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Maximizing quantum discord from interference in multi-port fiber beamsplitters

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Fourth-order interference is an information processing primitive for photonic quantum technologies, as it forms the basis of photonic controlled-logic gates, entangling measurements, and can be used to produce quantum correlations. Here, using classical weak coherent states as inputs, we study fourth-order interference in 4×4 multi-port beam splitters built within multi-core optical fibers, and show that quantum correlations, in the form of geometric quantum discord, can be controlled and maximized by adjusting the intensity ratio between the two inputs. Though these states are separable, they maximize the geometric discord in some instances, and can be a resource for protocols such as remote state preparation. This should contribute to the exploitation of quantum correlations in future telecommunication networks, in particular in those that exploit spatially structured fibers.

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INTRODUCTION

Quantum information promises to revolutionize the way in which information is transmitted, processed and stored, giving way to paradigms such as quantum cryptography and quantum computing. A key element in this effort is the transport of quantum states from one place to another. In this regard, to be viable, quantum communication will most likely need to employ the same technological infrastructure as classical telecommunications.

A main goal in telecommunications is to increase the transmission capacity of optical channels, as currently, data rates are nearing the physical limits that are possible in single-mode optical fibers, known as the “capacity crunch”^{1,2}. This has led to a number of interesting encoding schemes and technologies. One exciting solution to this problem is the use of multi-core optical fibers (MCF), composed of several fiber cores within the same cladding³. Multi-core optical fibers (MCFs) promise to have an even bigger impact on quantum information protocols⁴. For one, the relative phase fluctuations between quantum states propagating in different cores in the same cladding is much less than for multiple single-mode fibers^{5–7}. This has led to a number of MCF-based experiments involving quantum systems with dimension greater than two^{8–14}. However, a complete toolbox for the manipulation of photonic quantum information encoded in MCFs is still lacking. One important element, a fiber-embedded multi-core beam splitters (MCF-BS), has only been recently developed. These are multi-port interference devices that coherently combine light from input fiber cores, and can be used to decrease the optical depth of linear optical circuits^{15,16}, as well as to build multi-path interferometers with applications in optical metrology^{17,18}, for example. They have been employed in a few single-photon experiments^{10,12,13}.

An important application of optical beam splitters in the context of quantum information is the realization of two-photon interference¹⁹. It can be used to build photonic controlled-logic gates^{20,21}, and in projection onto entangled states^{22–26}. Moreover, two-photon interference can produce quantum correlations when the quantum state is post-selected in the number basis. We note

that this was the first source of polarization-entangled photon pairs from spontaneous parametric down-conversion^{27,28}.

Here we investigate multi-photon interference in a 4×4 multi-core fiber beam splitter (4CF-BS) that is entirely compatible with multi-core fiber infrastructure, and show how post-selected quantum correlations can be controlled and maximized by adjusting intensity ratio of the input laser pulses. The correlations arise from a fourth-order interference effect^{29,30}, where bunching and anti-bunching behavior is observed when photon pairs are detected in coincidence at different output ports. Our experiment is performed using two independent weak coherent states (WCS) at telecom wavelengths. There has been much recent interest in this scenario since the development of measurement-device independent quantum key distribution^{31,32}. When considering the complete Poissonian photon statistics, the input and output states are each a tensor product of coherent states, and thus present no correlations. However, in Ref. ³³, it was shown that using post-selection in photon number, the output state produced by equal intensity WCSs could in fact demonstrate non-classical correlations in the form of quantum discord^{34,35}. Though discord does not imply quantum entanglement³⁵, recent studies have shown that it plays an important role in quantum tasks such as quantum computing^{36,37}, remote state preparation (RSP)³⁸, quantum illumination and metrology^{39,40}, quantum cryptography⁴¹, quantum state discrimination^{42,43} and the quantum-classical transition^{44,45}. Moreover, a number of interesting dynamical features of discord have been explored^{46–52}. A more comprehensive review of discord and its relation to quantum phenomena and protocols can be found in Refs. ^{53–56}.

RESULTS

Experimental description

Let us first introduce the experimental setup, shown in Fig. 1. The source is a continuous wave laser, with a wavelength $\lambda = 1546$ nm, that is coupled through a single mode fiber (SMF) to a lithium niobate (LiNbO₃) intensity modulator (IM) to produce a

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Fig. 1 Illustration of the experimental setup. Weak coherent pulses are interfered incoherently on a four-core fiber beam splitter (4CF-BE). CW: continuous wave laser, IM: intensity modulator, Att: attenuator, BS: single-mode fiber 2×2 beam splitter, PC: polarizer controller, MUX: fiber multiplexer for single-core SMF fiber to multicore fiber, 4CF-BE: four-core beam splitter, PM: phase modulator, FG: function generator, D_j : triggered single photon detector, FPGA: field programmable gate array device.

Fig. 2 Examples of multi-core optical fiber devices. a 4×4 multi-core fiber beam splitter. **b** Single-mode to multi-core fiber multiplexer device.

train of 5 ns width quasi-Gaussian pulses at a repetition rate of 2.78 MHz. The pulses are attenuated and sent into a SMF 50/50 fiber beam splitter (BS0), resulting in two WCSs. One of them is delayed with respect to the second one by 72 m of fiber. A fiber polarization controller (PC) is used to configure the polarization between the two, and then the WCSs are sent into the 4CF-BE. Since the relative delay between the WCSs is much greater than the coherence length of the laser, they are mutually incoherent when the overlap at the 4CF-BE, so there is no second-order (single photon) interference between the overlapping pulses. However, fourth-order (two-photon) interference can occur^{24,29,30,33}.

The 4CF-BE is a tapered MCF device (see Fig. 2a), where the proximity of the cores produces high optical crosstalk, whose architecture has been used for high-quality single-photon (second-order) interference^{10,12,13}. An optical single-mode to multi-core fiber multiplexer device (MUX) is required to couple the WCS source to the 4CF-BE. As Fig. 2b shows, a MUX consists of a group of SMFs fused to MCF, where each SMF is oriented to map to a single core in the MCF, allowing access to any of these cores through an SMF fan-out. Hence, MUX1 connects both WCSs from the WCS source to the $i1$ and $i3$ inputs of the 4CF-BE.

The 4CF-BE outputs can be connected to two independent SMF-based two-arm interferometers, each of which is phase controlled with fiber-coupled LiNbO₃ phase modulators (PM0 and PM1).

Two-photon events are registered from coincidence detections (coincident counts) between four triggered single photon detectors (SPD). The detectors were configured with 5 ns detection windows and 10% detection efficiency. Polarization-noise in the SMF decreases the quality of overlap in each output BS and therefore reduce the visibility of fourth order interference. To delete polarization information^{57,58}, two additional PCs are used. The pulse generation via the IM in the WCSs source and the triggered measurements at the SPDs are synchronized by a field programmable gate array (FPGA). The laser pulses were attenuated to have an average number of photons per pulse of $\mu = 0.46$ at the detectors. This implies that 36.9% of the pulses will have one or more photons. Of these non-vacuum events, 78.8% correspond to pulses containing only a single photon, 18.1% to two photons, and only 3% to more than two photons. Given the pulse generation rate and the detection efficiency, about 25,000 single counts $\cdot s^{-1}$ are recorded from each detector (D_j with $j \in 0, 1, 2, 3$). Furthermore, the FPGA is configured to record all possible two-photon coincidence counts in a 1 ns wide coincidence-window, achieving a rate of ~ 400 total coincidence counts $\cdot s^{-1}$. With respect to the phase control in the two-arm interferometers, the PM0 modulator is driven by the same FPGA while the second (PM1) is independently controlled by a simple electrical signal generator (function generator in Fig. 1).

4 × 4 multi-port fiber beamsplitter

The two WCSs interfere at a 4 × 4 multicore fiber beam splitter. The transfer matrix relating the input and output modes of this device is well-described by¹⁰:

$$\mathbf{B}_4 = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix}. \quad (1)$$

We can denote the input and output creation operators using the vectors $\hat{\mathbf{a}}_{\text{in/out}}^\dagger = (\hat{a}_0^\dagger, \hat{a}_1^\dagger, \hat{a}_2^\dagger, \hat{a}_3^\dagger)$, so that $\hat{\mathbf{a}}_{\text{out}}^\dagger = \mathbf{B}_4 \hat{\mathbf{a}}_{\text{in}}^\dagger$.

Input and output state with post-selection

A coherent state with complex amplitude η can be written in the Fock basis as

$$|\eta\rangle = e^{-|\eta|^2/2} \sum_{n=0}^{\infty} \frac{\eta^n}{\sqrt{n!}} |n\rangle. \quad (2)$$

The Fock states are defined as $|n\rangle = (\hat{a}^\dagger)^n |0\rangle / \sqrt{n!}$, where $\hat{a}^\dagger$ is the photonic creation operator and $|0\rangle$ is the vacuum state. Let us consider two input weak coherent states with amplitudes $|\eta| \ll 1$ and $|\eta'| \ll 1$ that are mutually incoherent, input into modes $i1$ and $i3$ of $\mathbf{B}_4$. Considering that the probability that the coherent state contains n photons is $p_n = |\eta|^{2n} \exp(-|\eta|^2) / n!$, using the fact that $p_1 = |\eta|^2 p_0$ and $p_2 = |\eta|^4 p_0 / 2$ and ignoring terms containing more than two photons, the input state can be written as

$$\rho_{\text{in}} = C(|2, 0\rangle\langle 2, 0| + \gamma^2 |0, 2\rangle\langle 0, 2| + 2\gamma |1, 1\rangle\langle 1, 1|), \quad (3)$$

where $\gamma = |\eta'|^2 / |\eta|^2$ is the ratio between mean photon numbers of the input pulses, the bras and kets refer to input modes $i1$, $i3$ and C is a normalization constant. Since state (3) is a convex sum of different Fock product states, to calculate the output modes of the 4CF-BS, we can transform each component using matrix (1), and sum the results. For the two-photon state $|2, 0\rangle = (\hat{a}_1^\dagger)^2 |0, 0\rangle / \sqrt{2}$, the field operators transform as

$$(\hat{a}_1^\dagger)^2 \rightarrow \frac{1}{4} (\hat{a}_0^\dagger + \hat{a}_1^\dagger - \hat{a}_2^\dagger - \hat{a}_3^\dagger)^2. \quad (4)$$

Here and below we omit the subscripts “in” and “out”. For the state $|0, 2\rangle = (\hat{a}_3^\dagger)^2 |0, 0\rangle / \sqrt{2}$, likewise we have

$$(\hat{a}_3^\dagger)^2 \rightarrow \frac{1}{4} (\hat{a}_0^\dagger - \hat{a}_1^\dagger - \hat{a}_2^\dagger + \hat{a}_3^\dagger)^2. \quad (5)$$

Finally, for the state $|1, 1\rangle = \hat{a}_1^\dagger \hat{a}_3^\dagger |0, 0\rangle$, we have the corresponding 4CF-BS transformation

$$\hat{a}_1^\dagger \hat{a}_3^\dagger \rightarrow \frac{1}{4} (\hat{a}_0^{\dagger 2} - \hat{a}_1^{\dagger 2} + \hat{a}_2^{\dagger 2} - \hat{a}_3^{\dagger 2} - 2\hat{a}_0^\dagger \hat{a}_2^\dagger + 2\hat{a}_1^\dagger \hat{a}_3^\dagger). \quad (6)$$

Here the absence of the terms $\hat{a}_0^\dagger \hat{a}_1^\dagger$, $\hat{a}_0^\dagger \hat{a}_3^\dagger$, $\hat{a}_1^\dagger \hat{a}_2^\dagger$ and $\hat{a}_2^\dagger \hat{a}_3^\dagger$ is due to two-photon interference. That is, when the two photons are indistinguishable, these terms vanish due to destructive interference.

Using the above transformations in the input state (3), The output state is given by a 10×10 density matrix, which can be written in the basis of possible two-photon states in the four output cores of the 4CF-BS. For example, state $|2000\rangle$ correspond to two photons in output mode 0, state $|0020\rangle$ to two photons in output mode 2, and state $|1010\rangle$ to one photon in mode 0 and one photon in mode 2. The complete density matrix written in this number basis is provided in the Methods section.

Fourth order interference

Two-photon interference between the WCS inputs can be observed by registering two-photon coincidence events at detectors connected to different output cores of the 4CF-BS. Let us consider equal intensity WCSs, so that $\gamma = 1$. The probabilities

P_{jk} to detect one photon in output core j and the other in output mode k ($j < k$) are $P_{01} = P_{03} = P_{12} = P_{23} = 1/16$, and $P_{02} = P_{13} = 3/16$, and correspond to the diagonal elements of the 6×6 block in the lower right of the density matrix in Eq. (16). To see that these values correspond to two-photon interference, we can consider the case of distinguishable photons, in which all of the 16 two-photon events are equally likely. Since there are two events that result in one photon in mode j and another in mode k (e.g., P_{jk} results from photon 1 going to j and photon 2 to k , or vice versa), these output probabilities would be equal to $1/8$. Thus, comparing the case of indistinguishable photons with that of distinguishable ones, events with $P_{jk} = 1/16 < 1/8$ correspond to interference minima (destructive interference), and those with $P_{jk} = 3/16 > 1/8$ to interference maxima (constructive interference). The predicted interference visibility for WCSs is given by $\mathcal{V} = (\max - \min) / (\max + \min) = 1/2$. This is the classical limit for fourth-order interference, compared to the unit visibility which is achievable in principle with a pair of input Fock states¹⁹.

Fourth order interference was explored in our setup by adjusting the polarization state of one of the WCSs by rotating one paddle of a fiber PC in the WCSs source (see Fig. 1), so that we can change continuously between distinguishable and indistinguishable photons. As a function of a distinguishability parameter θ related to the overlap between polarization states, the output probability is

$$P_{jk} = \frac{1}{8} (1 \pm \mathcal{V} f(\theta)), \quad (7)$$

where the plus (minus) sign refers to constructive (destructive) interference and $f(\theta)$ describes the overlap between the polarization states of the two input WCSs. In these measurements, the output cores of the 4CF-BS were connected directly to the single-photon detectors using a MUX device. Coincidence counts C_{jk} between detectors D_j and D_k were recorded as a function of θ and are shown in Fig. 3. When the polarization states are orthogonal, input photons from different pulses are distinguishable, and all combinations of coincidence counts have the same count rates. When the polarization states are parallel, fourth-order interference occurs, resulting in an increase in coincidence counts C_{02} and C_{13} ,

Fig. 3 Experimental results showing interference of mutually incoherent WCSs at a 4CF beam splitter. **a** Single count probabilities remain unchanged under variation of polarization states. **b** Coincidence count probabilities P_{jk} between detectors D_j and D_k showing interference of mutually incoherent WCSs at a 4CF beam splitter. Constructive and destructive fourth-order interference can be observed in the coincidence probabilities between pairs of detectors as a function of the orthogonality of the polarization states, while the single count probabilities (top) remain unchanged. Error bars (smaller than symbols) correspond to Poissonian count statistics. The solid gray curves represent Gaussian curve fits to experimental data, and are intended merely as a guide for the eye.

and a suppression of the coincidence counts at other detector pairs. Shown also are the single counts at each detector, which remain constant as the polarization is varied. This confirms that the increase or suppression in coincidence counts is due in fact to fourth-order interference. Using the average values at the interference maximum and minimum, in a time integrations of 10s, we obtained a visibility of $\mathcal{V} = 0.48 \pm 0.02$, in agreement with the predicted value of $1/2$.

As Fig. 3 shows, photon coalescence occurs for four combinations of coincidence outputs. For example, the suppression of coincidence counts at output cores o0 and o1 indicates that it is more likely to have both photons exiting the 4CF-BS together in one core or the other in a two-photon wavepacket. Moreover, the quantum state given by matrix (16) contains coherences between these wavepackets. For example, if we isolate output events $|20\rangle_{01}$, $|11\rangle_{01}$, and $|02\rangle_{01}$ by post-selection on detections in output fibers o0 and o1, we have the normalized density operator

$$\rho_{2A} = \frac{1}{8} \begin{pmatrix} 3 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 3 \end{pmatrix}, \quad (8)$$

showing a non-zero coherence term between the $|20\rangle_{01}$ and $|02\rangle_{01}$ elements.

To test this two-photon coherence, we connect the output fibers o0 and o1 of the MUX to a 2×2 fiber-based BS (BS1) (as shown in Fig. 1), and then connect the output fibers of BS1 to detectors D0 and D1. If we assume that the relative phase between the two fibers before the BS1 is ϕ , and look at coincidence detections at D0 and D1 after BS1, the detection probability is

$$P_{2A}(\phi) = \frac{3}{4} + \frac{1}{4} \cos(2\phi). \quad (9)$$

We thus see oscillations with a frequency ($=2$) that is double that of single-photon interference, corresponding to self-interference of a two-photon wavepacket with wavelength $\lambda/2$ ^{59,60}. The maximum expected visibility is given by $\mathcal{V}_2 = (3/4 - 1/4)/(3/4 + 1/4) = 1/2$, which is below the known visibility bound for higher-order interference from classical light⁶¹.

As described above, the phase was modulated using phase modulator PM0 controlled by the FPGA. We registered single counts and coincidence counts as the phase was varied linearly over time at a rate of 14 mHz, shown in Fig. 4 as a function of the phase. To compare the period of the two-photon interference with that of the single photon interference, we created a slight imbalance in the intensities of the WCSs, so that low contrast single-photon interference fringe could be observed. Fitting the curves to a sinusoidal function, we obtain the mean single-photon period $T = 1.00 \pm 0.03$, while the two-photon interference has period $T_2 = 0.52 \pm 0.01$, confirming the factor of two in the oscillation frequency and the self-interference of a two-photon wavepacket. The visibility of the two-photon interference was $\mathcal{V}_2 \approx 0.52 \pm 0.3$, in agreement with the maximum predicted value, showing high-quality fourth-order interference. The near-perfect visibility serves as a benchmark for the quality of the 4CF-BS, which has split ratios very close to the theoretical value of 0.25.

Post-selected bipartite state and quantum correlations

As an application of two-photon interference at the 4CF-BS device, we now consider the bipartite quantum correlations that can be produced. To do so, we divide the output modes into two bipartitions. We post-select on events where one photon exits in partition A, composed of output modes o0 and o1, while the other exits in partition B, composed of modes o2 and o3 (see Fig. 1). Let us switch from the multiple-rail notation to a qubit notation: $|1000\rangle \equiv |0\rangle_A$, $|0100\rangle \equiv |1\rangle_A$, $|0010\rangle \equiv |0\rangle_B$ and $|0001\rangle \equiv |1\rangle_B$. Following the results from section Input and Output state with

Fig. 4 Single and coincidence count probabilities at detectors D0 and D1 when the phase in the corresponding 2×2 interferometer is varied over time. **a** Single count probabilities. **b** The coincidence counts show the double frequency corresponding to self-interference of a two-photon wavepacket. Error bars correspond to Poissonian count statistics.

Post-Selection, and the total density operator given in (15), the post-selected bipartite density operator is

$$\rho(\gamma)_{AB} = \frac{1}{4} \begin{pmatrix} t & u & u & v \\ u & w & w & u \\ u & w & w & u \\ v & u & u & t \end{pmatrix}, \quad (10)$$

where $t = (1 + 4\gamma + \gamma^2)/(1 + \gamma)^2$, $u = (1 - \gamma)/(1 + \gamma)$, $v = (1 - 4\gamma + \gamma^2)/(1 + \gamma)^2$, $w = (1 + \gamma^2)/(1 + \gamma)^2$. Normalization requires $t + w = 2$.

The output state depends explicitly upon the ratio γ between mean photon numbers in the input WCSs. Without loss of generality, we consider the range $0 \leq \gamma \leq 1$, as $\gamma > 1$ can be handled by simply interchanging η and η' . The purity is given by $\text{tr}(\rho(\gamma)_{AB}^2) = (\gamma^4 + 4\gamma^2 + 1)/(1 + \gamma)^4$, and reaches a minimum value of $3/8$ when $\gamma = 1$ (equal intensity WCSs). For $\gamma = 1$, $u = 0$ and we have

$$\rho(1)_{AB} = \frac{1}{8} \begin{pmatrix} 3 & 0 & 0 & -1 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ -1 & 0 & 0 & 3 \end{pmatrix}. \quad (11)$$

This state is diagonal in the Bell-state basis

$$\rho(1)_{AB} = \frac{1}{4} (|\Phi^+\rangle\langle\Phi^+| + 2|\Phi^-\rangle\langle\Phi^-| + |\Psi^+\rangle\langle\Psi^+|), \quad (12)$$

and an example of a so-called “X” state, with maximally mixed marginal density matrices $\rho_A = \rho_B = \mathbb{I}/2$, which have been widely studied in the literature^{62–64}.

In the general case, the post-selected output state $\rho(\gamma)_{AB}$, though separable, displays quantum correlations. To analyze this, it is useful to put the output state in its Bloch representation

$$\rho(\gamma)_{AB} = \frac{1}{4} \left(I_2 \otimes I_2 + u \sigma_x \otimes I_2 + u I_2 \otimes \sigma_x + \sum_j C_{jj} \sigma_j \otimes \sigma_j \right), \quad (13)$$

where I_2 is the 2×2 identity matrix and σ_i are the usual Pauli matrices with $i = x, y, z$. The state is characterized by local Bloch vectors $\mathbf{r}_A = \mathbf{r}_B = (u, 0, 0)$ such that the local density operators for

Fig. 5 Theoretically obtained properties of the post-selected bipartite output state as a function of the intensity mismatch γ . **a** The single-qubit coherences decrease as the intensity mismatch γ goes from 0 to 1. **b** The geometric discord is maximum for $\gamma \approx 0.435$, when the two input WCSs have unequal intensities.

qubits A and B are $\rho_A = \rho_B = (I_2 + u\sigma_x)/2$, giving single qubit purity $(1 + u^2)/2$. The single qubit coherence, $C_A(\gamma) = 2|\langle\sigma_+\rangle| = |u|$, is plotted in Fig. 5a, and is maximum for $\gamma = 0$ and vanishes for $\gamma = 1$. The correlations between the qubits are given by the matrix $\mathbf{C}$ with real elements $C_{ij} = \langle\sigma_i \otimes \sigma_j\rangle$. Direct calculation shows that $\mathbf{C} = \text{diag}(w + v, w - v, t - w)/2$ is a diagonal matrix. To evaluate quantum correlations, we use the geometric discord $\mathcal{D}^{65}$, since it can be calculated without numerical optimization, and can be directly related to figures of merit of quantum tasks³⁸. It is defined as: $\mathcal{D} = (\|\mathbf{r}_A\|^2 + \|\mathbf{C}_{ij}\|^2 - k_{\max})/2$, where $k_{\max}$ is the largest eigenvalue of the matrix $\mathbf{K} = \mathbf{r}_A \mathbf{r}_A^T + \mathbf{C} \mathbf{C}^T$, and $\|\mathbf{V}\|^2 = \text{tr}(\mathbf{V}^T \mathbf{V})$. For the state (10), we find

$$\mathcal{D}(\gamma) = \frac{1}{2}(C_{xx} + C_{xx}^2 + C_{yy}^2 + C_{zz}^2 - \max[C_{xx} + C_{xx}^2, C_{yy}^2]), \quad (14)$$

where we choose a scaling parameter so that $\mathcal{D} = 1$ for a maximally-entangled Bell state. Using the definitions of t, w, v just after Eq. (10), we see that $C_{xx} = u^2$ and $C_{yy}^2 = C_{zz}^2$, the latter implies that the value of $k_{\max}$ is between two quantities that depend on γ . A plot of the discord (14) for the state (10) is shown in Fig. 5b, in which one can see a “kink” at $\gamma \approx 0.435$. It is at this point that there is a change in which argument is maximum in Eq. (14). For values of $\gamma \leq 0.435$ we have $C_{xx}^2 \geq C_{yy}^2$ so that the discord takes the form $\mathcal{D}(\gamma) = C_{yy}^2$. Instead, for $\gamma \geq 0.435$ we have $C_{xx}^2 \leq C_{yy}^2$ so the discord is $\mathcal{D}(\gamma) = (C_{xx}^2 + C_{xx} + C_{yy}^2)/2$. Geometric discord is a minimum distance from the state to the entire set of zero-discord states, i.e., it is the distance to the closest classical state⁶⁵. Thus, for different values of γ , the state can be closer to different classical states. Therefore, the inflection point is where the state is equidistant to two different states of zero discord. Similar non-analytical points, corresponding to sudden changes of discord, have been studied in the context of quantum dynamics^{46–49}, quantum phase transitions^{50,51}, and the quantum-classical transition^{44,45}. For a more comprehensive survey of this topic, see the recent reviews^{54,55}. We note that in our case the kink appears as a function of the initial condition γ , and not due to some type of dynamics of the system^{46–49}. Thus, one can control the type of state produced using the intensity ratio γ .

The maximum value of the discord reaches $\mathcal{D} \sim 0.178$ for WCS intensity ratio $\gamma \sim 0.435$, versus $\mathcal{D} = 0.125$ when the intensities

Fig. 6 Geometric discord $\mathcal{D}$ for the post-selected bipartite output state as a function of the local coherence $\mathcal{C}_S$ ($S = A, B$). The dashed gray upper curve is the theoretical prediction from (14). The red and blue points correspond to experimental points obtained by considering the local marginals of A or B , respectively. The black curve is a theoretical model taking into account dephasing and polarization mode mismatch. The experimental error bars were obtained by Monte Carlo simulation of 500 runs obeying the same Poissonian count statistics.

are equal. We note that for a two-qubit separable state the maximum attainable value is $\mathcal{D}_{\text{sep,max}} = 1/4^{66}$. The increase of quantum discord among photon states propagating in optical fibers has direct appeal for future quantum networks. Moreover, the multi-port fiber beam splitter considered here compatible with the space-division multiplexing technology, which is the current candidate to increase data transmission rates in telecommunications.

Experimental evaluation of quantum correlations

Our system consists of two parts A and B , where each one is a spatial qubit spanned by the states corresponding to two 4CF-BS output cores. Projective measurements are performed on two two-arm interferometers connected to these output cores. These projective measures are on local bases composed of eigenstates of the form $|\phi\rangle = (|0\rangle \pm \exp(i\phi)|1\rangle)/\sqrt{2}$. The measurements are configured by controlling the phase in each interferometer through PM0 and PM1 (see Fig. 1). In this case, for $\phi = 0$ we can project onto the σ_x eigenstates, while for $\phi = \pi/2$ we project onto the eigenstates of σ_y . The elements of the correlation matrix and Bloch vectors were estimated followed the procedure described in the Methods section.

Our measurement setup allows us to easily estimate local coherences simultaneously with joint projective measurements. Following the theoretical model (10), the local coherences are monotonic functions of γ . Thus, for convenience, we experimentally estimated the geometric discord $\mathcal{D}$ as a function of the local coherence $\mathcal{C}_S$ ($S = A, B$). Figure 6 shows our experimental results. The gray dashed line corresponds to our theoretical model, combining Fig. 5a, b. Red and blue points correspond to discord calculated using the local marginals for systems A and B , respectively. These do not overlap with the ideal theoretical model, which is due to experimental imperfections such as phase fluctuations and polarization mode mismatch. To test this, we consider the initial state (10) generated with imperfect mode overlap (given by $2\mathcal{V}$, where $\mathcal{V} = 0.48$ is the two-photon interference visibility from section Fourth order interference), and phase damping corresponding to the imperfect single photon interference visibility ($\sim 0.9 \pm 0.03$) that we register when only one WCS is input to the interferometer. The model, described in more detail in the Methods section, corresponds to the black solid line in Fig. 6, and is compatible with the experimental data. The experimental points show that the quantum correlations can be increased by manipulating the intensity mismatch of the input WCSs.

DISCUSSION

We presented results demonstrating fourth-order interference of mutually incoherent classical laser pulses in a multi-port beam splitter device, embedded within a multi-core optical fiber. We observed photon/photon interference with visibility compatible with the ideal theoretical value. In addition, the self-interference of the output two-photon wavepackets from photon coalescence was also tested, also giving high-quality visibility. As an application of fourth-order interference in this type of multi-port device, we studied post-selected quantum correlations in the form of quantum discord. We showed that the geometric discord can be maximized by controlling the intensity mismatch ratio between the input weak coherent laser pulses, without altering their Poissonian statistics.

It has been shown that discord can be an interesting resource for quantum information even when the bipartite state is separable^{36,38}. A particular Werner state is often cited^{54,55,65} as the separable Bell-diagonal state (maximally mixed marginals) with largest geometric discord of $\mathcal{D}_{\text{Werner}} = 1/9$. Curiously, we note the state produced by two-photon interference with equal intensity WCSs as described by our theoretical model is also separable, Bell-diagonal, and gives a larger value $\mathcal{D}(1) = 1/8$. We note that this state has been previously produced in Ref. ³³ in a similar setup using fourth-order interference. As shown here, using WCSs with unequal intensities leads to states that are no longer Bell-diagonal, resulting in an even greater geometric discord than the two examples mentioned above, achieving a maximum of $\mathcal{D}(0.435) = 0.178$. Moreover, the set of states $\rho(\gamma \leq 0.435)_{AB}$ (where incidentally the majority of our experimental data resides) are of the class described in Ref. ³⁸, for which the geometric discord coincides with the fidelity gain in RSP protocols, and where it was demonstrated that separable states with discord can outperform some entangled states in this task. We note that the figure of merit, the RSP-fidelity, is equal to the discord, so that our separable state in Eq. (10) can achieve even higher fidelities than in³⁸. Thus, these results will be useful in quantum communications systems. In particular, our source and optical system is based on off-the-shelf telecommunications components, and our multi-core fiber beam splitter is compatible with space-division multiplexing infrastructure.

METHODS

Full output density matrix

The full density matrix containing all two-photon components can be written in terms of the states (modes in vacuum state are omitted here) $|2\rangle_{01}|2\rangle_{11}, |2\rangle_{21}, |2\rangle_{31}, |1\rangle_{01}, |1\rangle_{02}, |1\rangle_{03}, |1\rangle_{12}, |1\rangle_{13}, |1\rangle_{23}$. Explicitly, it is

$$\frac{1}{16} \begin{pmatrix} \frac{a}{2} & \frac{c}{2} & \frac{a}{2} & \frac{c}{2} & \frac{b}{\sqrt{2}} & -\frac{a}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & \frac{b}{\sqrt{2}} \\ \frac{c}{2} & \frac{a}{2} & \frac{c}{2} & \frac{a}{2} & \frac{b}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{a}{\sqrt{2}} & \frac{b}{\sqrt{2}} \\ \frac{a}{2} & \frac{c}{2} & \frac{a}{2} & \frac{c}{2} & \frac{b}{\sqrt{2}} & -\frac{a}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & \frac{b}{\sqrt{2}} \\ \frac{c}{2} & \frac{a}{2} & \frac{c}{2} & \frac{a}{2} & \frac{b}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{a}{\sqrt{2}} & \frac{b}{\sqrt{2}} \\ \frac{b}{\sqrt{2}} & \frac{b}{\sqrt{2}} & \frac{b}{\sqrt{2}} & \frac{b}{\sqrt{2}} & d & -b & -d & -d & -b & d \\ -\frac{d}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & -\frac{a}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & -b & a & b & b & c & -b \\ -\frac{b}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -d & b & d & d & b & -d \\ -\frac{b}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -\frac{b}{\sqrt{2}} & -d & b & d & d & b & -d \\ -\frac{c}{\sqrt{2}} & -\frac{a}{\sqrt{2}} & -\frac{c}{\sqrt{2}} & -\frac{a}{\sqrt{2}} & -b & c & b & b & a & -b \\ \frac{b}{\sqrt{2}} & \frac{b}{\sqrt{2}} & \frac{b}{\sqrt{2}} & \frac{b}{\sqrt{2}} & d & -b & -d & -d & -b & d \end{pmatrix}, \quad (15)$$

where $a = (2 + 8\gamma + 2\gamma^2)/M$, $b = (2 - 2\gamma^2)/M$, $c = (2 - 8\gamma + 2\gamma^2)/M$, $d = (2 + 2\gamma^2)/M$ and $M = 1 + 2\gamma + \gamma^2$.

When the input WCSs have equal intensity so that $\gamma = 1$, we have

$$\frac{1}{16} \begin{pmatrix} \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & 0 & -\frac{3}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{3}{\sqrt{2}} & 0 \\ \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & 0 & -\frac{3}{\sqrt{2}} & 0 & 0 & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{2} & \frac{3}{2} & -\frac{1}{2} & \frac{3}{2} & 0 & \frac{1}{\sqrt{2}} & 0 & 0 & -\frac{3}{\sqrt{2}} & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -1 & 0 & 1 \\ -\frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 & 3 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 & 1 & 0 & -1 \\ \frac{1}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & \frac{1}{\sqrt{2}} & -\frac{3}{\sqrt{2}} & 0 & -1 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -1 & -1 & 0 & 1 \end{pmatrix}. \quad (16)$$

Experimental measurement of the discord

To estimate the discord it is necessary to determine the individual Bloch vectors $\mathbf{r}_S^T = (\langle \sigma_x \rangle, \langle \sigma_y \rangle, \langle \sigma_z \rangle)$ and the correlation matrix C with elements $C_{ij} = \langle \sigma_i \otimes \sigma_j \rangle$ for $S = A, B$ and $i, j = x, y, z$. We now briefly describe the measurement process that was used to estimate the geometric discord.

In a previous work¹⁰, we characterized the 4CF-BS using process tomography, obtaining a fidelity of 0.995 ± 0.003 with the ideal unitary matrix given in (1), corresponding to split ratios of 0.25. This can also be observed in the near perfect four-photon interference described in section Fourth order interference, which validates our theoretical model for the two-photon state output from the 4CF-BS. The main source of errors in our setup corresponds to changes in the polarization state and phase fluctuations within the output fibers in the two-arm interferometers used for measurement. Both of these errors leads to phase damping (dephasing), which does not affect diagonal elements of the single photon and two-photon density matrices, and can be modeled by considering phase damping channels, characterized by a parameter δ ⁶⁷. The secondary source of error is mode mismatch at the 4CF-BS. We modeled mode mismatch by considering an auxiliary degree of freedom of each input mode, and then traced them out to obtain the relevant quantum state. With this, the two-photon density matrix becomes

$$\rho(\gamma)_{AB} = \frac{1}{4} \begin{pmatrix} t' & u\delta & u\delta & v'\delta^2 \\ u\delta & w' & w''\delta^2 & u\delta \\ u\delta & w''\delta^2 & w' & u\delta^2 \\ v'\delta^2 & u\delta & u\delta & t' \end{pmatrix}, \quad (17)$$

where $t' = (1 + g_+ \gamma + \gamma^2)/(1 + \gamma)^2$, $u = (1 - \gamma)/(1 + \gamma)$, $v' = (1 - g_- \gamma + \gamma^2)/(1 + \gamma)^2$, $w' = (1 + g_- \gamma + \gamma^2)/(1 + \gamma)^2$, $w'' = (1 - g_- \gamma + \gamma^2)/(1 + \gamma)^2$, and $g_{\pm} = 2(1 \pm \mathcal{V})$ is due primarily to imperfect polarization mode matching, characterized by the fourth-order interference visibility ($\mathcal{V} = 0.48 \pm 0.02$) obtained in section Fourth order interference. Here δ is a real parameter that describes the phase damping ($0 \leq \delta \leq 1$) due to fluctuations in the two-arm interferometers (see below).

Measurements with the two-arm interferometers are made with respect to zero relative local phase. That is, we use the local interference maxima at detector D0 for A and D2 for B to define the phase origin. This, together with the near-perfect 25% split ratio of our the 4CF-BS allows us to make a few approximations that simplify the measurement process. First, we approximate $\langle \sigma_z \rangle_S = 0$, as predicted by the near-perfect split ratio and our theoretical model. Second, we characterized the phase damping due to rapid phase fluctuations and changes in the polarization state by analyzing the single-photon interference visibility with only a single WCS input into the setup. We obtained visibilities $\sim 0.9 \pm 0.3$ in this case (averaged over roughly one hour), thus giving $\delta \sim 0.9$. We assume that any additional loss in single photon coherence is due to bipartite correlations. With this, using the density operator (10), we find that $C_{zz} \approx (1 - \bar{C}^2)/2$, where $\bar{C}$ is the single photon coherence, averaged over systems A and B. In addition, we take $C_{jz} = C_{zj} \approx 0$ for $j = x, y$, as predicted by the density operator (10). Finally, the two-path interferometers and phase modulators described in section Experimental Description are used to measured directly $\langle \sigma_i \rangle_S$, giving the single photon coherence $\mathcal{C}_S$, and the correlators C_{ij} for $i, j = x, y$.

DATA AVAILABILITY

The datasets generated during and/or analyzed during the current study are available from the corresponding author on reasonable request.

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AUTHOR CONTRIBUTIONS

S.P.W. and G.L. developed the main concept. J.C., S.P.W. and G.L. performed the experiment and analyzed the results. M.N.A-S. and S.P.W. developed the theory and calculations. All authors contributed to writing/revising the paper and are accountable for all aspects of the work.

COMPETING INTERESTS

The authors declare no competing interests.

ADDITIONAL INFORMATION

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