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Nonlinearity-mediated digitization and amplification in electromechanical phonon-cavity systems

Tongqiao Miao ¹, Xin Zhou¹, Xuezhong Wu^{1,2,3}, Qingsong Li¹, Zhanqiang Hou¹, Xiaoping Hu¹, Zenghui Wang ^{4,5} & Dingbang Xiao ^{1,2,3}

Electromechanical phonon-cavity systems are man-made micro-structures, in which vibrational energy can be coherently transferred between different degrees of freedom. In such devices, the energy transfer direction and coupling strength can be parametrically controlled, offering great opportunities for both fundamental studies and practical applications such as phonon manipulation and sensing. However, to date the investigation of such systems has largely been limited to linear vibrations, while their responses in the nonlinear regime remain yet to be explored. Here, we demonstrate nonlinear operation of electromechanical phononcavity systems, and show that the resonant response differs drastically from that in the linear regime. We further demonstrate that by controlling the parametric pump, one can achieve nonlinearity-mediated digitization and amplification in the frequency domain, which can be exploited to build high-performance MEMS sensing devices based on phonon-cavity systems. Our findings offer intriguing opportunities for creating frequency-shift-based sensors and transducers.

¹ College of Intelligence Science, National University of Defense Technology, 410073 Changsha, China. ² The Laboratory of Science and Technology on Integrated Logistics Support, National University of Defense Technology, 410073 Changsha, China. ³ MEMS Engineering Center of Hunan, 410100 Changsha, China. ⁴ Institute of Fundamental and Frontier Sciences, University of Electronic Science and Technology of China, 610054 Chengdu, China. ⁵ State Key Laboratory of Electronic Thin Films and Integrated Devices, University of Electronic Science and Technology of China, 610054 Chengdu, China. ⁶⁴ email: zenghui.wang@uestc.edu.cn; dingbangxiao@nudt.edu.cn

onlinearity is ubiquitous in real-world physical systems. Among the many man-made structures, resonant microelectromechanical and nanoelectromechanical systems (MEMS/NEMS) offer great opportunities for designing, tuning, and exploiting nonlinearities and have enabled the exploration of nonlinear processes and nonlinearity-dictated device properties, such as energy dissipation¹, energy quantization², superposition of states³, dynamic range⁴, and enhancement of frequency stability⁵. As the degree of freedom (DOF) in the system increases (i.e., more than one resonant mode or vibrating part), nonlinearity gives rise to unique phenomena such as synchronization^{6–8}, chaos⁹, internal resonance¹⁰, and generation of frequency comb^{11–13}, by enabling and modulating coherent energy transfer between the different DOFs¹⁴.

Among multi-DOF systems, electromechanical phonon-cavity systems¹⁵ offer the unique capability of phonon manipulation by parametrically coupling a mechanical resonance to a phonon cavity^{16,17}, in many aspects analogous to a photon-cavity^{18–22}, which has enabled a plethora of exotic physical phenomena such as self-cooling^{23,24}, induced transparency^{25,26}, parametric amplification²⁷, and quantum squeezing^{28,29}. These exquisite functions are realized through a pump signal in the cavity's sideband, which parametrically controls the dynamical coupling and backaction between the resonant mode and the cavity^{30–32}.

One key feature that differentiates the phonon-cavity systems is that they operate entirely in the electromechanical domain, and thus are much more advantageous for implementation using monolithic solid-state devices. Further, the characteristic frequency of the phonon cavity is typically in the same frequency band as the resonant mode (unlike that of a photon-cavity, typically at orders-of-magnitude higher frequency), which greatly simplifies the signal transduction, making such systems promising for frequency-shift-based applications^{8,33–35}, such as sensing.

However, in such parametrically-coupled systems, despite the nonlinear nature of the coupling between the different DOFs¹⁴, research to date has been largely confined to linear operations, i.e., with limited vibration amplitude, which has plagued the exploration of nonlinear processes in these systems. Here we study nonlinear operation in a microelectromechanical phonon-cavity system and show that the response clearly differs from that in the linear regime. More importantly, leveraging the nonlinearity-mediated bi-stability, we demonstrate digitization and amplification modes of signal sensing enhancement in the frequency domain, which can be further tuned by the degree of nonlinearity in the vibration response.

Results

The electromechanical resonator used in this study (Fig. 1a) is designed to exhibit a torsional mode (at frequency $\omega_1 = 2\pi \times$ 6969.1 Hz) and a flexural mode resonance (at frequency $\omega_2 = 2\pi \times$ 16649.6 Hz). In this work, we specifically design our device to facilitate the observation of dynamical coupling in the nonlinear regime. This is achieved by the small capacitance gap, large capacitance area, and high-quality factor of the resonator (see "Methods"). By activating different combinations of the 12 electrodes (Fig. 1b), we can selectively excite these two distinct modes in both linear and nonlinear regimes (Fig. 1c, d), as well as injecting a parametric pump signal that controls the coupling between these two modes (see "Methods" and Fig. S2 for additional details). Enabled by the capacitive transduction scheme, both resonant modes exhibit clear Duffing softening response^{36,37} (see SI 2.2), with clear nonlinearitymediated bi-stability (vertical jump in amplitude). We denote the critical frequency where such bi-stability occurs as $\omega_{\rm b}$, which depends on the driving amplitude and the sweep direction (Fig. S3).

The electromechanical structure is designed to facilitate coherent energy transfer between the two resonant modes via

parametric pump-controlled intermodal coupling. Among the two resonant modes, the flexural mode at frequency ω_2 serves as the phonon cavity, which can coherently exchange energy with the resonant mode of interest at ω_1 if the pump frequency ω_p aligns with the red sideband of the cavity at $\omega_{red} = \omega_2 - \omega_1$, or the blue sideband at $\omega_{blue} = \omega_2 + \omega_1$. Specifically, when only the mechanical mode at ω_1 is excited, pumping the red sideband leads to phonon removal at ω_1 (Fig. 1e), while pumping the blue sideband engenders phonon creation (Fig. 1f). This parametrically excited-modulated vibrational energy transfer forms the operation basis of phonon-cavity systems^{15,16,38}.

Such process is experimentally manifested as the modulation in vibrational amplitude around ω_1 when ω_p approaches either of the cavity sidebands. We first examine the parametric coupling to the phonon cavity with the resonant mode at ω_1 excited in the linear regime (Fig. 2a–d), when scanning the excitation signal frequency ω_d around ω_1 and the pump frequency ω_p around ω_{red} or ω_{blue} . We find that when $\omega_p \approx \omega_{red}$ the resonant amplitude shows a clear dip (Fig. 2a, b), while near $\omega_p \approx \omega_{blue}$ it clearly peaks (Fig. 2c, d), showing excellent agreement with results in the literature^{15,38}. This verifies that by using appropriate pump settings, our phonon cavity system can operate in the strong coupling regime with efficient energy transfer; and such phonon removal/creation processes can be efficiently controlled using the parametric pump.

We then investigate the phonon-cavity system with the resonant mode of interest excited into nonlinear vibration. As the driving voltage increases, the nonlinear resonant response develops bi-stability, and exhibits different behavior in upward (low to high) and downward (high to low) frequency sweeps (Fig. S3). We now scan ω_d around ω_1 with both upward and downward sweeps, while also tuning ω_p around ω_{red} or ω_{blue} . Both nonlinear responses show clear contrast with the linear case. In upward sweeps (Fig. 2e–h), the effect from the parametric pump is minimized, and the resonant responses remain largely unchanged as ω_p scans across both ω_{red} and ω_{blue} , showing minimal energy exchange with the phonon-cavity.

In downward sweeps, however, the choice of the parametric pump frequency ω_p has very strong and distinct effects on the resonant response: when pumping in the vicinity of the red side band (Fig. 2i, j), the bi-stability frequency $\omega_{\rm b}$ notably increases (moving towards ω_1) as ω_p approaches ω_{red} , producing a dip-like feature in the 2D color plot; when pumping around the blue side band (Fig. 2k, l), in contrast, the bi-stability frequency $\omega_{\rm b}$ clearly decreases (moving away from ω_1) with a sudden jump at $\omega_p \approx \omega_{\text{blue}}$, resulting in a spike-like feature in the 2D color plot (Fig. 2l). Interestingly, the effect of the parametric pump is clearly in the frequency domain, manifested as the modulation of $\omega_{\rm b}$; in the displacement domain, however, for all ω_p settings the nonlinear resonance peak has been following the same frequency response curve (the curved slope of the shark-fin-like Duffing resonance peak) in the frequency down-sweeps, regardless the position of $\omega_{\rm b}$. This is clearly visible in the 3D plots (Fig. 2i, k). Such behavior is in clear contrast to the linear case (Fig. 2a–d), where the parametric pump acts on the resonance of interest mostly in the displacement domain while causing little effect in the frequency domain.

To understand the unique behavior of the nonlinear response in our phonon-cavity system, we examine its equations of motion by introducing Duffing nonlinearity into this 2-DOF system^{10,39}:

$$\ddot{x}_{1} + \omega_{1}^{2}x_{1} = \varepsilon [-2\gamma_{1}\dot{x}_{1} - 2(c_{11}x_{1} + c_{12}x_{2})\cos(\omega_{p}t) + \alpha_{1}x_{1}^{3} + F_{d}\cos(\omega_{d}t)]$$

$$\ddot{x}_{2} + \omega_{2}^{2}x_{2} = \varepsilon [-2\gamma_{2}\dot{x}_{2} - 2(c_{21}x_{1} + c_{22}x_{2})\cos(\omega_{p}t)]$$
(1)

here x_i and ω_i (i = 1, 2) are the displacement and natural frequency of the resonant modes, respectively. Specifically, i = 1



corresponds to the mode of interest (torsional mode, which we intentionally drive into nonlinear vibration, with α_1 being the Duffing coefficient, and $F_d \cos(\omega_d t)$ being the harmonic driving force), and i = 2 corresponds to the phonon cavity (flexural mode, which remains undriven). The energy dissipation rates are given by γ_i , and c_{ii} (i, j = 1, 2) give the intra-modal (i = j) and

Fig. 1 Electromechanical phonon-cavity system with nonlinear responses.
a A false-color SEM image of the MEMS resonator designed to exhibit two prominent resonant modes.
b Schematic illustration of the electrode layout underneath the resonator body and the simplified measurement circuit diagram.
c, d Resonant responses (during downward frequency sweeps) of the two modes, showing nonlinear vibration with clear bi-stability (the vertical jump in amplitude), with mode shapes illustrated above.
e Schematic diagram of the phonon cavity system operating with the parametric pump in the cavity red sideband, and f blue sideband.

intermodal $(i \neq j)$ parametric coupling coefficients, with ω_p being the frequency of parametric pump. All these terms are normalized using the effective mass of the oscillator. ε is introduced as a scaling parameter, based on the assumption that all terms multiplied by ε are small compared with the terms on the left side of the equations. This assumption is true for steady-state vibration of high *Q* resonators, where the energy stored in the resonator (though constantly transforming between kinetic and potential energies) is much greater than dissipation or transfer between modes. This mathematical treatment allows us to apply multiplescale approximation using the standard multidimensional Newton-Raphson algorithm⁴⁰ (see SI 2.1 and 2.3–2.4 for details), from which we are able to numerically solve the above equations in the vicinity of the cavity sidebands ($\omega_p \approx \omega_{red}$ and $\omega_p \approx \omega_{blue}$), with the results shown in Fig. 3.

The numerical results show very good agreement with the experimental data (for easy comparison, see Fig. 2 2D color plots) by reproducing all the key features. Specifically, in the downward sweep under nonlinear excitation, the calculation clearly produces in the 2D color plots the dip-like feature for ω_p near ω_{red} , and the spike-like feature for ω_p near ω_{blue} . Such unique behavior can be qualitatively understood, in a simplified picture, by considering the nonlinearity-induced amplitude bi-stability in the resonance mode, together with its parametric-pump-controlled coupling to the phonon-cavity: in downward frequency sweeps, the resonant response initially follows the upper branch among the two stable solutions from the softening Duffing equation, before jumping to the lower branch at $\omega_{\rm b}$. When $\omega_{\rm p}$ approaches $\omega_{\rm red}$, the parametric pump causes phonon to be removed from the resonant mode (Fig. 1e), with the remaining ones insufficient to sustain the highamplitude vibration on the upper branch, thus expedites the jumping to the lower branch ($\omega_{\rm b}$ closer to ω_1). In contrast, when $\omega_{\rm p} \approx \omega_{\rm blue}$, phonons are created in the resonant mode (Fig. 1f), which help in sustaining the vibration on the upper branch and delaying the jump ($\omega_{\rm b}$ further away from ω_1).

It is also interesting to analyze the observed phenomena by considering two different effects. First, with the sudden jump between its two stable branches, Duffing bi-stability produces a distinct relationship between the amplitude (A) and frequency (f) of a resonator, leading to an A-f relationship. Second, in phononcavity systems, the amplitude of a given mode is affected by the frequency difference between some other modes (such as the pump and the cavity modes), which gives rise to an *f*-A effect. By operating a phonon cavity device in a nonlinear regime, one can excite both effects, resulting in an f-A-f transduction, enabling signal amplification entirely within the frequency domain. While quantitative explanations for specific details (such as the asymmetry in $\omega_{\rm p}$ for the dip-like and spike-like features) require more in-depth analyses and evaluation of multiple expressions derived from Eq. 1 (extensive details are offered in "Methods" and SI 2.3–2.4), the precise reproduction of experimental results allows us to further explore the unique nonlinear responses of our phonon-cavity system using numerical simulations.



Fig. 2 Linear versus nonlinear responses of the ω_1 **resonant mode parametrically coupled to the phonon cavity. a** Resonant response of the ω_1 mode in the linear regime under different pump frequency ω_p , when pumping near the red sideband ω_{red} of the phonon-cavity. **b** 2D color plot of the same data as in **a**; same hereafter. The 2D color plots allow key features in the frequency domain (along both ω_d and ω_p) to be better visualized, and enable efficient comparison to the theory plots in Fig. **3. c**, **d** Resonant response in the linear regime when pumping near the blue sideband ω_{blue} of the phonon-cavity. **e**-**h** Nonlinear response of the ω_1 mode with an upward frequency sweep, under different pump frequency ω_p near the red sideband ω_{red} (**e**, **f**) and blue sideband ω_{blue} (**k**, **l**), respectively, showing clear modulation of the bi-stability frequency ω_b with distinct patterns. Experimental details are provided in "Methods" and SI 1.2, with additional data in SI 3.1.

We now show that such tuning of nonlinearity-mediated bistability, via parametric coupling to the phonon cavity, can be exploited to realize distinct functions in the frequency domain, offering unique capabilities for sensing and signal transduction. In resonant electromechanical transducers, typically, a change in an input physical quantity (e.g., pressure, acceleration, logic state) engenders a frequency shift, which is then measured or processed. In our devices, the phonon-cavity (resonant mode at ω_2) can function as the front-end transducer, which converts the input signal to a shift in ω_2 (green signals in Fig. 4a, e). When a pump signal is present at the cavity sideband (blue arrow), the input at ω_2 can be coupled to the response at ω_1 . Here in Fig. 4 we discuss such processes with ω_p near the blue sideband (similar results are obtained for the red sideband, which are detailed in SI 3.2).

In the linear regime (Fig. 4a), the parametric coupling results in the mode at ω_1 a change in its amplitude (purple signal), as the input change in ω_2 is equivalent to a shift of the operation point (horizontal slices in Fig. 4b), with resulting traces presented in Fig. 4c, d for clear comparison. This is similar to a mode-localized resonant sensor⁴¹, which converts a shift in the frequency domain to a change in the displacement domain.

In the nonlinear regime (Fig. 4e), however, the input shift in ω_2 is not transduced to the displacement domain; in contrast, it is amplified within the frequency domain, resulting in a much larger shift in ω_b (purple signal), as clearly demonstrated in both experiment (Fig. 2l) and theory (Fig. 3f). Here, the key is to leverage the unique spike-like feature by operating near its sharp edge (horizontal slices in Fig. 4f) in order to achieve augmented output in the frequency domain (Fig. 4g, h). Given the unique response of ω_b in this region, we show that two intriguing sensing-related functions can be realized: digitization and amplification (Fig. 4i, which zooms into the boxed area of Fig. 4f).

In the first scenario, we utilize the horizontal red-blue boundary in Fig. 4i at $\omega_p = \omega_{blue}$, indicated by the double-



Fig. 3 Numerical solutions to the phonon-cavity system operating in linear and nonlinear regimes. All results are presented in 2D color plots. **a**, **b** Calculated response of the ω_1 mode in linear regime with pumping frequency ω_p near the (**a**) red sideband ω_{red} and **b** blue sideband ω_{blue} of the phonon-cavity. **c**, **d** Nonlinear response of the ω_1 mode with an upward frequency sweep, with pump frequency ω_p near the red sideband ω_{red} (**c**) and blue sideband ω_{red} (**c**) and blue sideband ω_{blue} (**d**), respectively. **e**, **f** Nonlinear response of the ω_1 mode with a downward frequency sweep, with pump frequency ω_p near the red sideband ω_{red} (**e**) and blue sideband ω_{blue} (**f**), respectively, showing clear modulation of the bi-stability frequency ω_b with distinct patterns. Calculation and plot details are provided in "Methods" and SI 2.3-2.4, with additional data in SI 3.1.

headed arrow. A small shift in ω_2 (0.01 Hz, for example, as used for Fig. 4j) can cause the operating point to cross this sharp edge, resulting in a discrete and sizable change (several Hz, as found in Fig. 4j) in the output ω_b value. The magnitude of discretization (shift in output ω_b) can be continuously and smoothly controlled by adjusting the pump strength V_p and the degree of nonlinearity (varying V_d), as shown in Fig. 4j. Such digitization of the signal could be potentially useful for sensing applications, by determining whether the signal exceeds a given threshold.

In the other scenario, we operate along the sloped red-blue boundary (ω_p slightly below ω_{blue}), outlined by the small circles in Fig. 4i. In this region, a change in ω_2 from the input engenders a continuous and amplified output (change in ω_b), for which we define the gain A as:

$$A = \frac{\Delta \text{Output}}{\Delta \text{Input}} = \frac{\Delta \omega_{\text{b}}}{\Delta \omega_2}$$
(2)

By exploring the parameter space, we observe a number of behaviors for this amplification effect. First, the gain (slope of the data in Fig. 4k, l) can be controlled by both the pump strength V_p and the degree of nonlinearity (adjusting V_d). We find that deeper nonlinear vibration or stronger parametric coupling can produce larger amplification (see SI 3.3 for additional details), even exceeding one order of magnitude (red data series). Second, there is a trade-off between gain and linear dynamic range: a larger gain is associated with a smaller dynamic range. By carefully adjusting V_d and V_p , one can choose the optimal operating condition.

While frequency-sweep measurements are used to explore the exquisite nonlinear dynamics in such phonon-cavity systems, towards device applications a more efficient readout scheme is required. Here we demonstrate both amplification and digitization operations with fast readout using a phase-locked loop (PLL).

The PLL set-up is shown in Fig. S8. A DC bias $V_{\rm f}$ is applied (blue lines) on the electrodes to shift the frequency of mode 2 (ω_2). In practice, the shift of ω_2 can be caused by external factors

such as temperature, pressure, acceleration, and angular velocity. Here, in order to demonstrate the feasibility of the proposed functions, we use the negative stiffness due to electrostatic force to simulate the frequency shift of mode 2 due to such external factors. To minimize the effect from environment variables, the MEMS resonator is measured in a vacuum with a constant temperature of 30 °C. The frequency output of mode 1 (ω_b) is monitored (purple lines) using PLL in addition to the frequency sweep measurements (frequency response analysis (FRA)).

The experimental results for the phonon-cavity system at blue and red pumps are shown in Fig. 5 and Fig. S9, respectively. Here we use the blue pump case as an example. Following the numerical simulation results in Fig. 4, we first perform a frequency sweep experiment (Fig. 5a) and verify the phonon cavity response as predicted in Fig. 4i. We then use the DC voltage $V_{\rm f}$ to control ω_2 , and monitor the frequency output $\omega_{\rm b}$ using PLL. The PLL data (Fig. 5b) clearly captures all the $\omega_{\rm b}$ values in Fig. 5a, again demonstrating the nonlinear behavior predicted in Fig. 4i. We note that with PLL $\omega_{\rm b}$ can be measured much faster than in frequency sweeps, and the bandwidth of the measurement system can be optimized by setting appropriate PID parameters and reference phase of mode 1.

To verify the signal sensing enchancement functions, we operate the phonon cavity system in corresponding parameter spaces. Figure 5c, d present the PLL experimental results of the digitization and amplification operations respectively, and both show good agreement with the anticipated behavior shown in Fig. 4j, k/l. We note that given the different operating parameters (such as V_d and V_p) used in simulation and experiments, the results may not exactly agree quantitatively, but qualitatively all key features and behaviors are successfully reproduced experimentally.

Similarly, the experimental results for the red pump case (Fig. S9) agree well with the numerical predictions (Fig. S6). While standalone instruments are used to demonstrate the PLL measurement scheme here, in practice with known design



parameters such PLL component can be efficiently realized in circuit designs, thus facilitating a high level of integration of such frequency-shift-based sensors.

These unique nonlinearity-mediated frequency-shift functions can be exploited for constructing phonon-cavity-based sensors and transducers. It is worth noting that operating devices in the frequency domain have a number of advantages, as "time and frequency are the most accurately measurable of all physical quantities"⁴². Our results show that, by harnessing nonlinearity in phonon-cavity system, intriguing opportunities Fig. 4 Signal transduction and amplification in the 2-DOF phonon cavity system. a Schematic illustration of the phonon-cavity's response (purple signal) to a shift in the input signal (green) under linear vibration, when the pump signal (blue) is aligned with the cavity's blue sideband. **b** 2D color plot of the phonon-cavity's response to the input perturbation under linear vibration. **c**, **d** Line plots taken from different operation points in **b**, showing a negligible shift in resonance peak frequency ω_{peak} . **e** Schematic illustration of the phonon-cavity's response (purple signal) to a shift in the input signal (green) under nonlinear vibration, when the pump signal (blue) is aligned with the cavity's blue sideband. f 2D color plot of the phonon-cavity's response to the input perturbation under nonlinear vibration. g, h Line plots taken from different operation points in **f**, showing a significant shift in $\omega_{\rm b}$. \mathbf{i} A zoomed-in view of the boxed area in \mathbf{f} , illustrating the regions used for the two frequency-domain operations. **j** The output shift in $\omega_{\rm b}$ for a given input shift $\Delta \omega_2 = 0.01 \text{ Hz}$ as functions of V_{d_1} under different V_p values. **k** The output shift in $\omega_{\rm b}$ versus input shift in ω_2 for different $V_{\rm p}$ values. Larger $V_{\rm p}$ corresponds to stronger parametric coupling. I The output shift in $\omega_{\rm b}$ versus input shift in ω_2 for different V_d values. Larger V_d corresponds to deeper nonlinearity. Plot details are provided in the "Methods" section.

emerge for high-performance frequency-shift-based sensors and transducers⁴³⁻⁵⁰. Specifically, compared with MEMS sensors and transducers using single-DOF nonlinear resonators⁵¹ or multiple coupled resonators^{52,53}, the nonlinear phonon-cavity system in this work could potentially offer greater responsivity, additional control, smaller device footprint, and simplified system design, thus resulting in improved performance in MEMS-based sensing and signal transduction applications.

Methods

Device design. The electromechanical structure is designed to facilitate coherent energy transfer between the two resonant modes via parametric pump-controlled intermodal coupling. A three-segment beam is used to couple the two masses together, facilitating intermodal coupling structurally. A large mass area and the small capacitive gap in the device design make it easier for the resonator to exhibit nonlinearity, without requiring excessive amplitude. The design of multiple groups of electrodes under the resonator allows us to efficiently adjust the frequency of the different modes and implement dynamic coupling at the same time. The quality factor of the resonator can be effectively controlled by the vacuum packaging process.

Device fabrication. The die embedding the vacuum-sealed MEMS resonators is electrically packaged in a ceramic leadless chip carrier with a pressure of 0.1 Pa. The structure layer is 40-µm-Si and the electrode layer is 6-µm-Si. The gap between the structure layer and the electrode layer is 2µm. The lateral size of the resonator is about 3 mm × 3 mm. The insulating layer underneath the electrode layer is 2-µm-SiO₂. The device is operated under room temperature. The device structure, including its cross-section, and the fabrication process are detailed in Fig. S1.

Experimental set-up. A number of instruments are used in the measurement. The bias voltages are generated by a low noise voltage source (ITECH IT6233). The drive and pump signals are provided by a two-channel lock-in amplifier (Zurich Instruments HF2LI). The response motion of the resonator is detected by a capacitance–voltage (*C*/V) converting scheme based on a charge amplifier and measured by the lock-in amplifier. Detailed measurement setup and wiring diagram are available in SI 1.2.

Resonance measurements. The linear and nonlinear responses of both resonant modes are first characterized in the absence of the parametric pump. The measurement condition for the dataset shown in Fig. 1 is as follows: the driving voltage of mode 1 and mode 2 are $V_d = 0.2$, 0.4, 0.6, 0.8, 1.0 mV_{pk} and $V_d = 1$, 2, 3, 4, 5 mV_{pk} respectively. The results in (c) and (d) are both measured with a frequency downward sweep.

The coherent energy transfer in the phonon cavity is characterized by scanning the parametric pump frequency near either of the cavity sidebands. The measurement and plotting details for Fig. 2 are as follows:

In the linear regime when $\omega_{\rm p} \approx \omega_{\rm red}$ (a, b), the device is measured with $V_{\rm d} = 0.05 \, {\rm mV_{plo}} \, V_{\rm p} = 5.0 \, {\rm V_{plo}}$ and plotted using $\omega_1 = 2\pi \times 6886.0 \, {\rm Hz}$, $\omega_{\rm red} = 2\pi \times 9592.0 \, {\rm Hz}$. In the linear regime when $\omega_{\rm p} \approx \omega_{\rm blue}$ (c, d), the device is measured with $V_{\rm d} = 0.05 \, {\rm mV_{plo}} \, V_{\rm p} = 2.0 \, {\rm V_{plo}}$ and plotted using $\omega_1 = 2\pi \times 6958.0 \, {\rm Hz}$, $\omega_{\rm blue} = 2\pi \times 23,882.0 \, {\rm Hz}$.

Fig. 5 Experimental demonstration of signal transduction and amplification using the nonlinear phonon-cavity system at the blue pump. a The frequency response curves as functions of ω_2 in downward

frequency sweep measurements. **b** Fast measurement of $\omega_{\rm b}$ as functions of ω_2 . The white circles are the PLL measured data, and the solid colors on both sides offer visual cues to the eye. **c** The output shift in $\omega_{\rm b}$ as functions of ω_2 in the digitization region measured by PLL. **d** The output $\omega_{\rm b}$ as functions of ω_2 in the amplification region measured by PLL.

In the nonlinear regime upward sweep, when $\omega_{\rm p}\approx\omega_{\rm red}$ (e, f), the device is measured with $V_{\rm d}=1.5~{\rm mV_{pk}}, V_{\rm p}=5.0~{\rm V_{pk}}$, and plotted using $\omega_1=2\pi\times 6886.0~{\rm Hz},$ $\omega_{\rm red}=2\pi\times 9592.5~{\rm Hz}$. When $\omega_{\rm p}\approx\omega_{\rm blue}$ (h, g), the device is measured with $V_{\rm d}=1.0~{\rm mV_{pk}}, V_{\rm p}=1.5~{\rm V_{pk}}$, and plotted using $\omega_1=2\pi\times 6958.0~{\rm Hz}, ~\omega_{\rm blue}=2\pi\times 23,575.0~{\rm Hz}$.

In the nonlinear regime downward sweep, when $\omega_{\rm p} \approx \omega_{\rm red}$ (i, j), the device is measured with $V_{\rm d} = 1.5~{\rm mV_{pk}},~V_{\rm p} = 5.0~{\rm V_{pk}}.$ and plotted using $\omega_1 = 2\pi \times 6886.0~{\rm Hz},~\omega_{\rm red} = 2\pi \times 9592.5~{\rm Hz}.$ When $\omega_{\rm p} \approx \omega_{\rm blue}$ (k, l), the device is measured with $V_{\rm d} = 1.0~{\rm mV_{pk}},~V_{\rm p} = 1.5~{\rm V_{pk}}.$ and plotted using $\omega_1 = 2\pi \times 6958.0~{\rm Hz},~\omega_{\rm blue} = 2\pi \times 23,575.0~{\rm Hz}.$

In all measurements in Fig. 2, the frequency scanning steps are the same: $\Delta \omega_d = 2\pi \times 0.05$ Hz, $\Delta \omega_p = 2\pi \times 0.2$ Hz. Note that the plot center frequencies are slightly adjusted for each panel for easy comparison across the different measurement conditions, in order to best illustrate the key findings. Similarly, the driving and pumping strength for the data presented in each panel are also chosen to best illustrate the key findings.

Numerical analysis. The numerical codes are constructed using C language. The numerical settings for producing the data in Fig. 3 are as follows:

In the linear regime when $\omega_{\rm p} \approx \omega_{\rm red}$ (a), the device response is calculated with $V_{\rm d} = 0.05 \ {\rm mV}_{\rm pk}, V_{\rm p} = 5.0 \ {\rm V}_{\rm pk}$, and plotted using $\omega_1 = 2\pi \times 6886.0 \ {\rm Hz}$, $\omega_{\rm red} = 2\pi \times 9592.0 \ {\rm Hz}$. In the linear regime when $\omega_{\rm p} \approx \omega_{\rm blue}$ (b), the device response is calculated with $V_{\rm d} = 0.05 \ {\rm mV}_{\rm pk}, V_{\rm p} = 2.0 \ {\rm V}_{\rm pk}$, and plotted using $\omega_1 = 2\pi \times 6958.0 \ {\rm Hz}$, $\omega_{\rm blue} = 2\pi \times 23,582.0 \ {\rm Hz}$.

In the nonlinear regime upward sweep, when $\omega_{\rm p} \approx \omega_{\rm red}$ (c), the device response is calculated with $V_{\rm d} = 1.5 \, {\rm mV}_{\rm pk}$, $V_{\rm p} = 5.0 \, {\rm V}_{\rm pk}$, and plotted using $\omega_1 = 2\pi \times 6886.0 \, {\rm Hz}$, $\omega_{\rm red} = 2\pi \times 9592.5 \, {\rm Hz}$. When $\omega_{\rm p} \approx \omega_{\rm blue}$ (d), the device response is calculated with $V_{\rm d} = 1.0 \, {\rm mV}_{\rm pk}$, $V_p = 2.0 \, {\rm V}_{\rm pk}$, and plotted using $\omega_1 = 2\pi \times 6958.0 \, {\rm Hz}$, $\omega_{\rm blue} = 2\pi \times 23,575.0 \, {\rm Hz}$.

In the nonlinear regime downward sweep, when $\omega_{\rm p} \approx \omega_{\rm red}$ (e), the device response is calculated with $V_{\rm d} = 1.5~{\rm mV_{pk}}, V_{\rm p} = 5.0~{\rm V_{pk}}$, and plotted using $\omega_1 = 2\pi \times 6886.0~{\rm Hz}, ~\omega_{\rm red} = 2\pi \times 9592.5~{\rm Hz}$. When $\omega_{\rm p} \approx \omega_{\rm blue}$ (f), the device response is calculated with $V_{\rm d} = 1.0~{\rm mV_{pk}}, V_{\rm p} = 2.0~{\rm V_{pk}}$, and plotted using $\omega_1 = 2\pi \times 6958.0~{\rm Hz}, ~\omega_{\rm blue} = 2\pi \times 23,575.0~{\rm Hz}$.

In all calculations presented in Fig. 3, the frequency scanning steps are the same: $\Delta \omega_d = 2\pi \times 0.05$ Hz, $\Delta \omega_p = 2\pi \times 0.2$ Hz. Note that the plot center frequencies are slightly adjusted for each panel for easy comparison across the different measurement conditions, in order to best illustrate the key findings. Similarly, the driving and pumping strength for the data presented in each panel are also chosen to best illustrate the key findings.

Experimental demonstration of digitization and amplification functions using PLL. The numerical settings for producing the data in Fig. 4 are as follows: In the linear operation (b), $V_d = 0.05 \text{ mV}_{pk}$, $V_p = 2.0 \text{ V}_{pk}$, $\omega_1 = 2\pi \times 6958.0 \text{ Hz}$, and $\omega_p = 2\pi \times 23,582.0 \text{ Hz}$. The frequency steps are $\Delta \omega_d = 2\pi \times 0.05 \text{ Hz}$, and $\Delta \omega_2 = 2\pi \times 0.2 \text{ Hz}$. The line plots (c, d) are taken from data in (b) with $\Delta \omega_2 = 0$ (c) and $\Delta \omega_2 = 2\pi \times 1.0 \text{ Hz}$ (d).

In the nonlinear operation with downward sweep (f), $V_d = 1.0 \text{ mV}_{pk}$,

 $V_{\rm p} = 2.0 \, V_{\rm pks} \, \omega_1 = 2\pi \times 6958.0 \, \text{Hz}$, and $\omega_{\rm p} = 2\pi \times 23,575.0 \, \text{Hz}$. The frequency steps are $\Delta \omega_{\rm d} = 2\pi \times 0.05 \, \text{Hz}$, and $\Delta \omega_2 = 2\pi \times 0.2 \, \text{Hz}$. The line plots (g, h) taken from data in (d) when $\Delta \omega_2 = 0$ (g) and $\Delta \omega_2 = 2\pi \times 1.0 \, \text{Hz}$ (h).

In the zoom-in plot (i), $V_d = 3.0 \text{ mV}_{pk}$, $V_p = 0.6 \text{ V}_{pk}$, $\omega_1 = 2\pi \times 6929.52 \text{ Hz}$, and $\omega_p = 2\pi \times 23,552.51 \text{ Hz}$. The frequency steps are $\Delta \omega_d = 2\pi \times 0.01 \text{ Hz}$, $\Delta \omega_2 = 2\pi \times 0.01 \text{ Hz}$.

The data of digitization in (j) are calculated for $V_p = 1.0, 1.5, 2.0 V_{pk}$ when V_d is scanned from 2.0 mV_{pk} to 3.0 mV_{pk}. The step of V_d is 0.1 mV_{pk}.

The data of amplification in (k) are calculated for V_d is 3.0 mV_{pk} and $V_p = 0.35$, 0.45, 0.50 V_{pk}. The step of $\Delta \omega_2$ is $2\pi \times 0.01$ Hz.

The data of amplification in (l) are calculated for V_p is 0.50 V_{pk} and $V_d = 2.7$, 2.9, 3.0 mV_{pk}. The step of $\Delta \omega_2$ is $2\pi \times 0.01$ Hz.

Experimental demonstration of digitization and amplification functions using PLL. The measurement settings for producing Fig. 5 are as follows:

In the nonlinear operation at blue pump with downward sweep (a),

 $V_{\rm d} = 1.25 \text{ mV}_{\rm pk}$, $V_{\rm p} = 1.5 \text{ V}_{\rm pk}$, $\omega_1 = 2\pi \times 6538.0 \text{ Hz}$, and $\omega_{\rm p} = 2\pi \times 23,048.0 \text{ Hz}$. The steps are $\Delta \omega_{\rm d} = 2\pi \times 0.02 \text{ Hz}$, and $\Delta V_{\rm f} = 0.05 \text{ V}_{\rm pk}$.

When using PLL to test the phonon-cavity system at blue pump (b), V = 1.25 mV = 1.5 V $= and w = 2\pi \times 23.048$ 0 Hz. V is scanned from

 $V_d = 1.25 \text{ mV}_{pk}$, $V_p = 1.5 \text{ V}_{pk}$, and $\omega_p = 2\pi \times 23,048.0 \text{ Hz}$. V_f is scanned from 0 to 1 V_{pk} and the step of V_f is 0.05 V_{pk} . The experiment of digitization at blue pump in (c), $V_p = 1.5 \text{ V}_{pk}$ when V_d is

scanned from 0.75 mV_{pk} to 1.25 mV_{pk}. The step of V_d is 0.05 mV_{pk}. The experiment of amplification at blue pump in (d), $V_d = 1.25$ mV_{pk},

 $V_p = 1.5 V_{pk}$, and $\omega_p = 2\pi \times 23,048.0$ Hz. V_f is scanned from 0.445 to 0.495 $_{pk}$ and the step of V_f is 5 mV_{pk}.

Frequency noise of the phonon-cavity system. The experiment result of frequency noise is shown in Fig. S10. Here, we measure the frequency noise of input signal ω_2 and output signal ω_b under different operation conditions: when there is no parametric pump when there is a blue pump and when there is a red pump applied to the phonon-cavity system.

The time-domain data is shown in Fig. S10a, and the calculated Allan deviation σ is shown in Fig. S10b. According to the fitting results, the frequency random walk and instability are calculated and shown in Table S2. The results show that the signal transduction through the phonon-cavity system does not increase the frequency noise. Interestingly, it shows that when operating under a blue pump, the noise performance can even improve.

Data availability

The datasets generated during and/or analyzed during the current study are available from the corresponding authors on reasonable request.

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Author contributions

T.M. conceived the study under the guidance of D.X. and Z.W. Y.K., D.X., and X.W. designed and fabricated the devices. D.X. and T.M. developed the measurement circuitry with the assistance of Z.H. and Q.L. T.M. and X.Z. conducted the measurements and processed the data. T.M. developed the theoretical framework under the supervision of X.H. and X.W. T.M., D.X., and Z.W. analyzed the data and prepared the manuscript, with input from all other authors. The project was planned by X.W.

Competing interests

The authors declare no competing interests.

Additional information

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Correspondence and requests for materials should be addressed to Zenghui Wang or Dingbang Xiao.

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