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Superconductivity in the doped quantum spin liquid on the triangular lattice

Hong-Chen Jiang ¹✉

Broad interest in quantum spin liquid (QSL) phases was triggered by the notion that they can be viewed as insulating phases with preexisting electron pairs, such that upon light doping they might automatically yield high temperature superconductivity. Yet despite intense experimental and numerical efforts, definitive evidence showing that doping QSLs leads to superconductivity has been lacking. We address the problem of a lightly doped QSL through a large-scale density-matrix renormalization group study of the t - J model on finite-circumference triangular cylinders with a small but nonzero concentration of doped holes. We provide direct evidences that doping QSL can naturally give rise to d -wave superconductivity. Specifically, we find power-law superconducting correlations with a Luttinger exponent, $K_{sc} \approx 1$, which is consistent with a strongly diverging superconducting susceptibility, $\chi_{sc} \sim T^{-(2-K_{sc})}$ as the temperature $T \rightarrow 0$. The spin–spin correlations—as in the undoped QSL state—fall exponentially which suggests that the superconducting pair-pair correlations evolve smoothly from the insulating parent state.

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INTRODUCTION

Quantum spin liquids (QSLs) are exotic phases of matter that exhibit a variety of novel features associated with their topological character^{1–5}. The simplest QSLs known theoretically are characterized by topological order and support fractional excitations such as spinons, which carry the spin-1/2 but not the charge of the electron. The close relationship between the QSL and the superconductor suggests that theoretically doping it might naturally lead to superconductivity^{5–20}. Indeed, this is the original dream of the resonating valence bond theory as a mechanism for high temperature superconductivity⁶. However, despite intense efforts during past three decades, definitive evidence showing that doping QSLs gives rise to superconductivity is lacking^{5,14,15,21–24}. This is partially due to the fact that the realization of QSLs is a great challenge to physicists, where candidate materials and systems are rare^{2–5}.

The spin-1/2 antiferromagnet on the triangular lattice with nearest-neighbor (NN) and next-nearest-neighbor (NNN) Heisenberg interactions is highly frustrated. A number of numerical simulations have provided strong evidence that its ground state is a QSL in the region of $0.07 \leq J_2/J_1 \leq 0.15$, which is sandwiched by a 120° magnetic order phase and a stripe magnetic order phase^{25–32}. Therefore, it can serve as an ideal platform to investigate the consequence of doping the QSL. Although it is still under debate whether this QSL is gapped or gapless in the 2D limit, it is consistent with a gapped (possibly “ Z_2 ”³³) spin liquid on finite width cylinders. This is evidenced by the observed nonzero spin-gap and exponentially falling spin–spin correlations where the correlation length is significantly shorter than the width (L_y in Fig. 1) of the cylinders, and short-range dimer–dimer and chiral–chiral correlations^{25–29}. Therefore, these leave little doubt that doping this state corresponds to doping a gapped QSL.

As the QSL has been realized on the spin-1/2 antiferromagnetic J_1 – J_2 Heisenberg model on the triangular lattice, a natural question is whether doping this QSL will give rise to superconductivity? Quasi-one-dimensional (1D) systems such as cylinders have become an important starting point to resolve this

problem which has essential degrees of freedom that allow for two-dimensional characteristics to emerge. Moreover, it can be accurately studied using the density-matrix renormalization group (DMRG)^{34–36}. According to the Mermin–Wagner theorem, a superconducting (SC) state that can be realized in quasi-1D systems such as cylinders has quasi-long-range SC correlation. The Luther–Emery (LE) liquid is an example of this³⁷, which has one gapless charge mode with quasi-long-range SC and charge-density-wave (CDW) correlations, but the spin–spin correlations fall exponentially^{38–44}.

In this paper, we directly address the question of doping the QSL on the triangular lattice by studying the t - J model defined in Eq. (6). On the cylinders we study, the undoped system is a fully gapped QSL. Upon lightly doping we find that the system still shows exponentially falling spin–spin correlations. Most importantly, we find that even at the smallest doping level δ and on our largest 6-leg cylinders, the SC correlations are strong and decay with a slow power law, $\Phi(r) \sim |r|^{-K_{sc}}$, with $K_{sc} \approx 1$. This slow decay implies a SC susceptibility that diverges as $\chi_{sc} \sim T^{-(2-K_{sc})}$ as the temperature $T \rightarrow 0$. Moreover, the SC correlations dominate over the CDW correlations in the sense that in all cases $K_c > K_{sc}$. This is suggestive that doping the QSL state on the triangular lattice could naturally give rise to superconductivity.

RESULTS

Principal results

We find that the system exhibits power-law CDW correlations corresponding to a local pattern of “partial-filled” charge stripes. For width $L_y = 4$ cylinders, the wavelength of the CDW, i.e., the spacings between two adjacent charge stripes, is $\lambda = 1/2\delta$ corresponding to an ordering wavevector $Q = 4\pi\delta$ with half a doped hole per unit cell (Fig. 2a). Equivalently, if we view the cylinder as a 1D system, this corresponds to two holes per 1D unit cell. For width $L_y = 6$ cylinders, the CDW has a wavelength of $\lambda = 1/3\delta$, and an ordering wavevector $Q = 6\pi\delta$ with only one third of a

¹Stanford Institute for Materials and Energy Sciences, SLAC National Accelerator Laboratory and Stanford University, Menlo Park, CA, USA. ✉email: hcjiang@stanford.edu

doped hole per unit cell (Fig. 2b). Similar with the 4-leg cylinders, this again corresponds to two holes per 1D unit cell.

Because $L_x \gg L_y$, our system can be thought of as quasi-1D, and we find that the ground state of the system is consistent with that of a LE liquid³⁷, which is characterized by power-law decaying CDW (Fig. 2) and SC correlations (Figs. 3, 4), but exponentially decaying spin-spin correlations (Fig. 5), with one gapless charge mode (Fig. 6). At long distance, the spatial decay of the CDW

Fig. 1 The t - J model on the triangular lattice. Periodic and open boundary conditions are imposed respectively along the directions specified by the lattice basis vectors $\mathbf{e}_y$ and $\mathbf{e}_x$. L_x and L_y are the number of sites in the $\mathbf{e}_x$ and $\mathbf{e}_y$ directions. t_1 (t_2) and J_1 (J_2) are hopping integral and spin exchange interactions between NN (NNN) sites. The electrons live at the vertices (filled circles), and a , b , and c denote the three different bonds.

correlation is dominated by a power-law with the Luttinger exponent K_c . The exponent K_c can be obtained by fitting the charge-density oscillations (Friedel oscillations) induced by the boundaries of the cylinder⁴⁵

$$n(x) = n_0 + \delta n * \cos(2k_F x + \phi) x^{-K_c/2}. \quad (1)$$

Here $n(x) = \langle \hat{n}(x) \rangle$, where $\hat{n}(x) = \sum_{y=1}^{L_y} \hat{n}(x, y) / L_y$ is the local rung density operator and x is the rung index of the cylinder. δn is a nonuniversal amplitude, ϕ is a phase shift, n_0 is the background density, and k_F is the Fermi wavevector. Note that a few data points (Fig. 2a, b, gray color) are excluded to minimize the boundary effect and improve the fitting quality. The extracted exponent K_c is similar on both width $L_y = 4$ and $L_y = 6$ cylinders, where we find $K_c > 1$ at all doping levels. Similarly, K_c can also be obtained from the charge density-density correlation which gives consistent results (see Supplementary Fig. 4).

At long distance, the SC pair-field correlation $\Phi(r)$, defined in Eq. (3), is also characterized by a power-law with the appropriate Luttinger exponent K_{sc} defined by

$$\Phi(r) \propto r^{-K_{sc}}, \quad (2)$$

where r is the distance between two Cooper pairs along the $\mathbf{e}_x$ direction. As shown in Fig. 3d, the extracted exponent in the long cylinder limit $L_x \rightarrow \infty$ is $K_{sc} < 1$ on $L_y = 4$ cylinders. This also holds true for $L_y = 6$ cylinders, again we find that $K_{sc} < 1$ when $L_x \rightarrow \infty$. For both cases, this implies a divergent SC susceptibility

Fig. 2 Charge-density profiles and Luttinger exponent K_c . Charge-density profiles $n(x)$ for (a) $L_y = 4$ and (b) $L_y = 6$ cylinders of length L_x at various doping levels δ . The extracted K_c using Eq. (1) for (c) $L_y = 4$ and (d) $L_y = 6$ cylinders as a function of $1/L_x$ and δ , as well as their extrapolated value in the long cylinder limit $1/L_x = 0$, i.e., $L_x \rightarrow \infty$. A few data points in light gray are removed to minimize the boundary effect. Error bars denote the numerical uncertainty and the dash-dotted lines denote the linear extrapolation.

Fig. 3 Superconducting correlations on $L_y = 4$ cylinders. SC correlations $\Phi_{ad}(r)$ on $L_y = 4$ cylinders of length L_x for (a) $\delta = 1/12$, (b) $\delta = 1/16$, and (c) $\delta = 1/24$ on double-logarithmic scales, where r is the distance between two Cooper pairs in the $\mathbf{e}_x$ direction. The solid lines denote power-law fitting $\Phi_{ad}(r) \sim r^{-K_{sc}}$. **d** Luttinger exponent K_{sc} as a function of δ and $1/L_x$, as well as their extrapolated value in the long cylinder limit $1/L_x = 0$, i.e., $L_x \rightarrow \infty$. Inset: The extrapolated exponents $K_{sc}(1/L_x = 0)$, $K_c(1/L_x = 0)$ in the long cylinder limit $1/L_x = 0$ and their product $K_{sc}K_c$ as a function of δ . Error bars denote the numerical uncertainty and the dash-dotted lines denote the linear extrapolation.

$\chi_{sc} \sim T^{-(2-K_{sc})}$ when the temperature $T \rightarrow 0$. As expected theoretically from the LE liquid³⁷, we find that the relation $K_c K_{sc} = 1$ holds within the numerical uncertainty and finite-size effect (Figs. 3d, 4d, insets) for $L_y = 4$ cylinders for doping levels $\delta \leq 1/12$ and $L_y = 6$ cylinders for doping levels $\delta \leq 1/18$. However, a notable deviation can be seen for the doping level $\delta = 1/12$ on $L_y = 6$ cylinders (Fig. 4d inset), we suspect that it might be attributed to the finite-size effect even after finite-size (L_x) extrapolation (Figs. 2d, 4d). This is also consistent with the central charge c shown in Fig. 6d, which also suffers from visible finite-size effect, although an apparent trend that c may approach to the expected value $c = 1$ in the long cylinder limit $L_x \rightarrow \infty$. In any case, the fact that $K_c > K_{sc}$ (Figs. 3d, 4d) demonstrates the dominance of the SC correlations $\Phi(r)$ on both $L_y = 4$ and $L_y = 6$ cylinders.

Charge-density wave order

To describe the charge-density properties of the ground state of the system, we have calculated the charge-density profile $n(x)$. Figure 2a shows the charge-density distribution $n(x)$ for width $L_y = 4$ cylinders, which is consistent with the “half-filled” charge stripe of wavelength $\lambda = 1/2\delta$, i.e., spacings between two adjacent stripes, for all doping concentration δ with half a doped hole per unit cell. The charge-density profile $n(x)$ for width $L_y = 6$ cylinders is shown in Fig. 2b, which is however consistent with “third-filled”

charge stripes for all δ with only one third of a doped hole per unit cell. As a result, it has a shorter wavelength $\lambda = 1/3\delta$ on $L_y = 6$ cylinders than the “half-filled” stripes on $L_y = 4$ cylinders at the same doping concentration δ .

At long distance, we find that the CDW correlations decay with a power-law with the Luttinger exponent K_c . The exponent K_c , which was obtained by fitting the data points using Eq. (1), is shown in Fig. 2c for width $L_y = 4$ cylinders and Fig. 2d for width $L_y = 6$ cylinders as a function of L_x and δ . For all cases, we find that K_c increases with L_x , which is consistent with weaker CDW correlations in the longer cylinders. A notable difference between the $L_y = 4$ and $L_y = 6$ cylinders is the doping dependence of K_c as shown in Figs. 3d, 4d. On the $L_y = 4$ cylinders, the value of K_c decreases with the increase of δ on width $L_y = 4$ cylinder. On the contrary, K_c increases with the increase of δ on width $L_y = 6$ cylinders. However, in any case, our results show that $K_c > 1$, or more precisely $K_c > 1.2$ in the long cylinder limit $L_x \rightarrow \infty$ (Fig. 2c, d), for all doping levels on both $L_y = 4$ and $L_y = 6$ cylinders. K_c can also be obtained directly from the charge-density-density correlation, which produces similar results (see Supplementary Fig. 4).

Superconducting correlation

To test the possibility of superconductivity, we have also calculated the equal-time SC pair-field correlations. As the ground state of the system with even number of doped holes is always

Fig. 4 Superconducting correlations on $L_y = 6$ cylinders. SC correlations $\Phi_{aa}(r)$ on $L_y = 6$ cylinders of length L_x for (a) $\delta = 1/12$, (b) $\delta = 1/18$, and (c) $\delta = 1/24$ on double-logarithmic scales, where r is the distance between two Cooper pairs in the $\mathbf{e}_x$ direction. The solid lines denote power-law fitting $\Phi_{aa}(r) \sim r^{-K_{sc}}$. **d** Luttinger exponent K_{sc} as a function of δ and $1/L_x$ as well as their extrapolated value in the long cylinder limit $1/L_x = 0$, i.e., $L_x \rightarrow \infty$. Inset: The extrapolated exponents $K_{sc}(1/L_x = 0)$, $K_c(1/L_x = 0)$ in the long cylinder limit $1/L_x = 0$ and their product $K_{sc}K_c$ as a function of δ . Error bars denote the numerical uncertainty and the dash-dotted lines denote the linear extrapolation.

found to have spin 0, we focus on spin-singlet pairing. A diagnostic of the SC order is the pair-field correlator, defined as

$$\Phi_{a\beta}(r) = \frac{1}{L_y} \sum_{y=1}^{L_y} \langle \Delta_a^\dagger(x_0, y) \Delta_\beta(x_0 + r, y) \rangle. \quad (3)$$

Here $\Delta_a^\dagger(x, y) = \frac{1}{\sqrt{2}} [c_{(x,y),\uparrow}^\dagger c_{(x,y)+a,\downarrow}^\dagger - c_{(x,y),\downarrow}^\dagger c_{(x,y)+a,\uparrow}^\dagger]$ is the spin-singlet pair-field creation operator, where the bond orientations are designated $a = a, b$, and c (Fig. 1), (x_0, y) is the reference bond with $x_0 \sim L_x/4$, and r is the distance between two bonds in the $\mathbf{e}_x$ direction.

Due to the presence of CDW modulations (Fig. 2), SC correlations $\Phi_{a\beta}(r)$ exhibit similar spatial oscillations with $n(x)$. For a given cylinder of length L_x , we determine the decay of SC correlations with reference bond located at the peak position around $x_0 \sim L_x/4$ of the charge-density distribution $n(x)$ to minimize the boundary effect^{42,43}. This gives accurate values of $\Phi_{a\beta}(r)$ for reliable finite-size scaling. Examples of $\Phi_{aa}(r)$ are given in Figs. 3, 4 for width $L_y = 4$ and $L_y = 6$ cylinders, respectively. Further details are provided in the Supplementary Fig. 2.

As the ground state of the system is a spin-singlet state, theoretically, the pair symmetry of the SC correlations will be consistent with either s -wave, $d \pm id$ or d -wave^{46,47}. In the following we show that the pair symmetry is only consistent with the d -wave. First of all, our results show that the ground state

wavefunction of the system is purely real and the imaginary part of all the SC correlations $\Phi_{a\beta}(r)$ is zero. As a result, the pair symmetry is inconsistent with $d \pm id$ -wave. Second, the SC correlations change sign between different bonds, i.e., $\Phi_{ab} \sim -0.4\Phi_{aa}$ (see Supplementary Fig. 3), so the pair symmetry is also inconsistent with s -wave. Therefore, the only option left is the d -wave, which is expected to survive in two dimensions⁴⁷. This is indeed consistent with our results, where we find that the SC correlations are stronger on a bonds but weaker on other bonds, i.e., $\Phi_{bb} \sim \Phi_{cc} \sim 0.2\Phi_{aa}$ on both the $L_y = 4$ and $L_y = 6$ cylinders. Taken together theory and numerical results, we therefore conclude that the pairing symmetry of the SC correlations is consistent with the d -wave.

Similar to the CDW correlations, the SC correlations $\Phi(r)$ of the system on both width $L_y = 4$ (Fig. 3) and $L_y = 6$ cylinders (Fig. 4) also decay with a power law, whose exponent K_{sc} was obtained by fitting the results using Eq. (2)^{42–44}. The fact that the observed $K_{sc} < 1$ holds for all the cases when $L_x \gg L_y$ undoubtedly demonstrates the dominance of the SC correlations over the CDW correlations since $K_c > 1 > K_{sc}$. These findings are in stark contrast to the case of doping QSL on the Kagome lattice^{22,24}, where the SC correlations decay exponentially but the CDW correlations have 2D long-range order. This indicates that the lattice geometry may play a critical role in giving rise to superconductivity in doping a QSL.

Fig. 5 Spin–spin correlations. **a** Examples of spin–spin correlation functions $F(r)$ at doping level $\delta = 1/12$ for $L_y = 4$ and $L_y = 6$ cylinders on the semi-logarithmic scale. Solid lines denote exponential fitting $F(r) \sim e^{-r/\xi_s}$, where r is the distance between two sites in the $\mathbf{e}_x$ direction. **b** The spin–spin correlation length ξ_s as a function of δ for both $L_y = 4$ and $L_y = 6$ cylinders. Error bars denote the numerical uncertainty.

Spin–spin correlation

To describe the magnetic properties of the ground state, we calculate the spin–spin correlation functions defined as

$$F(r) = \frac{1}{L_y} \sum_{y=1}^{L_y} |\langle \vec{S}_{x_0,y} \cdot \vec{S}_{x_0+r,y} \rangle|, \quad (4)$$

where $\vec{S}_{x,y}$ is the spin operator on site $i = (x, y)$. (x_0, y) is the reference site with $x_0 \sim L_x/4$ and r is the distance between two sites in the $\mathbf{e}_x$ direction. It is worth noting that the oscillations have different wavelength between small r region (short distance) and large r region (long distance), and there are no antiphase domain walls in the spin–spin correlation $\langle \vec{S}_{x_0,y} \cdot \vec{S}_{x_0+r,y} \rangle$. Following similar procedure as for $n(x)$ and $\Phi(r)$, we first extrapolate $F(r)$ to the limit $\epsilon \rightarrow 0$ for a given cylinder of length L_x and then perform finite-size scaling as a function of L_x . As shown in Fig. 5a, $F(r)$ decays exponentially as $F(r) \sim e^{-r/\xi_s}$ at long-distances, with a correlation length $\xi_s = 1 \sim 8$ lattice spacings for the $L_y = 4$ cylinders and $\xi_s = 2 \sim 7$ lattice spacings for the $L_y = 6$ cylinders (Fig. 5b) for doping concentration $0 \leq \delta \leq 1/12$. Therefore, the spin–spin correlations are also short-ranged which is similar with the QSL at half-filling, where the spin–spin correlations also decay exponentially.

Central charge

A key feature of the LE liquid is that it has a single gapless charge mode with central charge $c = 1$, which can be obtained by calculating the von Neumann entropy $S = -\text{Tr} \rho \ln \rho$, where ρ is the reduced density matrix of a subsystem with length x . For critical systems in $1 + 1$ dimensions described by a conformal field theory, it has been established^{48,49} that for an open system of length L_x ,

$$S(x) = \frac{c}{6} \ln \left[\frac{4(L_x+1)}{\pi} \sin \frac{\pi(2x+1)}{2(L_x+1)} |\sin k_F| \right] + \tilde{A} \frac{\sin[k_F(2x+1)]}{\pi \sin \frac{\pi(2x+1)}{2(L_x+1)} \sin k_F} + \tilde{S}, \quad (5)$$

where $\tilde{A}$ and $\tilde{S}$ are model dependent fitting parameters, and k_F is the Fermi momentum. Examples of $S(x)$ are shown in Fig. 6a, b for the $L_y = 4$ and $L_y = 6$ cylinders, respectively. For a given cylinder of length L_x , a few data points in blue close to the ends are excluded in the fitting to minimize the boundary effect. The extracted central charge c is shown in Fig. 6c, d as a function of L_x and δ . Although the value of c deviates notably from $c = 1$ on short

cylinders, apparently it approaches $c = 1$ with the increase of L_x by taking into account the numerical uncertainty and finite-size effect. The result is consistent with one gapless charge mode with $c = 1$, which provides additional evidence for the presence of the LE liquid in doping the QSL on the triangular lattice.

DISCUSSION

Taken together, our results show that the ground state of doping QSL on the triangular lattice is consistent with a LE liquid with a finite gap in the spin sector. Although both CDW and SC correlations decay as a power-law, the fact that $K_c > 1 > K_{sc}$ on both the 4-leg and 6-leg cylinders when $L_x \gg L_y$ at all doping levels undoubtedly demonstrates the dominance of the SC correlations with divergent SC susceptibility. To the best of our knowledge, this is the direct numerical realization of dominant superconductivity in doping a time-reversal symmetric QSL, which provides compelling evidence that doping a QSL could naturally give rise to superconductivity^{6–9}. In this paper, we have focused on the lightly doped case, it will also be interesting to study the higher doping case as well as the consequence of doping different phases on the triangular lattice such as the magnetic ordered phase^{50,51}. Answering these questions may lead to better understanding of the mechanism of high temperature superconductivity.

METHODS

Model Hamiltonian

We employ DMRG³⁴ to study the ground state properties of the hole-doped t - J model on the triangular lattice, defined by the Hamiltonian

$$H = - \sum_{ij\sigma} t_{ij} (\hat{c}_{i\sigma}^\dagger \hat{c}_{j\sigma} + h.c.) + \sum_{ij} J_{ij} \left(\vec{S}_i \cdot \vec{S}_j - \frac{\hat{n}_i \hat{n}_j}{4} \right). \quad (6)$$

Here $\hat{c}_{i\sigma}^\dagger$ ($\hat{c}_{i\sigma}$) is the electron creation (annihilation) operator on site $i = (x_i, y_i)$ with spin σ . $\vec{S}_i$ is the spin operator and $\hat{n}_i = \sum_\sigma \hat{c}_{i\sigma}^\dagger \hat{c}_{i\sigma}$ is the electron number operator. The electron hopping amplitude t_{ij} is equal to t_1 (t_2) if i and j are NN (NNN) sites as shown in Fig. 1. J_1 and J_2 are the spin superexchange interactions between NN and NNN sites, respectively. The Hilbert space is constrained by the no-double occupancy condition, $n_i \leq 1$. At half-filling, i.e., $n_i = 1$, Eq. (6) reduces to the spin-1/2 antiferromagnetic J_1 - J_2 Heisenberg model.

Fig. 6 Entanglement entropy S and central charge c . S is shown at doping level $\delta = 1/12$ for (a) $L_y = 4$ and (b) $L_y = 6$ cylinders. Extracted central charge c using Eq. (5) for (c) $L_y = 4$ and (d) $L_y = 6$ cylinders as a function of L_x and δ . Note that a few data points in blue from each end are removed to minimize boundary effects. Error bars denote the numerical uncertainty.

Numerical details

The lattice geometry used in our simulations is depicted in Fig. 1, where $\mathbf{e}_x = (1, 0)$ and $\mathbf{e}_y = (1/2, \sqrt{3}/2)$ denote the two basis vectors. We take the lattice geometry to be cylindrical with periodic (open) boundary condition in the $\mathbf{e}_y$ ($\mathbf{e}_x$) direction. Here, we focus on cylinders with width L_y and length L_x , where L_x and L_y are number of sites along the $\mathbf{e}_x$ and $\mathbf{e}_y$ directions, respectively. The total number of sites is $N = L_x \times L_y$ with $N_e = N$ electrons at half-filling. The doping level of the system is defined as $\delta = N_h/N$, where $N_h = N - N_e$ is the number of doped holes.

For the present study, we focus on the lightly doped case with $\delta \leq 1/12$ on width $L_y = 4$ cylinders of length up to $L_x = 144$ and width $L_y = 6$ cylinders of length up to $L_x = 72$. We set $J_1 = 1$ as an energy unit and $J_2 = 0.11$ such that the system is deep in the QSL phase at half-filling^{25–30}. We consider $t_1 = 3$ and $t_2 = t_1 \sqrt{J_2/J_1} = 0.995$ to make a connection to the corresponding Hubbard model⁵². The results also hold for other t_1 and t_2 . We perform up to 100 sweeps and keep up to $m = 12,000$ states for width $L_y = 4$ cylinders with a typical truncation error $\epsilon < 3 \times 10^{-7}$, and up to $m = 30,000$ states for width $L_y = 6$ cylinders with a typical truncation error $\epsilon < 4 \times 10^{-6}$. This leads to excellent convergence for our results when extrapolated to $m = \infty$ limit. Further details of the numerical simulation are provided in the Supplementary Information sections I, II, and III.

DATA AVAILABILITY

The author declares that the main data supporting the findings of this study are available within the article and its Supplementary Information file. Extra data are available from the corresponding author upon reasonable request.

CODE AVAILABILITY

The codes implementing the calculations of this study are available from the corresponding author upon request.

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REFERENCES

- Anderson, P. Resonating valence bonds: a new kind of insulator?. *Mater. Res. Bull.* **8**, 153–160 (1973).
- Balents, L. Spin liquids in frustrated magnets. *Nature* **464**, 199–208 (2010).
- Savary, L. & Balents, L. Quantum spin liquids: a review. *Rep. Prog. Phys.* **80**, 016502 (2016).
- Zhou, Y., Kanoda, K. & Ng, T.-K. Quantum spin liquid states. *Rev. Mod. Phys.* **89**, 025003 (2017).
- Broholm, C. et al. Quantum spin liquids. *Science* **367**, eaay0668 (2020).
- Anderson, P. W. The resonating valence bond state in La_2CuO_4 and superconductivity. *Science* **235**, 1196–1198 (1987).
- Kivelson, S. A., Rokhsar, D. S. & Sethna, J. P. Topology of the resonating valence-bond state: Solitons and high- T_c superconductivity. *Phys. Rev. B* **35**, 8865–8868 (1987).
- Rokhsar, D. S. & Kivelson, S. A. Superconductivity and the quantum hard-core dimer gas. *Phys. Rev. Lett.* **61**, 2376–2379 (1988).
- Laughlin, R. B. Superconducting ground state of noninteracting particles obeying fractional statistics. *Phys. Rev. Lett.* **60**, 2677–2680 (1988).
- Laughlin, R. B. The relationship between high-temperature superconductivity and the fractional quantum hall effect. *Science* **242**, 525–533 (1988).

11. Wen, X. G., Wilczek, F. & Zee, A. Chiral spin states and superconductivity. *Phys. Rev. B* **39**, 11413–11423 (1989).
12. Emery, V. J., Kivelson, S. A. & Zachar, O. Spin-gap proximity effect mechanism of high-temperature superconductivity. *Phys. Rev. B* **56**, 6120–6147 (1997).
13. Senthil, T. & Lee, P. A. Cuprates as doped $U(1)$ spin liquids. *Phys. Rev. B* **71**, 174515 (2005).
14. Lee, P. A., Nagaosa, N. & Wen, X.-G. Doping a mott insulator: Physics of high-temperature superconductivity. *Rev. Mod. Phys.* **78**, 17–85 (2006).
15. Fradkin, E., Kivelson, S. A. & Tranquada, J. M. *Colloquium*: Theory of intertwined orders in high temperature superconductors. *Rev. Mod. Phys.* **87**, 457–482 (2015).
16. Venderley, J. & Kim, E.-A. Density matrix renormalization group study of superconductivity in the triangular lattice hubbard model. *Phys. Rev. B* **100**, 060506 (2019).
17. Venderley, J. & Kim, E.-A. Evidence of pair-density wave in spin-valley locked systems. *Sci. Adv.* **5**, eaat4698 (2019).
18. Jiang, Y.-F. & Jiang, H.-C. Topological superconductivity in the doped chiral spin liquid on the triangular lattice. *Phys. Rev. Lett.* **125**, 157002 (2020).
19. Song, X.-Y., Vishwanath, A. & Zhang, Y.-H. Doping the chiral spin liquid: Topological superconductor or chiral metal. *Phys. Rev. B* **103**, 165138 (2021).
20. Jiang, H.-C. & Kivelson, S. A. High temperature superconductivity in a lightly doped quantum spin liquid. Preprint at <https://arxiv.org/abs/2104.01485> (2021).
21. Kelly, Z. A., Gallagher, M. J. & McQueen, T. M. Electron doping a kagome spin liquid. *Phys. Rev. X* **6**, 041007 (2016).
22. Jiang, H.-C., Devereaux, T. & Kivelson, S. A. Holon wigner crystal in a lightly doped kagome quantum spin liquid. *Phys. Rev. Lett.* **119**, 067002 (2017).
23. Peng, C., Jiang, Y.-F., Devereaux, T. P. & Jiang, H.-C. Precursor of pair-density wave in doping Kitaev spin liquid on the honeycomb lattice. *npj Quantum Mater.* **6**, 64 (2021).
24. Peng, C., Jiang, Y.-F., Sheng, D.-N. & Jiang, H.-C. Doping quantum spin liquids on the kagome lattice. *Adv. Quantum Technol.* **4**, 2000126 (2021).
25. Zhu, Z. & White, S. R. Spin liquid phase of the $S=\frac{1}{2}J_1 - J_2$ Heisenberg model on the triangular lattice. *Phys. Rev. B* **92**, 041105 (2015).
26. Hu, W.-J., Gong, S.-S., Zhu, W. & Sheng, D. N. Competing spin-liquid states in the spin- $\frac{1}{2}$ Heisenberg model on the triangular lattice. *Phys. Rev. B* **92**, 140403 (2015).
27. Zheng, W., Mei, J.-W. & Qi, Y. Classification and Monte Carlo study of symmetric Z_2 spin liquids on the triangular lattice. Preprint at <https://arxiv.org/abs/1505.05351> (2015).
28. Saadatmand, S. N. & McCulloch, I. P. Symmetry fractionalization in the topological phase of the spin- $\frac{1}{2}J_1 - J_2$ triangular Heisenberg model. *Phys. Rev. B* **94**, 121111 (2016).
29. Gong, S.-S., Zhu, W., Zhu, J.-X., Sheng, D. N. & Yang, K. Global phase diagram and quantum spin liquids in a spin- $\frac{1}{2}$ triangular antiferromagnet. *Phys. Rev. B* **96**, 075116 (2017).
30. Gong, S.-S., Zheng, W., Lee, M., Lu, Y.-M. & Sheng, D. N. Chiral spin liquid with spinon Fermi surfaces in the spin-1/2 triangular Heisenberg model. *Phys. Rev. B* **100**, 241111(R) (2019).
31. Iqbal, Y., Hu, W.-J., Thomale, R., Poilblanc, D. & Becca, F. Spin liquid nature in the heisenberg $J_1 - J_2$ triangular antiferromagnet. *Phys. Rev. B* **93**, 144411 (2016).
32. Hu, S., Zhu, W., Eggert, S. & He, Y.-C. Dirac Spin Liquid on the Spin-1/2 Triangular Heisenberg Antiferromagnet. *Phys. Rev. Lett.* **123**, 207203 (2019).
33. Moessner, R. & Sondhi, S. L. Resonating valence bond phase in the triangular lattice quantum dimer model. *Phys. Rev. Lett.* **86**, 1881–1884 (2001).
34. White, S. R. Density matrix formulation for quantum renormalization groups. *Phys. Rev. Lett.* **69**, 2863–2866 (1992).
35. Schollwöck, U. The density-matrix renormalization group in the age of matrix product states. *Ann. Phys. (N. Y.)* **326**, 96–192 (2011).
36. Stoudenmire, E. & White, S. R. Studying two-dimensional systems with the density matrix renormalization group. *Annu. Rev. Condens. Matter Phys.* **3**, 111–128 (2012).
37. Luther, A. & Emery, V. J. Backward scattering in the one-dimensional electron gas. *Phys. Rev. Lett.* **33**, 589–592 (1974).
38. Balents, L. & Fisher, M. P. A. Weak-coupling phase diagram of the two-chain Hubbard model. *Phys. Rev. B* **53**, 12133–12141 (1996).
39. Seidel, A. & Lee, D.-H. The Luther-emery liquid: Spin gap and anomalous flux period. *Phys. Rev. B* **71**, 045113 (2005).
40. Dolfi, M., Bauer, B., Keller, S. & Troyer, M. Pair correlations in doped hubbard ladders. *Phys. Rev. B* **92**, 195139 (2015).
41. Dodaro, J. F., Jiang, H.-C. & Kivelson, S. A. Intertwined order in a frustrated four-leg t-J cylinder. *Phys. Rev. B* **95**, 155116 (2017).
42. Jiang, H.-C., Weng, Z.-Y. & Kivelson, S. A. Superconductivity in the doped t-J model: results for four-leg cylinders. *Phys. Rev. B* **98**, 140505 (2018).
43. Jiang, H.-C. & Devereaux, T. P. Superconductivity in the doped Hubbard model and its interplay with next-nearest hopping t' . *Science* **365**, 1424–1428 (2019).
44. Jiang, Y.-F., Zaanen, J., Devereaux, T. P. & Jiang, H.-C. Ground state phase diagram of the doped Hubbard model on the four-leg cylinder. *Phys. Rev. Res.* **2**, 033073 (2020).
45. White, S. R., Affleck, I. & Scalapino, D. J. Friedel oscillations and charge density waves in chains and ladders. *Phys. Rev. B* **65**, 165122 (2002).
46. Feiguin, A. E., White, S. R., Scalapino, D. J. & Affleck, I. Pairing symmetry and josephson current in doped 2-leg t-J ladders. *Phys. Rev. Lett.* **101**, 217001 (2008).
47. Raghu, S., Kivelson, S. A. & Scalapino, D. J. Superconductivity in the repulsive Hubbard model: an asymptotically exact weak-coupling solution. *Phys. Rev. B* **81**, 224505 (2010).
48. Calabrese, P. & Cardy, J. Entanglement entropy and quantum field theory. *J. Stat. Mech. Theory Exp.* **2004**, P06002 (2004).
49. Fagotti, M. & Calabrese, P. Universal parity effects in the entanglement entropy of XX chains with open boundary conditions. *J. Stat. Mech. Theory Exp.* **2011**, P01017 (2011).
50. Oike, H., Miyagawa, K., Taniguchi, H. & Kanoda, K. Pressure-induced mott transition in an organic superconductor with a finite doping level. *Phys. Rev. Lett.* **114**, 067002 (2015).
51. Oike, H. et al. Anomalous metallic behaviour in the doped spin liquid candidate $K-(\text{ET})_4\text{Hg}_{2.89}\text{Br}_8$. *Nat. Commun.* **8**, 756 (2017).
52. Misumi, K., Kaneko, T. & Ohta, Y. Mott transition and magnetism of the triangular-lattice Hubbard model with next-nearest-neighbor hopping. *Phys. Rev. B* **95**, 075124 (2017).

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AUTHOR CONTRIBUTIONS

H.C.J. conceived the study, performed numerical simulations, analyzed data, and wrote the paper.

COMPETING INTERESTS

The author declares no competing interests.

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Correspondence and requests for materials should be addressed to H.-C.J.

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