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Resistive switching in nano-structures

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Solid state memory and switching devices aimed at replacing the flash memory technology operate by switching from the high to low resistance when conductive filaments are created in response to the electric pulse. The filaments are identified with either structurally different protrusions or purely electronic conductive pathways. The former can appear via the field induced nucleation (FIN), while the latter do not require phase transformations and are attributed to certain types of temperature and bias dependent conductivity. The existing understanding of those processes ignores features related to extremely small linear sizes of nano-structures. Such are, for example, the device sizes smaller than critical nucleation radii, and/or the electron energy relaxation lengths exceeding the structure dimensions. This paper develops a theory of switching under nano-size conditions. We show how the structure thinness can make FIN a truly threshold phenomenon possible only for voltage (not the field) exceeding a certain critical value. We predict the possibility of threshold switching without memory for certain thickness dependent voltages. The thermal runaway mechanism of electronic switching is described analytically leading to results consistent with the published numerical modeling. Our predictions offer possible experimental verifications deciding between FIN and thermal runaway switching.

The presence of conductive filaments (CFs) is critically important for functionality of phase change memory (PCM)¹, resistive random access memory (RRAM)², and threshold switches (TS)³. Also, CFs are responsible for the breakdown phenomena in gate dielectrics⁴.

While the important role of CFs is commonly recognized, their underlying physics is not sufficiently understood. The two types of mechanisms of CF formation have been proposed in the literature: (1) the direct electric field induced nucleation (FIN) of a new phase in the form of conductive filamentary pathway characterized by its critical length and nucleation barrier^{5,6}, and (2) the electronic filament precursor raising the local temperature enough to trigger a phase transformation or remain as such for the case of TS⁷.

We recall that CFs in the above mentioned devices are initially created by the electro-forming process that requires a certain forming voltage $\sim 1 - 4$ V. A formed structure is then modified (using additional electric stimuli) in such a way as to introduce a relatively narrow insulating gap across the filament. Repeatedly closing and opening that gap makes the device switching between two distinct states underlying its functionality. As the order of magnitude estimates, we mention the original CF length of $H \sim 10 - 20$ nm, and the insulating gap width $h_{\text{gap}} \sim 1 - 3$ nm⁸⁻¹⁰. The voltages required to create and bridge the insulating gaps in CFs are typically several times lower than the forming voltages¹⁰⁻¹².

Here we consider a constrained FIN where the volume available for nucleation is limited. That condition can be important for the modern nanometer devices where conductive bridges form through narrow, $h_{\text{gap}} \sim 1 - 3$ nm gaps, while the critical nucleation length $h_c \gg h_{\text{gap}}$, say, $h_c \sim 10$ nm^{5,6}, as illustrated in Fig. 1. It can be relevant for the electroforming processes as well if the dielectric layer thickness is small enough, $\lesssim 10$ nm.

The paper is organized as follows. In Sec. II we apply the standard field induced nucleation theory to the narrow ($h_c \gg h_{\text{gap}}$) gap case. Sec. III will introduce a different FIN scenario driven by the free energy originating from the effective capacitor formed by the CF tip and the opposite electrode. Sec. IV discusses the interplay between the two FIN scenarios. The thermal runaway electronic mechanism of switching is analyzed in Sec. V. Sec. VI contains final conclusions.

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Figure 1. Left: critical length (h_c) conductive embryo formed in a uniform electric field within a gap of height H significantly exceeding h_c . That condition corresponding to that of the standard FIN theory can relate to a pristine (before ‘forming’) device structure. Right: a conductive filament with an insulating gap of thickness h_{gap} in a ‘formed’ device. A small conductive embryo will nucleate inside the gap of $h_{\text{gap}} < h_c$ i. e. beyond the limits of applicability of the standard FIN. Dashed arrows represent the electric field.

The Standard FIN Scenario

We recall that FIN is a process where a small metallic needle-shaped embryo nucleates in a non-conductive host due to the polarization energy gain in a strong external electric field E . Similar to other nucleation processes, FIN is characterized by the critical nucleation length h_c and nucleation barrier W representing respectively the length above which the embryo grows spontaneously, and its corresponding energy. Along the standard lines, the latter quantities are defined without any spacial constraints.

Assuming a high aspect ratio nucleus, $h \gg 2r$ its free energy is presented as,

$$F(h, r) = 2\pi r h \gamma + \mu \pi r^2 h - \alpha h^3 \epsilon E^2. \quad (1)$$

where E is the strength of a uniform field before the embryo nucleation [The field becomes non-uniform upon the metal embryo nucleation, which however is accounted for by the field dependent term in Eq. (1); see e.g. 14, p. 17]. Here, γ is the surface tension and μ is the chemical potential difference between the two phases. The third term in Eq. (1) represents the electrostatic energy gain and is written in a truncated form where α stands for a combination of numerical and logarithmic multipliers. For example, $\alpha = \{12 \ln[(2h/r) - 1]\}^{-1}$ for the ellipsoidal CF, and $\alpha = \{\ln[(2h/r) - 7]\}^{-1}$ for the cylindrical CF, assuming $h \gg r^{5,13}$. In what follows we neglect the logarithmic dependence of α treating it as a constant smaller than one, say, $\alpha \sim 0.1$.

FIN theory describes the case of strong fields where the bulk term [second on the right-hand-side of Eq. (1)] is insignificant and omitted compared to the field term. As a result, nucleation in TS ($\mu > 0$) and non-volatile memory ($\mu < 0$) is described similarly¹⁴. The difference appears beyond the present scope, in that, upon the field removal, the formed needle-shaped nucleus is stable in the memory and unstable in TS devices.

The free energy of Eq. (1) has a stationary point, which is a maximum and thus unrelated to the transition barrier between the initial state ($r = h = 0$) and the new phase state where F decreases with h and r . It was shown that such a barrier is provided by the absolute minimum of $F(h, r)$ located at the boundary region of the minimum allowed value of r (in sub-nanometer range) denoted here as r_{min} . The latter is determined by the conditions of electric and mechanical integrity for CF⁵.

We note that the inequality $h \gg 2r$ underlying Eq. (1) reduces here to $h \gg 2r_{\text{min}}$. While r_{min} is in a sub-nanometer range, the latter inequality may be not very strong with the left-hand side by only a numerical factor (say, 3–5) exceeding the right one. Our results then remain valid approximately not changing any qualitative conclusions.

For $r = r_{\text{min}}$, the free energy is a minimum at

$$h_c = \sqrt{\frac{2\pi r_{\text{min}} \gamma}{3\epsilon \alpha}} \frac{1}{E} \quad (2)$$

corresponding to the barrier height

$$W(E) = F[h_c(E), r_{\text{min}}] = \frac{2^{5/2} (\pi r_{\text{min}} \gamma)^{3/2}}{3^{3/2} (\alpha \epsilon)^{1/2} E} \quad (3)$$

as illustrated in Fig. 2.

For the future references, it is convenient to represent the nucleation barrier through the electric potential $U = EH$,

Figure 2. The free energies of a conductive embryo vs. its height in the electric fields of three different strengths, corresponding (from right to left) to the unstable, metastable, and thermodynamically stable embryos. The initial state of the system corresponds to $h \approx 0$ (to the accuracy of thermal energy fluctuation $\sim kT$). W is the nucleation barrier height, W_{ms} is the barrier determining the lifetime of a metastable embryo. The inset diagram illustrates how the transition creates a state (in TS) that is unstable upon the field removal.

$$W(U) = W_0 \frac{U_0}{U} \quad (4)$$

with

$$W_0 = 2\pi\gamma r_{\min} H, \quad \text{and} \quad U_0 = \sqrt{\frac{8\pi r_{\min} \gamma}{27\alpha\epsilon}}. \quad (5)$$

Because the nucleation time $\tau = \tau_0 \exp(W/kT)$ where $\tau_0 = \text{const}$, it follows that FIN takes place under any voltage U over the time that exponentially decreases with U as

$$\tau \propto \exp[(W_0/kT) (U_0/U)]. \quad (6)$$

This type of dependence was experimentally observed^{15–18}.

Considering FIN in a narrow gap, the weak field region, $E < E_{ms}$ [explicitly given in Eq. (7)] can be determined by the condition $h_c > h_{\text{gap}}$. In that region, the free energy within the gap is increasing with h making embryos unstable. On the other hand, under higher fields, $E > E_s$ [also, explicitly given in Eq. (7)], the barrier is at shorter distance than h_c , and yet, the energy $F(h_{\text{gap}}, r_{\min})$ is positive. Therefore, for the field strengths in the interval,

$$E_{ms} \equiv \sqrt{\frac{2\pi r_{\min} \gamma}{3\epsilon\alpha h_{\text{gap}}^2}} < E < \sqrt{\frac{2\pi r_{\min} \gamma}{\epsilon\alpha h_{\text{gap}}^2}} \equiv E_s, \quad (7)$$

a metastable embryo within a gap ($h_c < h_{\text{gap}}$) can be formed. Shown in Fig. 2 the barrier determining that embryo lifetime is given by

$$W_{ms} = W(E) - F(h_{\text{gap}}) \quad (8)$$

with $W(E)$ and $F(h)$ from Eqs (1) and (3). The metastable embryo lifetime is given by

$$\tau_{ms} = \tau_0 \exp(W_{ms}/kT) \quad (9)$$

with $W_{ms} = 0$ when $E = E_{ms}$.

The upper characteristic field E_s in Eq. (7) is determined by the condition $F(h_{\text{gap}}) = 0$, i.e. the embryo remains stable as long as the field E is applied. When the field is removed, the second term in Eq. (1) disappears, and the embryo free energy linear in h makes it decay as illustrated in Fig. 2 relevant for TS functionality.

It follows from the above that while the nucleation of unconstrained CF is possible for the field of any strength (with exponentially field dependent nucleation times), the confined CFs require the field strength $E > E_{ms}$ and the corresponding voltages $U > U_{ms} = E_{ms} h_{\text{gap}}$. Similarly, stable CFs can nucleate when $U > U_s = E_s h_{\text{gap}}$. It follows from Eq. (7) that both U_s and U_{ms} are independent of h_{gap} and $U_s = \sqrt{3} U_{ms}$. Therefore, as opposed to the case of unconstrained CF, the confinement seems to make nucleation a truly threshold voltage phenomenon.

Interestingly, the latter expressions for U_s and U_{ms} predict numerical values consistent with the data in the order of magnitude^{19–24},

Parameter	H, nm^a	$r_{\min}, \text{nm}^b$	ϵ^c	$\gamma, \text{dyn/cm}^d$	α^e	$\kappa, \text{cm}^2/\text{s}^f$
Value	3	0.3	25	10–100	0.1	0.1

Table 1. Some parameters related to FIN. ^aFollowing published estimates^{8–10}. ^bWe use $r_{\min}$ discussed in the early work on FIN⁵. ^cWe use the dielectric permittivity of HfO₂. ^dBecause the values of interfacial energies in materials undergoing FIN are not available, we use the ballpark of typical values for a variety of other systems^{44,45}. ^eSee the discussion after Eq. (1). ^fThermal diffusivity estimated for HfO₂ based RRAM devices⁶.

$$U_s = \sqrt{\frac{2\pi\gamma r_{\min}}{\epsilon\alpha}} = \left(\frac{3^{3/2}}{2}\right) U_0 \sim 0.3 - 1 \text{ V}. \quad (10)$$

Here and in what follows we use the numerical values listed in Table 1.

Also, we note that the transition from the regime of constant field to constant voltage with dielectric thinning below 10 nm was observed in PCM devices²⁵, although its original explanation was different. It was demonstrated that in 30 nm PCM structures switching is dominated by the field strength rather than voltage²⁶.

Another Scenario of FIN

The preceding section analysis tacitly assumed the polarization energy of a metal needle remaining cubic in length regardless of its closeness to opposite electrode. That assumption remains valid through almost the entire gap, since the electric field is strongly different from the uniform only in a small region $\sim r^2/h \ll h_{\text{gap}}$ around the tip²⁷.

However, there is another gap related effect significantly contributing to the free energy: capacitive interaction between the CF tip and its opposite electrode. That interaction can be thought of as due to a flat plate capacitor formed by the tip's end face and the opposite electrode, which may have a much larger area (for example in micron range) that can be approximated as infinite compared to the tip area. However the field distribution is strongly dominated by the smallest of the two plates, as was confirmed in our numerical COMSOL modeling. Using the flat plate capacitor formula leads to the energy contribution,

$$F_C = -\frac{CU^2}{2} \approx -\frac{\epsilon r^2 U^2}{8h_{\text{gap}}} \quad (11)$$

where C is the flat plate capacitor capacitance, U is the voltage between the electrodes. Note that assuming other geometrical shapes of the tip (with the infinite opposite electrode) lead to only slightly different results^{28,29}, where, for example, r is replaced with the electrode size in Eq. (11) for the case of hyperboloid of revolution²⁹.

A comment is in order to explain the negative sign of F_C in Eq. (11). In the process of changing the gap capacitance ΔC with its width h_{gap} , we assume that the system remains at constant voltage, so the capacitor energy changes by $U^2 \Delta C/2$. However, simultaneously the charge $\Delta Q = U \Delta C$ passes through the power source, which requires the energy $-U \Delta Q = -U^2 \Delta C$ making the total energy change equal $-U^2 \Delta C/2$.

Based on Eqs (1) and (11) and assuming the TS case, the free energy of CF can be approximated as

$$F(h, r) = 2\pi\gamma rh - \alpha h^3 \epsilon (U/H)^2 - \frac{\epsilon r^2 U^2}{8(H-h)}. \quad (12)$$

A major feature added here is a sharp energy decrease for relatively small gap widths $h_{\text{gap}} \ll H$ due to interaction between the CF tip and opposite electrode. We have verified the above heuristic free energy form of Eq. (12) with COMSOL modeling for a variety of CF dimensions; one example is presented in Fig. 3 for a cylinder shaped CF of radius $r = 0.5 \text{ nm}$ between two circular electrodes of radii 10 nm each. Overall, the analytical form of free energy in Eq. (12) provides a description within $\sim 10\%$ of relative error.

The free energy of Eq. (12) has a stationary point at certain h and r that can be determined analytically. Here we skip the corresponding cumbersome equations noting that, for that point, $D \equiv \partial^2 F / \partial h^2 \partial r^2 - (\partial^2 F / \partial h \partial r)^2 > 0$ and $\partial^2 F / \partial h^2 < 0$, $\partial^2 F / \partial r^2 < 0$, which identifies it as a maximum.

The case of a maximum stationary point in free energy landscape is similar to that of the standard field induced transformation scenario described in the preceding section: the phase transformation barrier is determined by the minimum acceptable radius $r_{\min}$. Its related gap width is given by $dF(h, r_{\min})/dh = 0$ with F from Eq. (12). Because the polarization term [second on the right hand side of Eq. (12)] is relatively unimportant for $H - h \ll H$, we get,

$$H - h = H \frac{U}{2U_\gamma} \quad \text{where} \quad U_\gamma \equiv \sqrt{\frac{4\pi\gamma H^2}{\epsilon r_{\min}}}. \quad (13)$$

The corresponding barrier height is given by,

$$W = W_0(1 - U/U_\gamma), \quad (14)$$

leading to the nucleation time exponentially dependent on bias. We conclude that the above introduced capacitive interaction acts as a sort of clutch decreasing CF energy in a narrow interval of its heights close to the opposite

Figure 3. The free energy of CF in a narrow gap between two electrodes. Symbols represent the results of the COMSOL numerical modeling for 0.5 nm radius CF between the two coaxial circular metal electrodes of 10 nm radius each. The solid line is a fit by Eq. (12). The dashed line is the free energy corresponding to the same surface tension and polarization, but without the capacitive interaction.

electrode. The corresponding nucleation barrier decreases with voltage linearly unlike the standard FIN dependence in Eq. (6).

Note that the parameters from Table 1, yield a numerical estimate of the characteristic voltage $U_\gamma \sim 3$ V. The corresponding gap width from Eq. (13) is typically in sub-nanometer range. For small enough voltages, Eq. (13) predicts $h_{\text{gap}} \lesssim 1$ Å, which raises a question on applicability of this theory, since that narrow gaps would allow significant electric currents, energy dissipation, and electric shorting.

To understand the role of the latter effects, we note the very short thermalization time $\tau_T \sim h_{\text{gap}}^2/\kappa \lesssim 0.1$ ps where κ is the thermal diffusivity ensuring local quasi-equilibrium. The dissipation will increase the quasi-equilibrium temperature, thus accelerating the nucleation. On the other hand, the concomitant electric shorting, would decrease the electrostatic energy of the entire device towards its final value corresponding to the fully formed CF. That decrease aggravates the free energy falloff again helping nucleation.

The quantitative description of the above mentioned transient temperature increase and shorting effect fall beyond the present scope. Therefore, we should admit a degree of uncertainty making the proposed FIN scenario questionable for small enough voltages. If that scenario does not work, the structures with nano-gaps can work only as TS and not memory when their applied biases are below U_{ms} . The viability of ~ 1 Å gaps between the nucleated CF and opposite electrode may depend on the morphology of materials involved.

Interplay of Two FIN Scenarios

The interplay between the standard and here developed FIN models is described by two relations,

$$\frac{U_\gamma}{U_s} = \sqrt{2\alpha} \frac{H}{r_{\min}} \quad (15)$$

and

$$\frac{W_{\text{new}}}{W_{\text{stand}}} = \left(\frac{3^{3/2}}{2} \right) \frac{U(1 - U/U_\gamma)}{U_s} \quad (16)$$

where the nucleation barriers W_{stand} and W_{new} are given respectively by Eqs (4) and (14). Comparing the barrier shapes in Figs 2 and 3 we then conclude that the standard FIN scenario dominates (i. e. $W_{\text{stand}} < W_{\text{new}}$) when $U > U_c \equiv 2/(3)^{3/2} U_s$. That is the same range of voltages where the standard FIN theory predicts the voltage dependence of τ in Eq. (6). We recall, in addition, that the transition between the two FIN scenarios has a threshold nature at voltage $U_{ms} = E_{ms} h_{\text{gap}}$ [see Eq. (7)]. That is slightly above U_c , since, based on their definitions, $U_c = (2/3) U_{ms}$. Such a behavior is illustrated in Fig. 4 (Left).

It should be noted in addition that the criterion of nucleation in the standard FIN scenario depends on the post-nucleation CF evolution and conditions determining the desired lifetime of the initially metastable CF created under voltage above U_{ms} . In general, the sufficient voltage is determined by the condition that the nucleated CF survives the desired time as specified in Eq. (9). However, the results presented in Sec. III show that the tip-electrode interaction can transform the metastable CF into a stable one as illustrated in Fig. 4 (Right).

Figure 4. *Left:* A sketch of the FIN barrier voltage dependence, linear in low voltage region and hyperbolic for higher voltages. *Right:* The filament free energies with (solid line) and without (dashed line) the tip electrode interaction [the last term in Eq. (12)] taken into account.

The above consideration predicts that, given all other factors, the switching voltage should decrease when the gap H decreases. Indeed, using $\tau = \tau_0 \exp(W/kT)$ along with Eq. (14), the voltage capable of triggering nucleation over time τ becomes,

$$U(\tau) = \left[H - \frac{kT}{2\pi\gamma r_{\min}} \ln \left(\frac{\tau}{\tau_0} \right) \right] \sqrt{\frac{4\pi\gamma}{\epsilon r_{\min}}}, \quad (17)$$

It increases with H for any given exposure time. The latter prediction is consistent with the data^{10–12} showing how the switching voltage in a formed structure is by a factor 2–4 lower than the forming voltage.

Electronic Switching

A number of recent publications^{30–35} theoretically studied purely switching (without any structural transformations) that might pertain to threshold switches operating as selector devices in modern solid state memory arrays. Their underlying thermal runaway scenario utilizes a non-linear current voltage characteristic where the conductivity is thermally and field activated,

$$\sigma = \sigma_0 \exp(-w/kT) \quad \text{with} \quad w = w_0 - \delta w(E). \quad (18)$$

Here $\delta w(E)$ is the field induced decrease in the activation energy of conductivity. A local lateral variation of current will then generate excessive local heat and temperature additionally increasing the current density at that location, etc.; hence, thermal electron instability evolving into a narrow filament carrying high electric current. The formation of such a filament is identified as switching.

Because the underlying modeling^{30–35} remains numerical, here, we give a simple analytical treatment allowing to examine the corresponding parameter ranges. We start with the standard 3D heat transfer equation $c\partial T/\partial t = \chi \nabla^2 T + \sigma E^2 = 0$ where the thermal capacitance (c), thermal conductivity (χ), and electric conductivity taking their respective values in the semiconductor and metal electrode materials forming a structure with axial symmetry shown in Fig. 5.

Given the ambient temperature T_0 at the external surfaces and the Neumann boundary conditions at the electrode-semiconductor interfaces, it was shown³⁶ that averaging the semiconductor temperature along the transversal direction (between the electrodes) reduces the heat transport equation to the form

$$\frac{\epsilon_s}{\chi_s} \frac{\partial T}{\partial t} = \nabla^2 T + \beta^2 (T_0 - T) + \frac{\sigma E^2}{\chi_s}, \quad \beta^2 \equiv \frac{2\chi_e}{\chi_s H L}. \quad (19)$$

Here T depends on the radial coordinate r , indexes s and e denote respectively the semiconductor and electrode materials. β is a reciprocal thermal length characterizing the decay of radial nonuniformities. Note that the second and third terms on the right-hand-side represents respectively heat dissipation and evolution.

Equation (19) along with Eq. (18) allows the standard linear stability analysis constituting a natural approach to studying run-away phenomena. Along the standard lines³⁷, it introduces small deviations from the average quantities, then linearizes the equations with respect to the deviations, and establishes the nature of temporal evolution of the latter (growing or decaying) by introducing their Fourier expansions.

More specifically, we introduce the average lateral temperature $\bar{T}$ and substitute

$$\delta T \equiv T - \bar{T} = \sum_q T_q \exp(i\omega_q t - iqr) \quad (\ll \bar{T}), \quad (20)$$

into Eqs (18) and (19). Linearizing the latter yields,

Figure 5. A sketch of a semiconductor layer (transparent) between two metal electrodes (grey) of thickness L each. The bottom view shows a hot spot caused by the runaway instability. χ_s and χ_e are respectively the thermal conductivities of the semiconductor and electrode materials, σ is the semiconductor conductivity, E is the electric field, and T_0 is the temperature on the external electrode surfaces.

$$(\bar{T} - T_0) \frac{\beta^2 \chi_s}{\sigma_0 E^2} = \exp\left(-\frac{w}{k\bar{T}}\right) \quad (21)$$

and

$$\frac{c_s i\omega_q}{\chi_s} = -q^2 - \beta^2 + \frac{\sigma_0 E^2}{\chi_s} \frac{w}{k\bar{T}^2} \exp\left(-\frac{w}{k\bar{T}}\right). \quad (22)$$

The instability takes place for positive real values of $i\omega$ [the corresponding temporal exponents in Eq. (20) increase]. For the modes with q below a certain wave number q_0 , the latter criterion reduces to the form,

$$\lambda \equiv \frac{\bar{T} - T_0}{\bar{T}} \frac{w}{k\bar{T}} \frac{\beta^2}{\beta^2 + q_0^2} \geq \lambda_{cr} = 1. \quad (23)$$

While derived differently, the inequality in Eq. (23) is similar to the classical criterion^{38,39} $\lambda \geq \lambda_{cr} \approx 0.88$ for 1D thermal instability with $q_0 = 0$ and $\beta = H/2$.

A graphical representation in Fig. 6 shows that when the field is low enough, Eq. (21) has three solutions: low (T_l) corresponding to the uniform current flow, high (T_h) for the hot spot, and intermediate (T_{unst}), which is unstable as seen from the relationship between the heat evolution and dissipation [cf. the note after Eq. (19)]. That conclusion agrees with the general phenomenological analysis of thermal instabilities⁴⁰. Furthermore, the condition for critical field (straight line tangential to the exponential curve in Fig. 6) self-consistently coincides with the criterion in Eq. (23) when $q = 0$.

The above simplistic analysis provides approximate numerical results given the same parameters as in the published numerical modeling^{30–35}. For example, assuming values³⁴ $E = 10^8$ V/m, $\sigma_0 = 10^7$ S/m, $\chi_s = 0.12$ Wm⁻¹K⁻¹, $H = 10$ nm, $L = 30$ nm, $\chi_e/\chi_s = 170$, $R = 60$ nm, yields $\beta^2 \approx 10^{18}$ m⁻², and $w/k\bar{T} = \ln\{\sigma_0 E^2 / [\beta^2 \chi_s (\bar{T} - T_0)]\} \approx 7$. Here we have rather arbitrarily approximated $(\bar{T} - T_0) = T_0$ under the logarithm, which has no significant effect when it appears with a multiplier that is by many orders of magnitude bigger than $(\bar{T} - T_0)$. Substituting into Eq. (23) gives $7(\bar{T} - T_0)/\bar{T} = 1$, i. e. the switching temperature $\bar{T} \approx 340$ K, in fare agreement with numerical simulations³⁴.

Furthermore, the estimated $\bar{T} \approx 340$ K and $w/k\bar{T} = 7$ yield $w \approx 0.21$ eV, which must be interpreted as the barrier for the above used $E = 10^8$ V/m. Any assumptions about that barrier field dependence are not necessary as long as w remains an adjustable parameter. That pertains to all the preceding numerical modeling^{30–35} carried out under the assumptions of $\delta w(E)$ following the Pool-Frenkel law. We would like to emphasize that thermal

Figure 6. A sketch of the graphical solution of Eq. (21) where the straight line and the curve represent its left- and right-hand sides. Note that the curve is S-shaped and thus crosses the straight line three times, because it describes the exponent linear in $-1/T$ (rather than T).

runaway switching will take place for any model of thermally activated conductivity with or without field or voltage dependent barrier. In particular, a close fitting of switching currents with the Pool-Frenkel based modeling curves does not appear indicative of that law. Of course, the switching field will depend $\delta w(E)$ when the value of zero field barrier w_0 is postulated.

The case of very thin dielectrics requires additional comments when the electron transport is ballistic violating the assumption of power generation inside the structure. That assumption underlies the term proportional to σE^2 in Eq. (19); it was adopted in all the preceding modeling work^{30–35}. The switching processes of interest take place under voltages of ~ 1 V, which means that electrons dissipate the energy of ~ 1 eV (between the electrode quasi-Fermi levels). For the case of ballistic transport, it will be injected in the opposite electrode as the electron kinetic energy with the relaxation length $\mathcal{L}$ is on the order of tens of nanometers even for amorphous materials⁴¹. Since $\mathcal{L} \gg H$, the heat is generated in the layer of thickness $\sim \mathcal{L}$, and its transport is dominated by the conductive material. When that type of transport takes place, the mechanism of thermal runaway electronic switching outlined in the beginning of this section becomes irrelevant.

Some additional observations are as follows.

- (i) Using the available data, Eq. (23) shows that switching takes place when the barrier is still significant, $w/kT \gg 1$. With that in mind, one can approximate the required barrier suppression vs. gap width, $\delta w(E) = \text{const} + kT \ln H$, which dependence may be too weak to resolve experimentally.
- (ii) Unlike FIN, the runaway model per se does not predict any delay time between voltage pulse and switching. The observed delay was attributed in that model to the thermalization time³⁰ or left without explicit interpretation³⁵.
We note here that the thermalization attributed delay time must be controlled by the conductivity, which is activated over the characteristic time $\tau \propto \exp[-\delta w(E)/T]$ [cf. Eq. (18)]. That delay time is exponential in voltage as intended to fit the data. That leads to the prediction that the delay time is reciprocal of the voltage dependent conductivity, which is open for experimental verifications. Note that FIN predictions for $\tau(U)$ are quite different as specified in Eqs (6) and (14).
- (iii) The above analysis will describe the finite area effects when we set $q_0 = 2\pi/R$ where R is the device radius taking into account, along the general lines, that the strongly oscillating terms with $q > 2\pi/R$ in the expansion of Eq. (20) are immaterial. With that in mind, Eqs (21) and (23) predict that switching requires temperature increase $(\bar{T} - T_0)$ that are by the factor of $1 + 4\pi^2(\beta^2 R^2)^{-1}$ greater than that for infinitely large devices. Since $(\bar{T} - T_0) \propto E^2$, we conclude that the switching fields and potentials scale as $\sqrt{1 + 4\pi^2(\beta^2 R^2)^{-1}}$ with device size, which is at least qualitatively consistent with the results of numerical modeling³¹. That scaling becomes practically important for nano-sized devices. No such scaling is predicted by FIN where switching field remains independent of device diameter.

The above (ii) and (iii) can be used to experimentally decide between the switching mechanisms of FIN and thermal runaway. A general concern about the latter arises for the case of submicron and especially nanometer thick amorphous structures with transversal conduction varying by many orders of magnitude between different spots⁴², which strongly affect the thermal runaway mechanism⁴³.

Conclusions

We have shown that,

- (1) FIN in narrow (shorter than the critical nucleation length) gaps between two metal electrodes exhibits new trends governed by certain characteristic voltages in the practically interesting range.
- (2) There exists the characteristic voltage U_{ms} independent of the gap width, below which the nucleation barrier decreases with voltage linearly, while it is reciprocal of voltage above U_{ms} , and the transition is of threshold nature independent of the gap width.
- (3) For some materials and structure thicknesses, nucleation of CF can result in functionality of only TS without memory.
- (4) Our analytical consideration of thermal runaway switching provides results consistent with the earlier published numerical modeling and leads to several predictions offering experimental verifications deciding between the mechanisms of FIN and thermal runaway.

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Author Contributions

V.G.K. wrote the main manuscript text and D.N. carried out COMSOL numerical modeling. Both authors critically read and contributed to the manuscript preparation, and also reviewed the manuscript.

Additional Information

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