Balancing energy and mass with neutrons

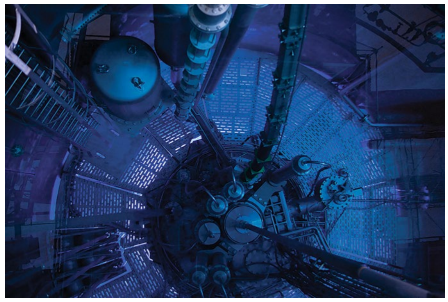
Michael Jentschel and Klaus Blaum explain why the most famous equation of physics needs checking — and how to do it.

Einstein's energy–mass equivalence principle $E = mc^2$ is among the most famous formulas of science. Despite its simplicity, a direct experimental test is quite difficult; any failure of this equation would hint at a breakdown of special relativity.

A direct validation would be achieved by a measurement of the energy released during annihilation of matter and antimatter. Since the energy equivalent of one gram of matter is an explosion of 20 kilotons of TNT, any precision annihilation experiment needs to be limited to very light particles only. With this in mind, the annihilation of an electron–positron pair with the emission of two photons with an energy of 511 keV each is the most promising choice, and measurements based on this process have been carried out[1]. However, at typical temperatures for this type of experiment, light particles move very fast. Therefore, the measured photon energies are Doppler-broadened and precision is limited to a few parts per million.

Alternatively, one could consider mass and energy variation during a nuclear reaction. This is somewhat less direct, but allows working with heavier and therefore slower particles — the neutron capture reaction is a good candidate. Before the reaction, neutrons and atoms have thermal energies of only a few meV, while after neutron capture the newly formed isotope releases energy as gamma rays summing up to a total energy of several MeV. This promises a reasonably small energy uncertainty (on the order of 10^{-9}). Also, inside high-flux neutron reactors, such as the one at the Institut Laue–Langevin (ILL) in Grenoble, France (pictured), one has sufficiently high neutron capture rates to make precision experiments statistically feasible. Although this approach enables more precise experiments, it requires a change of paradigm: we accept Einstein's equation to compare the mass defect of a compound system with the change in energy of its internally bound particles.

The most precise way to measure gamma-ray energies is through the determination of their diffraction angles in a double perfect-crystal spectrometer: gamma-ray wavelengths are compared to the lattice spacing in a perfect crystal of silicon. The latter can be taken with eight-digit significance from



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experimental work within the Avogadro project[2,3], an on-going effort in the framework of the redefinition of the kilogram. The diffraction angle measurements can be done with seven-digit accuracy[4]. The mass defect is obtained from high-precision atomic-mass measurements using Penning traps. In these devices, charged particles are trapped by carefully prepared electric and magnetic fields and made to perform periodic motions. The charge-to-mass ratio of each ion determines the cyclotron period of motion. Subsequent measurements of the cyclotron frequencies of X^+ (the ion of interest) and $^{12}C^{+6}$ (the reference ion) in the same trap yields the relative atomic mass $A_r(X^+)$ with up to 11-digit significance!

Translated to the observables introduced above, Einstein's equation reads as $[A_r(n) + A_r(^{N}X) - A_r(^{N+1}X)] = 10^3 N_A hc^{-1} \Sigma_i \lambda_i (E\lambda)$, with the sum running over the wavelengths λ_i of the gamma emitted in the de-excitation of the nucleus after neutron capture. The equation links atomic masses and wavelengths through a combination of fundamental constants: the product of the Avogadro and the Planck constant $N_A h$ is known with more than ten significant digits from other precision measurements, while the speed of light in vacuum c is defined and adds no uncertainty. The equation still contains the relative atomic mass of the uncharged neutron Ar(n), which one cannot measure directly with a Penning trap; it is initially extracted from measurements of the masses $A_r(^{1,2}H)$ and the wavelength of the deuterium line $\lambda(E_{2.2\,\mathrm{MeV}})$. Measurements with an additional isotope pair complete the energy–mass equivalence test.

The mass defect is only a small fraction of nuclear masses. Therefore, each mass needs to be determined three orders of magnitude more accurately than the wavelengths. This necessitates an 11-digit precision of the Penning trap (demonstrated for a few isotopes) to match the required eight digits for the crystal diffraction. On the other hand, the diffraction of gamma rays at perfect crystals is inefficient with only one photon per 10^{11} accepted for diffraction. Only a few isotope combinations provide sufficient gamma-ray activity inside the reactor to achieve the required statistical significance. For the combination of both experimental techniques only three isotope pairs have so far been suitable: $^{1,2}H$, $^{32,33}S$ and $^{28,29}Si$. More precise results are expected in the near future to come from $^{35,36}Cl$. With the existing data, it has been possible to demonstrate the equality of mass and energy at the level of $1.4(4.4) \times 10^{-7}$ (ref. [5]).

It is fascinating that two completely independent experimental techniques achieve agreement on such a level. The wavelength measurements still contribute the largest error in this test today. The new crystal spectrometer GAMS6 under commissioning at the ILL is aimed at achieving another order of magnitude. Recent Penning-trap measurements show further improvement in mass precision of light particles and make more isotope combinations accessible[6]. With these new instruments it will be possible to work towards an even more precise verification of Einstein's simple equation. ❒

Michael Jentschel[1]* and Klaus Blaum[2]*
[1]*Institut Laue–Langevin, Grenoble, France.*
[2]*Max-Planck-Institute for Nuclear Physics, Heidelberg, Germany.*
**e-mail: jentsch@ill.fr; klaus.blaum@mpi-hd.mpg.de*

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