# Conformational properties of a polymer tethered to an interacting flat surface 

Hong $\mathrm{Li}^{1}$, Chang-Ji Qian ${ }^{1}$, Li-Zhen Sun ${ }^{2}$ and Meng-Bo Luo ${ }^{2}$<br>The conformational properties of a lattice self-avoiding polymer chain tethered to an interacting and impenetrable flat surface were simulated using a dynamic Monte Carlo method. The results show that the conformational size reaches a minimum at the critical adsorption point (CAP) and that the scaling behavior of the polymer at the CAP is the same as that in the bulk solution. The results provide a new method to determine the CAP of polymer chains.

Polymer Journal (2010) 42, 383-385; doi:10.1038/pj.2010.9; published online 17 March 2010
Keywords: critical adsorption point; Monte Carlo simulation; self-avoiding chain

## INTRODUCTION

The adsorption of polymer chains on surfaces by means of physical or chemical interactions is an important subject in polymer and biological sciences. A polymer chain may adsorb or desorb, depending on interactions with the surface. This phenomenon is relevant to many technological applications such as in polymer compatibilizers, colloid stabilizers and polymeric surfactants. ${ }^{1-3}$ In many biological systems, ligands are attached to a surface through flexible tether chains. ${ }^{4,5}$ The conformation of the attached tether chains will affect the binding of the ligand to a receptor and will thus influence the whole biological process. ${ }^{6}$ The adsorption phenomenon is also a part of many physical systems; for example, polymer chains grafted to colloid particles and block copolymers at liquid-air interfaces. ${ }^{7}$ The adsorption of polymers has attracted a large number of theoretical and experimental studies. ${ }^{7-12}$ The growing interest in polymers interacting with substrates warrants a thorough understanding of the static and dynamic properties of the tethered chain.

A mathematical model often used for studying the adsorption of tethered polymers on surfaces is the self-avoiding walk (SAW) chain in a three-dimensional (3D) simple cubic lattice that interacts with a flat surface and is restricted to lie on one side of the surface. ${ }^{13-17}$ Every walk step that contacts the surface is assigned an interaction energy $\varepsilon$ (in units of $k_{\mathrm{B}} T$, where $k_{\mathrm{B}}$ is the Boltzmann constant and $T$ is the temperature). It is well accepted that a single chain adsorbed on an attractive surface exhibits a phase transition from a desorbed state to an adsorbed state when the adsorption strength increases beyond a critical value. The CAP $\varepsilon_{c}$ was estimated to be about -0.29 for polymers on the simple cubic lattice. ${ }^{9-12,16,18,19}$

In this work, we studied how the conformational properties of the chain change from a desorbed state for $\varepsilon>\varepsilon_{c}$ to an adsorbed state for $\varepsilon<\varepsilon_{\mathrm{c}}$. We found that both the mean square end-to-end distance $\left\langle R^{2}\right\rangle$
and the mean square radius of gyration $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle$ have a minimum near $\varepsilon_{\mathrm{c}}$. The conformation size $\left\langle R^{2}\right\rangle$ and $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle$ increase rapidly when $\varepsilon<\varepsilon_{\mathrm{c}}$. The scaling law $\left\langle R^{2}\right\rangle \propto n^{2 v_{R}}$ holds for every interaction energy $\varepsilon$ we investigated. The exponent $v_{R}$ is dependent on $\varepsilon$. We found a minimum $v_{R} \approx 0.6$, equal to that in bulk solvent, at a CAP $\varepsilon_{\mathrm{c}} \approx-0.29$. This result provides a new method to determine $\varepsilon_{\mathrm{c}}$ in experiments and simulations.

## MODEL AND CALCULATION METHOD

Simulations were performed on a simple cubic lattice. The flat surface was assumed to be infinitely large, impenetrable to the polymer chain, and located at $z=0$, with direction $z$ being perpendicular to the flat surface. The lattice layers above the surface were numbered $z=1,2, \ldots$. A SAW polymer chain comprises $n$ identical, consecutively linked monomers with a bond length of one lattice unit. The first monomer of the chain was fixed at position $(0,0,1)$ and was considered to be adsorbed to the impenetrable surface. We used a large simulation box to eliminate the size effect. The simulation sizes in the $x$ and $y$ directions were $L_{x}=L_{y}=n$, and periodic boundary conditions were used in the $x$ and $y$ directions. The length in the $z$ direction was $L_{z}=n$. Therefore, there was no size effect in the simulations.

At the beginning of the simulation, we grew a tethered SAW chain using the Rosenbluth-Rosenbluth method ${ }^{20}$ with the first monomer fixed at $(0,0,1)$. After creating a whole chain, we let it undergo a series of Brownian motions resulting from random collisions between chain beads and solvent molecules. The movement of the polymer chain was restricted to the upper half-space with $z>0$. The Brownian movement was achieved by a dynamic Monte Carlo technique that has been described in detail previously. ${ }^{21}$ In the dynamic model, a monomer is chosen randomly and one of the three elementary motions-the end-monomer, normal-monomer or $90^{\circ}$ crankshaft motion-is attempted. This trial move is accepted if the following three conditions are satisfied: (1) self-avoidance is obeyed, (2) the new site locates at $z>0$, and (3) the Boltzmann factor $\exp (-\Delta E)$ is greater than a random number uniformly distributed over the interval $(0,1)$, where $\Delta E$ (in unit of $k_{\mathrm{B}} T$ ) is the energy shift due to the

[^0]change in the monomer site. In one Monte Carlo step, $n$ monomers in the chain attempt to move once, on average.

The simulation was started from $\varepsilon=0$ with a desorbed state to $\varepsilon=-0.6$ with a highly adsorbed state by slowly decreasing $\varepsilon$. At each $\varepsilon$, the chain was relaxed for a time period $\tau=2.5 n^{2.13}$, which was found to be long enough to achieve an equilibrium state. ${ }^{22}$ In the present calculation, 10000 independent runs with different starting configurations and random number series were performed. All quantities were thus averaged over these 10000 runs.

## RESULTS AND DISCUSSION

We calculated the mean-square end-to-end distance $\left\langle R^{2}\right\rangle$ at different interactions. Here, the end-to-end distance was $R=\left|\vec{r}_{n}-\vec{r}_{1}\right|$, where $\vec{r}_{1}$ and $\vec{r}_{n}$ are the position vectors of the first and last monomers. The dependence of $\left\langle R^{2}\right\rangle / n$ on the interaction $\varepsilon$ is plotted in Figure 1 for a variety of chain lengths. We found that each curve had a minimum. Interestingly, the minimum was located near the $\mathrm{CAP} \varepsilon_{\mathrm{c}}=-0.29$ of the model system. ${ }^{12,16}$ Above $\varepsilon_{\mathrm{c}}\left\langle R^{2}\right\rangle$ increased slowly with $\varepsilon$, while below $\varepsilon_{\mathrm{c}}$ it grew rapidly with the decrease in interaction. The minimum $\left\langle R^{2}\right\rangle$ became more obvious with increasing chain length. This result indicates that one can locate $\varepsilon_{c}$ from the mean-square end-to-end distance of a single long chain length, which is more convenient than the scaling method in which different chain lengths are needed. The end-to-end distance of a long chain length can be measured from the angular distribution of the intensity of the scattered light, which can be studied easily by experimentation. ${ }^{23}$

The mean-square radius of gyration $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle$ was also computed. The radius of gyration is defined as $R_{\mathrm{G}}^{2}=\frac{1}{n} \sum_{i=1}^{n}\left(\vec{r}_{i}-\vec{r}_{c}\right)^{2}$, where $\vec{r}_{i}$ and $\vec{r}_{c}$ are the position vectors of the $i$ th monomer and the mass center of the chain, respectively. The dependence of $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle$ on interactions is plotted for chain lengths $n=50,100,200$ and 400 in Figure 2. We found that $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle$ behaved similarly to $\left\langle R^{2}\right\rangle$. The minimum of $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle$ was also near the CAP $\varepsilon_{\mathrm{c}}=-0.29$. However, the minimum of $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle$ was not as obvious as that of $\left\langle R^{2}\right\rangle$. Thus, we focus mainly on $\left\langle R^{2}\right\rangle$ in the following analysis.
It is well known that the conformation size scales with the chain length. For example, $\left\langle R^{2}\right\rangle$ is in a scaling law with the chain length $n$ according to the equation

$$
\begin{equation*}
\left\langle R^{2}\right\rangle \propto n^{2 v_{R}} \tag{1}
\end{equation*}
$$

For a free SAW chain in a simple cubic lattice used as a model of the chain in bulk solution, the exponent $v_{R}$ is about 0.6 . For a tethered polymer chain, the expected asymptotic behavior above the CAP ( $\varepsilon>\varepsilon_{c}$ ) is the same as for polymers in bulk. At the CAP, the scaling behavior is also valid, even for finite chain lengths. ${ }^{16}$ For a small chain length $n$ and at $\varepsilon$ above the CAP, the perpendicular component $\left\langle R^{2}\right\rangle_{z}$ is influenced by the surface and the correction to scaling becomes important. ${ }^{16}$ At $\varepsilon=0,\left\langle R^{2}\right\rangle_{z}$ can be expressed as

$$
\begin{equation*}
\left\langle R^{2}\right\rangle_{z}=A_{z} n^{2 v_{R}}\left(1+c / n^{\psi}\right) \tag{2}
\end{equation*}
$$

with $c<0$, found by assuming $v_{R}$ is the bulk value. ${ }^{16}$ The scaling behavior of the parallel component $\left\langle R^{2}\right\rangle_{x y}$ is not affected by the surface.

For a finite chain length, we have examined whether the scaling law in Equation (1) still exists for a tethered chain on an interacting surface. In this way, we assumed the exponent $v_{R}$ was not always the bulk value. The results are presented in the inset of Figure 3; such a scaling relation always exists at interactions above and below $\varepsilon_{\mathrm{c}}$. The dependence of $v_{R}$ on the interaction $\varepsilon$ is plotted in Figure 3. We found that $v_{R}$ also has a minimum near $\varepsilon_{c}$. The minimum value is very close to 0.6 , a typical value for a SAW chain in bulk solution. This result is consistent with the idea that the chain at the CAP behaves in the same


Figure 1 Mean-square end-to-end distance $\left\langle R^{2}\right\rangle$ vs interaction $\varepsilon$ for chain length $n=50,100,200$ and 400.


Figure 2 Mean-square radius of gyration $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle$ vs interaction $\varepsilon$ for chain length $n=50,100,200$ and 400.
manner as a chain in bulk solution. ${ }^{14}$ This provides an alternative method to locate $\varepsilon_{c}$.

The dependence of the mean-square radius of gyration $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle$ on the chain length $n$ was also found to obey the same scaling law: $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle \propto n^{2 v_{S}}$. We found the behavior of the exponent $v_{S}$ to be similar to that of $v_{R}$. We also found that $v_{S}$ has a minimum at about $\varepsilon_{\mathrm{c}}=-0.29$.

In this model system, a chain exhibits a transition from a desorbed state at $\varepsilon>\varepsilon_{\mathrm{c}}$ to an adsorbed state at $\varepsilon<\varepsilon_{\mathrm{c}}$. The transition can clearly be seen from the behavior of the parallel and perpendicular components of $\left\langle R^{2}\right\rangle,\left\langle R^{2}\right\rangle_{x y}$ and $\left\langle R^{2}\right\rangle_{z}$, respectively. Figure 4 shows the dependence of $\left\langle R^{2}\right\rangle_{x y}$ and $\left\langle R^{2}\right\rangle_{z}$ on the interaction $\varepsilon$. At $\varepsilon=0$, $\left\langle R^{2}\right\rangle_{z}>\frac{1}{2}\left\langle R^{2}\right\rangle_{x y}$, indicating that the $z$ component is enlarged by the excluded volume effect of the surface. The decrease of $\left\langle R^{2}\right\rangle_{z}$ and the increase of $\left\langle R^{2}\right\rangle_{x y}$ at $\varepsilon<0$ show that the chain tends to approach the surface because of the attraction of the surface. $\left\langle R^{2}\right\rangle_{z}$ tends to zero for $\varepsilon<\varepsilon_{c}$, which means that the chain is very close to the surface and is almost two-dimensional. The value of $v_{R}$ at $\varepsilon=-0.6$ is quite close to 0.75 , the value of a two-dimensional SAW chain. ${ }^{24}$ Our results reveal a conformational transition of the polymer from a threedimensional state to a two-dimensional state at $\varepsilon_{\mathrm{c}}$.

We found that a rapid decrease in $\left\langle R^{2}\right\rangle_{z}$ occurs at $\varepsilon_{\mathrm{c}}=-0.29$, as shown in Figure 4. It is clear that the decrease in $\left\langle R^{2}\right\rangle_{z}$ at $\varepsilon_{\mathrm{c}}=-0.29$ was larger than the increase in $\left\langle R^{2}\right\rangle_{x y} .\left\langle R^{2}\right\rangle_{x y}$, however, increased very fast below $\varepsilon_{\mathrm{c}}$. Therefore, one can find a minimum conformation size near $\varepsilon_{\mathrm{c}}$, as shown in Figures 1 and 2.


Figure 3 Plot of the exponent $v_{R}$ vs the interaction $\varepsilon$. The inset presents the log-log dependence of the mean-square end-to-end distance $\left\langle R^{2}\right\rangle$ on the chain length $n$ at different interactions: $\varepsilon=0,-0.3,-0.4$ and -0.5 . The solid lines in the inset are linear fits.


Figure 4 Plot of $\left\langle R^{2}\right\rangle_{x y}$ and $\left\langle R^{2}\right\rangle_{z}$ against the interaction $\varepsilon$ for chain length $n=400$.

## CONCLUSION

Dynamic Monte Carlo simulations were carried out for a lattice selfavoiding polymer chain with one end tethered to an interacting, flat surface. The conformational properties of the tethered chain were dependent on the polymer-surface interaction $\varepsilon$. In the vicinity of the CAP $\varepsilon_{\mathrm{c}}=-0.29$, the conformational size $\left\langle R^{2}\right\rangle$ and $\left\langle R_{\mathrm{G}}{ }^{2}\right\rangle$ were minimized. The scaling exponent $v_{R}$ also reached a minimum near $\varepsilon_{c}$. The minimum $v_{R}$ was about 0.6 , the same as that of a chain in bulk solution. The chain transitioned from a three-dimensional extended
state to a two-dimensional adsorbed state at $\varepsilon_{\mathrm{c}}$. Our results provide an alternative way to determine the CAP.

## ACKNOWLEDGEMENTS

This work was supported by the National Natural Science Foundation of China (Grant No. 20771092) and the Natural Science Foundation of Zhejiang Province, China (Grant No. Y607142).

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[^0]:    ${ }^{1}$ Department of Physics, Wenzhou University, Wenzhou, China and ${ }^{2}$ Department of Physics, Zhejiang University, Hangzhou, China
    Correspondence: Professor C-J Qian, Department of Physics, Wenzhou University, Wenzhou 325035, China.
    E-mail: kjqian838@vip.sina.com
    Received 23 November 2009; revised 18 January 2010; accepted 20 January 2010; published online 17 March 2010

