC}^\dagger(t) \right),$$

and $|\varphi(t)\rangle_{ABC}$ at t is defined as

$$\begin{aligned} |\varphi(t)\rangle_{ABC} &= \frac{1}{\sqrt{2}} (U_{AC}(t) (|\psi_+\rangle |+\rangle) + U_{AC}(t) (|\phi_+\rangle |-\rangle)) \\ &= \frac{1}{\sqrt{2}} \left(\cos(t) \left(\frac{1}{2} (|010\rangle + |011\rangle + |100\rangle + |101\rangle) \right) + \cos(t) \left(\frac{1}{2} (|000\rangle - |001\rangle + |110\rangle - |111\rangle) \right) \right. \\ &\quad \left. - i \sin(t) \left(\frac{1}{2} (|111\rangle + |110\rangle + |001\rangle + |000\rangle) \right) + i \sin(t) \left(\frac{1}{2} (|101\rangle - |100\rangle + |011\rangle - |010\rangle) \right) \right), \end{aligned}$$

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