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Non-equilibrium Model for Nanofluid Free Convection Inside a Porous Cavity Considering Lorentz Forces

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In current article, transportation of CuO nanoparticles through a porous enclosure is demonstrated. The enclosure has complex shaped hot wall. Porous media has been simulated via two temperature equations. Magnetic force impact on nanofluid treatment was considered. Control volume based finite element method has been described to solve current article in vorticity stream function form. Single phase model was chosen for nanofluid. Nanofluid characteristics are predicted via KKL model. Roles of solid-nanofluid interface heat transfer parameter (Nhs), porosity, Hartmann and Rayleigh numbers have been illustrated. Outputs illustrated that conduction mode reduces with augment of Ra. Increasing magnetic forces make nanofluid motion to decrease. Temperature gradient of nanofluid decreases with augment of Nhs. Reducing porosity leads to enhance in Nusselt number.

Solar power collectors and drying technologies are two common uses for porous enclosure. In some application, researchers should consider two- temperature model. Alsabery *et al.*¹ demonstrated nanoparticle transportation in a tilted porous cavity. They indicated that convective flow is significantly influenced by the permeable layer augmentation. Lu *et al.*² investigated about nanofluid radiative three dimensional flow containing gyrotactic microorganism with anisotropic slip. They considered the effect of activation energy. Zaimi *et al.*³ illustrated nanofluid boundary layer movement on a porous plate. Sheikholeslami and Shehzad⁴ showed the two temperature model for nanoparticle migration inside a permeable medium. They revealed that Nu increases with decrease of porosity. Khan *et al.*⁵ reported the nanofluid mixed convection over an oscillating vertical plate. They utilized Laplace transform method to solve the governing equations.

Sheikholeslami and Shehzad⁶ investigated the role of radiation on nanoparticle treatment. They found that Nu decreases with reduce of radiation parameter. Haq *et al.*⁷ used carbon nanotubes to improve convective heat transfer over plate with slip flow. Carbon Nanotubes has been dispersed in to engine oil by Haq *et al.*⁸ is examined of magnetic forces. Tripathi *et al.*⁹ illustrated viscous dissipation and Hall effects on nanofluid rotating flow. Selimefendigil and Oztop¹⁰ depicted impact of inclination on hydrothermal behavior. They found that titled angle can be used as control parameter.

Promvonge *et al.*¹¹ applied new way in a duct to improve the thermal characteristics. Sheikholeslami¹² described the impact of electric field on nanofluid free convection. He proved that Nusselt number enhances by adding electric field. Aman *et al.*¹³ illustrated the nanofluid thermal improvement in migration of CNTs nanoparticles. Sheikholeslami and Seyednezhad¹⁴ illustrated nanofluid Electrohydrodynamic flow in a permeable enclosure. Different applications of Fe₃O₄-water nanofluid were categorized by Sheikholeslami and Rokni¹⁵. Akbar *et al.*¹⁶ showed the role of Hartmann flow on nanoparticles migration in a duct. Najib *et al.*¹⁷ demonstrated the impact of chemical factor on flow style. They found that the Nu augments with augment of curvature. Various articles were been available about nanoparticle migration through porous media^{18–21}.

Current publication is about nanoparticle migration in a porous enclosure with two temperature model via CVFEM considering magnetic force. Results illustrate the roles of significant parameters on contours.

Explanation of Geometry

Figure 1 illustrates the details of current geometry. The hot wall can formulate by:

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Figure 1. (a) Geometry and the boundary conditions with (b). A sample triangular element and its corresponding control volume.

Coefficient values	CuO–Water
a_1	−26.593310846
a_2	−0.403818333
a_3	−33.3516805
a_4	−1.915825591
a_5	6.42185846658E-02
a_6	48.40336955
a_7	−9.787756683
a_8	190.245610009
a_9	10.9285386565
a_{10}	−0.72009983664

Table 1. The coefficient values of CuO–Water nanofluid.

	$\rho(\text{kg/m}^3)$	$C_p(\text{J/kgK})$	$k(\text{W/m.k})$	$\beta \times 10^5(\text{K}^{-1})$	$d_p(\text{nm})$	$\sigma(\Omega \text{ m})^{-1}$
Water	997.1	4179	0.613	21	—	0.05
CuO	6500	540	18	29	29	10^{-10}

Table 2. Thermo physical properties of water and nanoparticles.

Mesh size in radial direction $\times$ angular direction				
61×181	71×211	81×241	91×271	101×301
1.36733	1.36828	1.36967	1.36971	1.36989

Table 3. Comparison of the average Nusselt number Nu_{ave} for different grid resolution at $Ra = 1000$, $Ha = 20$, $\varepsilon = 0.9$, $Nhs = 1000$ and $\phi = 0.04$.

Figure 2. Comparison of the present solution with previous work (Kim *et al.*²⁴) for different Rayleigh numbers when $Ra = 10^5$, $Pr = 0.7$; (b) comparison of the temperature on axial midline between the present results and numerical results obtained by Khanafer *et al.*²⁵ for $Gr = 10^4$, $\phi = 0.1$ and $Pr = 6.2$ (Cu–Water).

$$r_{out} = r_{in} + A \cos(N(\zeta - \zeta_0)) \quad (1)$$

r_{out} , r_{in} , A , N are outer and inner radius, amplitude, number of undulation. The porous enclosure is full of nano-fluid and influenced by magnetic force.

CVFEM and Explanation

Formulation. According to existence of magnetic force and two temperature model for porous medium the basic formulas are:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (2)$$

$$-\frac{\partial P}{\partial x} + \sigma_{nf} B_0^2 [\nu (\sin \gamma)(\cos \gamma) - u(\sin \gamma)^2] - \frac{\mu_{nf}}{K} u = 0 \quad (3)$$

Ha	$Gr = 2 \times 10^4$		$Gr = 2 \times 10^5$	
	Present	Rudraiah <i>et al.</i> ²⁶	Present	Rudraiah <i>et al.</i> ²⁶
0	2.5665	2.5188	5.093205	4.9198
10	2.26626	2.2234	4.9047	4.8053
50	1.09954	1.0856	2.67911	2.8442
100	1.02218	1.011	1.46048	1.4317

Table 4. Nu_{ave} for various Gr and Ha at $Pr = 0.733$.

Figure 3. Streamlines (Ψ), isotherms for the nanofluid (θ_{nf}) and the solid (θ_s) at $Ra = 100$, $Ha = 0$, $\phi = 0.04$.

Figure 4. Streamlines (Ψ), isotherms for the nanofluid (θ_{nf}) and the solid (θ_s) at $Ra = 100$, $Ha = 20$, $\phi = 0.04$.

$$\frac{\mu_{nf}}{K}v = -\frac{\partial P}{\partial y} + (T - T_c)g\rho_{nf}\beta_{nf} + \sigma_{nf}B_0^2[u(\cos\gamma)(\sin\gamma) - v(\cos\gamma)^2] \quad (4)$$

$$\frac{1}{\varepsilon}\left(u\frac{\partial T_{nf}}{\partial x} + v\frac{\partial T_{nf}}{\partial y}\right) = \frac{h_{nfs}}{(\varepsilon)(\rho C_p)_{nf}}(-T_{nf} + T_s) + \frac{k_{nf}}{(\rho C_p)_{nf}}\left(\frac{\partial^2 T_{nf}}{\partial y^2} + \frac{\partial^2 T_{nf}}{\partial x^2}\right) \quad (5)$$

Figure 5. Streamlines (Ψ), isotherms for the nanofluid (θ_{nf}) and the solid (θ_s) at $Ra = 500$, $Ha = 0$, $\phi = 0.04$.

$$\frac{k_s}{(\rho C_p)_s} \left(\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} \right) + \frac{h_{nfs}}{(1 - \epsilon)(\rho C_p)_s} (T_{nf} - T_s) = 0 \quad (6)$$

$(\rho C_p)_{nf}$, $(\rho\beta)_{nf}$, ρ_{nf} , σ_{nf} and k_{nf} , μ_{nf} can define as²²:

$$(\rho C_p)_{nf} = \phi(\rho C_p)_p + (1 - \phi)(\rho C_p)_f \quad (7)$$

Figure 6. Streamlines (Ψ), isotherms for the nanofluid (θ_{nf}) and the solid (θ_s) at $Ra = 500$, $Ha = 20$, $\phi = 0.04$.

$$(\rho\beta)_{nf} = (1 - \phi)(\rho\beta)_f + \phi(\rho\beta)_p \quad (8)$$

$$\rho_{nf} = \rho_f(1 - \phi) + \rho_p\phi \quad (9)$$

$$\frac{\sigma_{nf}}{\sigma_f} = \frac{(MM - 1)3\phi}{(MM + 2) + \phi(1 - MM)} + 1, \quad MM = \frac{\sigma_p}{\sigma_f} \quad (10)$$

Figure 7. Streamlines (Ψ), isotherms for the nanofluid (θ_{nf}) and the solid (θ_s) at $Ra = 1000$, $Ha = 0$, $\phi = 0.04$.

$$\frac{k_{nf}}{k_f} = 1 - 3 \frac{\left(1 - \frac{k_p}{k_f}\right)\phi}{\left(1 - \frac{k_p}{k_f}\right)\phi + \left(\frac{k_p}{k_f} + 2\right)} + \sqrt{\frac{\kappa_b T}{\rho_p d_p}} c_{pf} (5 \times 10^4) g'(d_p, T, \phi) \rho_f \phi$$

$$g'(d_p, T, \phi) = (a_2 \ln(d_p) + a_1 + a_3 \ln(\phi) + a_5 \ln(d_p)^2 + a_4 \ln(d_p) \ln(\phi)) \ln(T) + (a_9 \ln(d_p) \ln(\phi) + a_{10} \ln(d_p)^2 + a_7 \ln(d_p) + a_6 + a_8 \ln(\phi))$$
(11)

Figure 8. Streamlines (Ψ), isotherms for the nanofluid (θ_{nf}) and the solid (θ_s) at $Ra = 1000$, $Ha = 20$, $\phi = 0.04$.

$$\mu_{hf} = \frac{\mu_f}{(1 - \phi)^{2.5}} + \frac{\mu_f}{k_f} (k_{Brownian}/Pr) \quad (12)$$

where ϕ is nanofluid volume fraction. Required characteristics and parameters are illustrated in Tables 1 and 2²². Considering following definitions:

Figure 9. Contour plots for effects of Ra , Ha , ε , Nhs on average Nusselt number.

$$\begin{aligned} v &= -\frac{\partial \psi}{\partial x}, (X, Y) = (x, y)/L, \quad u = \frac{\partial \psi}{\partial y}, \\ \theta_{nf} &= (T_{nf} - T_c)/(T_h - T_c), \quad \Psi = \psi/\alpha_{nf}, \quad \theta_s = (T_s - T_c)/(T_h - T_c), \end{aligned} \quad (13)$$

Final equations are:

$$\begin{aligned} \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} &= \frac{A_6}{A_5} Ha \left[\frac{\partial^2 \Psi}{\partial Y^2} (\sin^2 \gamma) + \frac{\partial^2 \Psi}{\partial X^2} (\cos^2 \gamma) + 2 \frac{\partial^2 \Psi}{\partial X \partial Y} (\sin \gamma)(\cos \gamma) \right] \\ &\quad - \frac{A_3 A_2}{A_4 A_5} \frac{\partial \theta_{nf}}{\partial X} Ra \end{aligned} \quad (14)$$

$$\varepsilon \left(\frac{\partial^2 \theta_{nf}}{\partial Y^2} + \frac{\partial^2 \theta_{nf}}{\partial X^2} \right) + Nhs(\theta_s - \theta_{nf}) = -\frac{\partial \theta_{nf}}{\partial Y} \frac{\partial \Psi}{\partial X} + \frac{\partial \Psi}{\partial Y} \frac{\partial \theta_{nf}}{\partial X} \quad (15)$$

Figure 10. 3D plots for effects of Ra , Ha , ε , Nhs on average Nusselt number.

$$\varepsilon \left(\frac{\partial^2 \theta_s}{\partial Y^2} + \frac{\partial^2 \theta_s}{\partial X^2} \right) + Nhs \delta_s (\theta_{nf} - \theta_s) = 0 \quad (16)$$

where:

$$\begin{aligned} A_1 &= \frac{\rho_{nf}}{\rho_f}, \quad Ra = (\rho\beta)_f \frac{Kg L \Delta T}{\alpha_f \mu_f}, \quad A_5 = \frac{\mu_{nf}}{\mu_f}, \\ A_2 &= \frac{(\rho C_p)_{nf}}{(\rho C_p)_f}, \quad \delta_s = k_{nf} / [k_s (1 - \varepsilon)], \quad A_4 = \frac{k_{nf}}{k_f}, \\ A_3 &= \frac{(\rho\beta)_{nf}}{(\rho\beta)_f}, \quad Ha = \frac{\sigma_f K B_0^2}{\mu_f}, \\ Nhs &= h_{nf} L^2 / k_{nf}, \quad A_6 = \frac{\sigma_{nf}}{\sigma_f}, \end{aligned} \quad (17)$$

Boundary conditions are:

$$\begin{aligned}
 \theta_{nf} = \theta_s = 0.0 & \quad \text{on outer wall} \\
 \Psi = 0.0 & \quad \text{on all walls} \\
 \theta_{nf} = \theta_s = 1.0 & \quad \text{on inner wall}
 \end{aligned} \tag{18}$$

Nu_{loc} and Nu_{ave} are:

$$Nu_{loc} = \left(\frac{k_{nf}}{k_f} \right) \frac{\partial \theta_{nf}}{\partial r} \tag{19}$$

$$Nu_{ave} = \frac{1}{2\pi} \int_0^{2\pi} Nu_{loc} dr \tag{20}$$

CVFEM. The innovative in which triangular element is used and upwind method is applied for advection term is CVFEM (Fig. 1(b)). Gauss-Seidel is the name of the method which is used for final step as mentioned in ref.²³.

Mesh Independent Test and Validation

Obviously, the final outputs should not alter by changing mesh size. So, this test should be done for various cases as illustrated Table 3. Also, we should be sure about accuracy of written code by applying this code for previous published problem. Table 4 and Fig. 2 illustrates nice accuracy^{24–26}.

Results and Discussion

In current article, migration of nanoparticle in a porous medium which is described by new porous model is presented. Numerical method is applied to display the roles of the porosity ($\varepsilon = 0.3$ to 0.9), Rayleigh number ($Ra = 100,500$ and 10^3), Hartmann number ($Ha = 0$ to 20) and Nhs ($Nhs = 10$ to 1000).

Influences of Nhs , Ra , ε and Ha on isotherms for solid (θ_s), streamlines (Ψ) and nanofluid (θ_{nf}) were depicted in Figs 3, 4, 5, 6, 7 and 8. When buoyancy force is weak, conduction mode is significant and the impacts of other variables are negligible. In this case, (θ_{nf}) contours are similar to (θ_s) contours. As Ra increase, thermal plume can be seen close to the vertical centerline and (θ_{nf}) contours become more complex but (θ_s) contours have no changes. Velocity of nanofluid decrease with augment of magnetic forces and the (θ_{nf}) contours are stratified with enhance of Ha . (Ψ_{max}) enhances with augment of Nhs due to stronger convective flow. As ε enhances, the pores volume through the enclosure augments. Therefore, convective flow becomes stronger. Besides, the impact of ε on isotherms is as same as Nhs .

Figures 9 and 10 depict the impact of Nhs , ε , Ha and Ra , on Nu_{ave} . Nu_{ave} can calculate as:

$$\begin{aligned}
 Nu_{ave} = & 1.64 + 1.3Ra^* - 0.16Ha^* - 0.12\varepsilon - 1.24Nhs^* \\
 & - 0.3Ra^*Ha^* - 0.34Ra^*\varepsilon - 0.55Ra^*Nhs^* + 0.19Ha^*\varepsilon + 0.28Ha^*Nhs^* \\
 & + 0.34\varepsilon Nhs^* \\
 & - 0.08(Ra^*)^2 + 0.74(Nhs^*)^2 - 0.036(Ha^*)^2 - 0.2(\varepsilon)^2
 \end{aligned} \tag{21}$$

Coefficient of determination is 0.98 for this correlation. Also, in this formula we have: $Ha^* = 0.1Ha$, $Nhs^* = 0.001Nhs$, $Ra^* = 0.001Ra$. Nu_{ave} is a reducing function of Nhs because temperature gradient decreases with augment of this factor. Decreasing porosity makes Nu to enhance which is similar of Ha impact. Nu_{ave} decreases with reduce of Rayleigh number.

Conclusions

In current research, migration of CuO nanoparticles is simulated via Non-equilibrium. Innovative method is applied to show the impacts of porosity, buoyancy, magnetic forces and the Nhs . Results show that Nu_{ave} reduces with augment of Ha , ε , Nhs . Convective flow enhances with increase of Nhs and ε while it reduces with enhance of magnetic force. When $Ra = 1000$, $\varepsilon = 0.3$, increasing Nhs leads to 17.06 percent decrement of Nu_{ave} in absence of magnetic field. Impact of Nhs is negligible in existence of Lorentz force. Also Nu_{ave} decreases about 53% with augment of Hartmann number.

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Author Contributions

I.K. formulated the problem. M.S. computed the numerical results. I.K. and M.S. wrote the main manuscript text.

Additional Information

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