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Comment

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# Mott physics in the multiflavored age

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The multiflavor Mott insulators, whose local Hilbert space consists of multiple degrees of freedom, occur widely in both quantum materials and ultracold atom systems. This Comment recommends the review article by Chen and Wu that is, to the author's knowledge, the first one to deal with all aspects and physical realizations of the multiflavor Mott insulators.

In 1937, de Boer and Verwey discovered that nickel oxide, which should be a metal according to band theory, was an insulator[1]. The first detailed theory of this family of systems, known as Mott insulators, was put forward by Mott more than a decade later[2]. If the number of electrons per unit cell is an odd integer, and if the hopping between sites is small enough, Coulomb repulsion can stabilize a state in which electrons are localized. This theory has two testable consequences: First, it should be possible to induce a transition to a metallic state by applying pressure since this will increase the hopping; Secondly, contrary to band insulators, Mott insulators do not have a gap to all excitations, but only to charge excitations. The spins of the electrons are not frozen, and the minimal model to describe their fluctuations is the Heisenberg model

$$H = \sum_{i,j} J_{ij} \vec{S}_i \cdot \vec{S}_j, \tag{1}$$

where the components of $\vec{S}_i$ are spin-$S$ angular momentum operators. This is the basis of the microscopic theory of antiferromagnetism[3], and Mott insulators have actually long been referred to as magnetic insulators.

In many circumstances, however, this minimal description is not sufficient. It assumes that the local degrees of freedom can be accurately grouped into the $2S+1$ states of a spin-$S$ angular momentum and that the coupling is just a scalar product between these angular momenta, resulting in rotational SU(2) symmetry. This can only be true if there is a single way to build a spin-$S$ from the $2S$ spins-1/2 of a partially filled level due to the Hund's rule coupling, a condition that requires that the number of spins-1/2 is equal to the number of atomic levels. This is possible, but not generic. For instance, crystal field splitting typically splits the five $3d$ levels into multiplets with two-fold or three-fold degeneracy, and if the number of spins-1/2 does not match this degeneracy, there is another degree of freedom that keeps track of the occupied orbitals. The resulting model, put forward in the seventies, is known as the Kugel-Khmoskii model[4], and it takes the generic form

$$H_{KK} = \sum_{i,j} J_{ij} H^S_{ij} H^T_{ij}, \tag{2}$$

where $H^S_{ij}$ is a spin Hamiltonian for which a good minimal model is, up to constant, a Heisenberg model, while $H^T_{ij}$ is a pseudo-spin Hamiltonian, where the pseudo-spin $\vec{T}_i$ describes the orbital degrees of freedom. The pseudo-spin Hamiltonian $H^T_{ij}$ is usually less symmetric than the spin one $H^S_{ij}$.

Over the last 25 years, two major developments have taken place. First of all, it has been realized that the Kugel-Khomskii model is relevant in other contexts, the role of the orbital degree of freedom being played by another degree of freedom. For example, in spin-1/2 Mott insulators living on a lattice consisting of weakly coupled triangles, the relevant low-energy model is based on the ground-state manifold of a triangle which consists of two degrees of freedom, the total spin, and a chirality degree of freedom that keeps track of the symmetry of the ground state with respect to the 3-fold rotation axis of the triangle[5].

Another major conceptual development has been to realize that, in many circumstances, the local degrees of freedom are better described not as a representation of the SU(2) symmetry group, but of another symmetry group, for instance, SU(N). Consider the case of three local degrees of freedom. This can be considered as a spin-1 of SU(2), or as the fundamental representation of SU(3). The resulting minimal models are very different. For a spin-1, the Heisenberg model has no matrix elements between the states $+1$ and $-1$, while for the fundamental representation of SU(3), the SU(3) Heisenberg model is simply a quantum permutation between all flavors, $-1$, 0 and $+1$. The connection with SU(N) was first realized in the context of the orbital degeneracy. Indeed there is a symmetric version of the Kugel-Khomskii model that is equivalent to the SU(4) Heisenberg model in the fundamental representation. However, the SU(N) point of view acquired a lot of visibility when it was realized that it is the relevant description of the Mott phase of $N$-flavor fermions loaded in optical lattices.

The review article by Chen and Wu is, to my knowledge, the first one to deal with all aspects and physical realizations of the problem. This was a real challenge because of the diversity of the settings and the physical properties, but the result was worth the effort. The authors have managed to describe in detail the physical origin of the extra degrees of freedom in many contexts, and they have nicely illustrated the wealth of physical properties, which range from orbital ordering to baryon-like physics when local degrees of freedom combine into the singlet representation of SU(N). Last but not least, they found a great name, *multiflavor Mott insulators*, to describe this broad and multifaceted family of Mott insulators[6].

Progress in science often occurs through cross-fertilization, and physics is no exception. Review articles such as the present one are extremely useful to bridge gaps and to transfer knowledge between communities. I enthusiastically recommend this review article to any physicist interested in Mott insulators with complex local degrees of freedom.

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## Competing interests
The author declares no competing interests.

## Additional information
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