Dimensional Properties of Polypeptides in the Helix–Coil Transition Region II. Intrinsic Viscosity

Jong Ryul KIM and Taikyue REE

Department of Chemistry, Korea Advanced Institute of Science and Technology,
P.O. Box 150, Cheongryangni, Seoul 131, Korea

ABSTRACT: The theory presented in the preceidng paper (Part I) for the molecular dimensions, $\langle s^2 \rangle_{ol}$ and $\langle s^2 \rangle_I$, of polypeptides in the helix–coil transition region is applied here to determine the intrinsic viscosity $[\eta]_I$ (the $[\eta]$ in the transition region) as a function of the helical content f_H which varies with the cooperativity parameter (eq I-16). Using the Kurata–Yamakawa theory, the linear expansion factor α_s was converted to the hydrodynamic expansion factor α_η. Thus, with $\alpha_{\eta I}$ (the α_η in the transition region) and $\langle s^2 \rangle_{ol}$ (eq I-10), the $[\eta]_I$'s of polypeptides in the helix–coil transition region were calculated (eq 3), and compared with experimental results for various kinds of polypeptide–solvent systems with good agreement (Figures 3 and 4).

KEY WORDS   Solution Properties / Helix–Coil Transition / Synthetic Polypeptide / Helical Content / Intrinsic Viscosity / Cooperativity Parameter / Kurata–Yamakawa Theory /

This paper is a continuation of the preceding one (referred to as Part I, herein) where the expression eq I-10 (eq 10 in Part I), correlating the unperturbed dimension $\langle s^2 \rangle_{ol}$ of polypeptides with helical content f_H, was derived, and in which were also presented the equations by which the changing trend of the linear expansion factor α_{sI} in the helix–coil transition region (see eq I-18 and I-20) could be calculated. This paper predicts the intrinsic viscosity $[\eta]_I$, i.e., $[\eta]$ in the helix–coil transition region, from $\langle s^2 \rangle_{ol}$ and α_{sI}. $[\eta]_I$ is of great importance for explaining the behavior of a polypeptide chain in dilute solution with conformation change, but no theoretical attempt has been made along this line. By using $\langle s^2 \rangle_{ol}$ and α_{sI} with the aid of the Kurata–Yamakawa theory,[1] which correlates α_s with the hydrodynamic expansion factor α_η, the relation of $[\eta]_I$ vs. f_H is derived in this paper, and compared with experiment, e.g., PBLG in DCA–EDC (ethylene dichloride)[2,3] and poly(β-benzyl L-aspartate) (PBLA) in DCA–CHCl$_3$[4] and in m-cresol.[4]

THEORY

Flory and Fox[5] proposed the empirical equation

$$[\eta] = \Phi \frac{(6\langle s^2 \rangle_o)^{3/2}}{M} \alpha_s^3 = \Phi_o \frac{(6\langle s^2 \rangle_o)^{3/2}}{M} \alpha_\eta^3 \tag{1}$$

where $[\eta]$ is the intrinsic viscosity or the limiting viscosity number, α_η, the hydrodynamic expansion factor or the expansion factor for viscosity defined by $\alpha_\eta^3 \equiv [\eta]/[\eta]_\Theta$ ($[\eta]_\Theta = [\eta]$ in the unperturbed state), M, the molecular weight of the polymer, Φ_o, the Flory viscosity constant, a universal constant independent of the nature of polymers, and Φ, the viscosity constant which is a decreasing function[5] of the excluded volume effect.

$$\alpha_\eta^3 = \alpha_s^{2.43} \tag{2}$$

Using the second equality of eq 1 and eq I-10, $[\eta]_I/[\eta]_C$ may be written as

$$\frac{[\eta]_I}{[\eta]_C} = \left(\frac{\alpha_{\eta I}}{\alpha_{\eta C}}\right)^3 \left(\frac{1+f_H\cos 80°}{1-f_H\cos 80°}\right)^{3/2} \quad (3)$$

where $[\eta]_I$ and $[\eta]_C$ are $[\eta]$ for the intermediate chain in the helix–coil transition region and the random-coil, respectively, $\alpha_{\eta I}$ and $\alpha_{\eta C}$ are α_η for the intermediate chain and the random-coil.

From eq 2 and I-18

$$\left(\frac{\alpha_{\eta I}}{\alpha_{\eta C}}\right)^3 = \left(\frac{\alpha_{s I}}{\alpha_{s C}}\right)^{2.43} = \alpha_h^{2.43} \quad (4)$$

Substituting eq 4 into 3 leads to

$$\frac{[\eta]_I}{[\eta]_C} = \alpha_h^{2.43} \left(\frac{1+f_H\cos 80°}{1-f_H\cos 80°}\right)^{3/2} \quad (5)$$

RESULTS AND DISCUSSION

Changing Trend of the Expansion Factor

The quantities, f_H and $[\eta]_I/[\eta]_C$, were calculated from eq I-16 and 5, respectively, by eq I-14 and I-15 for $n=950$ at various given f with $\sigma=2.0\times 10^{-4}$.[8] The results are presented in Table I. In this table, it may be noted that $[\eta]_I$ increases slowly over the range of f_H from zero to about 0.85, but steeply from $f_H \simeq 0.85$ to 1.0. This is due to the changing trend of α_{sI} with f_H (see Table I of Part I and Figure 1). That is, $[\eta]_I/[\eta]_C$ is a function of α_h (eq 5) which equals α_{sI}/α_{sC} (eq 4) and behaves as shown in Figure 1.

Values of α_{sI}/α_{sC} ($=\alpha_h$) over the whole range of f_H for $n=950$ with the values of $\sigma=0.5\times 10^{-4}$, 1.0×10^{-4}, and 2.0×10^{-4}, were calculated from eq I-20 in collaboration with eq I-14 to I-16, and the results are shown in Figure 1. The quantity Φ was obtained from the first equality of eq 1 using the data of $[\eta]$ and $\langle s^2 \rangle$ ($=\alpha_s^2 \langle s^2 \rangle_o$); it generally decreased[6] with increasing α_s in accordance with the relation $\Phi = \Phi_o \times \alpha_s^{-0.57}$ obtained from eq 1 and 2. Norisuye[3] determined Φ of two high-molecular weight PBLG samples in DCA containing 8.3 wt% of CHL in the thermal

Table I. Dependence of f_H and $[\eta]_I/[\eta]_C$ on f for the chain of $n=950$ taking $\sigma=2.0\times 10^{-4}$

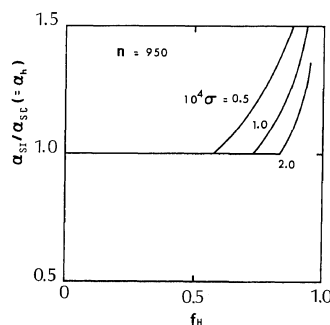| f	f_H	$[\eta]_I/[\eta]_C$
0.050	0.048	1.0255
0.150	0.142	1.0768
0.250	0.234	1.1297
0.350	0.325	1.1848
0.450	0.417	1.2429
0.525	0.486	1.2889
0.600	0.556	1.3373
0.675	0.628	1.3888
0.750	0.702	1.4439
0.825	0.778	1.5038
0.875	0.832	1.5472
0.900	0.860	1.6972
0.925	0.889	2.0110
0.950	0.919	2.5055
0.975	0.952	3.4955

Figure 1. Dependence of the expansion factor α_{sI} on the helical content f_H for $n=950$ with $\sigma=0.5\times 10^{-4}$, 1.0×10^{-4}, and 2.0×10^{-4}.

helix–coil transition region. The value of σ of the polypeptide–solvent system was evaluated[7] as 0.72×10^{-4}. According to Norisuye,[3] Φ is almost constant in the region of $0 \le f_H <$ about 0.7, and decreases sharply as f_H approaches unity. Combining this experimental fact with the above Φ vs. α_s relation, it is evident that α_{sI} is almost constant in the region from $f_H=0$ to about 0.7, but sharply increases in the region of $f_H > 0.7$ for the system of $\sigma=0.72\times 10^{-4}$. This is in good agreement with the results shown in Figure 1.

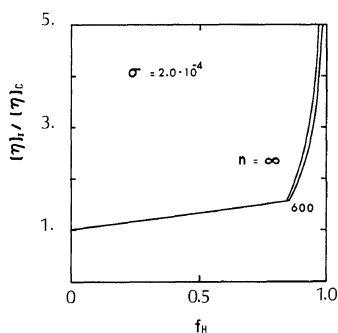The cooperativity parameter σ has an important effect on α_{sI} as shown in Figure 1, from

Figure 2. Dependence of $[\eta]_I/[\eta]_C$ on the helical content f_H with $\sigma = 2.0 \times 10^{-4}$ for $n = \infty$ and 600.

Figure 3. Variation in $[\eta]_I$ with the helical content f_H. The experimental points[2] are for PBLG ($\bar{N}_w = 1830$ and 1150) in DCA–EDC mixtures.

which it can be seen that a smaller σ gives a larger α_{sI} at a given helical content f_H. This is understandable since a smaller σ yields a smaller g_H (the average number of helical sequences, eq I-14) and produces a larger n_h (the average number of residues per helical sequence, see eq I-15) in the chain for fixed f_H and n. Consequently, α_h ($= \alpha_{sI}/\alpha_{sC}$) becomes larger according to eq I-20. This reasonably explains the data shown in Figure 1.

The values of $[\eta]_I/[\eta]_c$ for $n = 950$ were calculated from eq 5, and are shown in Table I as previously mentioned. By a similar method used for Table I, the values of $[\eta]_I/[\eta]_C$ and f_H were calculated for $n = 600$, and the results are shown in Figure 2. For $n = \infty$, eq I-21 was used with eq 5. The calculated results for $n = \infty$ also appear in Figure 2. Figure 2 shows that the dependence of $[\eta]_I/[\eta]_C$ on the chain length n is small; i.e., α_h (or α_{sI}/α_{sC}) does not depend very much on n (see eq 4 and 5). This is quite different from the effect of σ, which yields a great difference as shown in Figure 1.

Comparison with Experiment in the Transition Region

Teramoto et al.[2] measured $[\eta]_I$ vs. f_H for several samples of PBLG in DCA–EDC mixed solvents of different composition. Their experimental $[\eta]_I$ values vs. f_H for the samples of $\bar{N}_w = 1830$ and 1150 are shown in Figure 3.
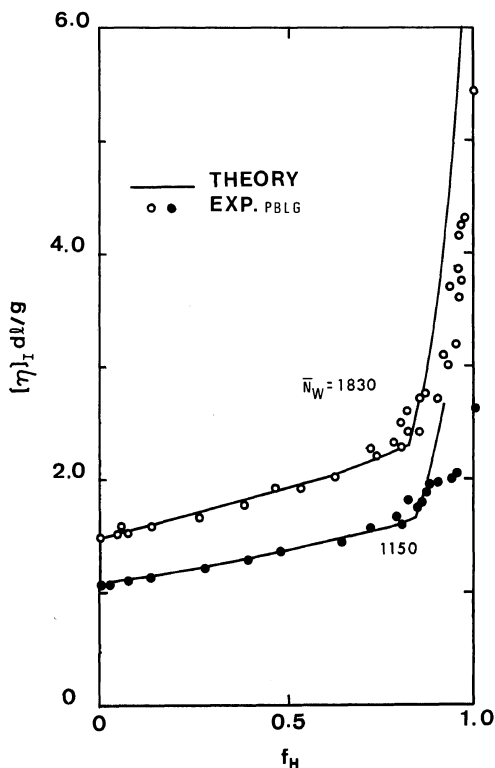
Zimm and Bragg[8] obtained $\sigma = 2.0 \times 10^{-4}$ by a curve–fitting method using the optical rotation data in the helix–coil transition region of PBLG with various molecular weights in DCA–EDC mixed solvents. We calculated $[\eta]_I/[\eta]_C$ from eq 5 using $\sigma = 2.0 \times 10^{-4}$, and applying the method used for obtaining Table I. Our results are shown by full curves in Figure 3, where the values of $[\eta]_C$ are equal to 1.49 and 1.01 dl g^{-1} (Teramoto et al.'s data) for $\bar{N}_w = 1830$ and 1150, respectively. The experimental $[\eta]_I$ values are compared with our theoretical values in Figure 3 with good agreement. Norisuye[3] repeated the same kind of experiment for PBLG samples with the chain lengths different from Teramoto et al.'s[2] in mixed solvents of DCA and EDC. Norisuye's data exhibit almost the same behavior
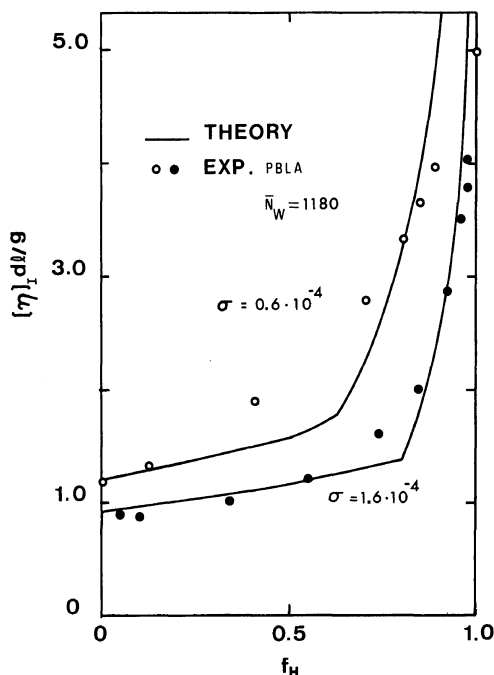
Figure 4. Variation in $[\eta]_I$ with the helical content f_H for PBLA ($\bar{N}_w = 1180$). Experimental points:[4] open circles, data for a mixed solvent DCA–CHCl (5.7 vol% DCA); filled circles, for *m*-cresol.

of $[\eta]_H/[\eta]_C$ *vs.* f_H as Teramoto *et al.*'s shown in Figure 3.

Hayashi *et al.*[4] obtained experimental data of $[\eta]_I$ *vs.* f_H for PBLA of $\bar{N}_w = 1180$ in *m*-cresol and in a mixture of chloroform and DCA (5.7 vol% DCA). The cooperativity parameter was distinctly smaller in the latter ($\sigma = 0.6 \times 10^{-4}$)[4] than former solvent ($\sigma = 1.6 \times 10^{-4}$).[4] Their experimental results are shown in Figure 4, where the data clearly exhibit the dependence of the relation of $[\eta]_I$ *vs.* f_H on σ. Our calculated results from eq 5 with $\sigma = 0.6 \times 10^{-4}$ and $[\eta]_C = 1.30$ dl g^{-1} (both

Hayashi *et al.*'s data) for PBLA in a mixture of CHCl$_3$ and DCA (5.7 vol%) are shown in Figure 4, where the α_h factor in eq 5 is calculated from eq I-20 in collaboration with eq I-14 to I-16. In Figure 4, our results calculated with $\sigma = 1.6 \times 10^{-4}$ and $[\eta]_C = 0.90$ dl g^{-1} (both Hayashi *et al.*'s data[4]) for PBLA in *m*-cresol are shown by a full curve. The agreement between our theoretical results and the experimental data is fairly good, and indicates the correct prediction of the σ effect on the behavior of $[\eta]_I$ *vs.* f_H.

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