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Relativistic Oscillation of Charged Particles in Laser Fields and Pair Production

ELECTRON-POSITRON pairs may be produced by plasma electrons in extremely high intensity laser fields (ref. 2 and N. G. Basov and O. N. Krokhin, personal communication). At these high intensities, the electron follows the relativistic equations of motion.

$$\frac{d}{dt} \left(m_0 v_y \left(1 - \frac{v_x^2 + v_y^2}{c^2} \right)^{-\frac{1}{2}} \right) = e E_y \quad (1a)$$

$$\frac{d}{dt} \left(m_0 v_x \left(1 - \frac{v_x^2 + v_y^2}{c^2} \right)^{-\frac{1}{2}} \right) = e v_y B_z \quad (1b)$$

where $E_y = E_0 \sin \omega t$ and $B_z = (E_0/c) \sin \omega t$ are the electric and magnetic field of a propagating laser beam of frequency ω of linear vacuum polarization. Dielectric effects at the cutoff density² are neglected and that of vacuum polarization at higher intensities than considered here, therefore, no phase shift between E and B will occur. e and m_0 are the charge and mass of the particle.

Equation (1) will be effective for electrons (protons) from CO₂ laser intensities I^* of $1.2 \times 10^{16} \text{ W cm}^{-2}$ ($4.2 \times 10^{22} \text{ W cm}^{-2}$) and for neodymium glass lasers of $1.2 \times 10^{18} \text{ W cm}^{-2}$ ($4.2 \times 10^{24} \text{ W cm}^{-2}$). The difficulty of a rigorous treatment of the highly nonlinear system of differential equations (1) may explain the use of nonrelativistic solutions in some papers³. I report here the exact solution of equation (1) for the maximum kinetic energy E_k of the oscillating particles (in excess to the rest energy) of

$$E_k = m_0 c^2 ((1 + e^2 E_0^2 m_0^{-2} \omega^{-2} c^{-2})^{\frac{1}{2}} - 1) \quad (2)$$

For $|v| \ll c$

$$E_k = \frac{1}{2} e^2 E_0^2 m_0^{-1} \omega^{-2} \quad (3)$$

depending on the mass of the particle. In the pure relativistic case for intensities $I \gg I^*$

$$E_k = e c E_0 / \omega \quad (4)$$

showing independence of the mass.

A further limitation to higher intensities for the particle motion described is the multipole emission of radiation. Therefore one must compare the kinetic energy of equation (4) with energy S emitted per oscillation⁴.

$$S = 1/3 (e^2/c^2) (\omega^3/(2\pi)^3) (x_0^2/(1 - v^3/c^2)^3) \quad (5)$$

where x_0 is the elongation of the oscillation. An upper bound for the multipole radiation to be significant is $2 \times 10^3 I^*$.

Therefore I find an exact formula for the number N_p of pair production³ for intensities I with $I^* < I < 2 \times 10^3 I^*$.

$$N_p = \pi^{-1} (e^2/m_e^2) \alpha^2 N^2 c \ln^3(1 + 0.5 e c E_0 / \omega) \nu \tau$$

where α is the fine structure constant, N the density of hydrogen atoms in the focus volume V of a laser pulse of length τ .

Because of the logarithmic function there is no very serious deviation from the results of Bunkin and Kazakov³. The exclusion of multipole radiation may be interpreted as diminishing the cross section of production of $e\bar{e}$ relative to that of pp at laser intensities exceeding $10^{23} \text{ W cm}^{-2}$ for CO₂ lasers and $10^{25} \text{ W cm}^{-2}$ for neodymium glass lasers.

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New Rule for X-ray Spin Doublets of Multiply Ionized Atoms

SPIN doublet formation has been observed in X-ray spectra of atoms multiply ionized in their deep layers¹⁻³. But investigation of this phenomenon is at a very early stage. Valuable regularities and rules of the kind which are well known for X-ray spin doublets of atoms singly ionized in their inner shells (diagram line spectra) are not yet known for X-ray spin doublets of multiply ionized atoms. The variation of $\Delta\lambda$ with Z is, however, analogous for X-ray spin doublets of singly and multiply ionized atoms. The other important rule which is well known^{5,6} for X-ray spin doublets of singly ionized atoms is "constancy of $\Delta\nu/R$ for constant Z ".

On examining the K and L diagram line spectra of an element it seems that the two K lines (KL_{II}, KL_{III}) are due to transitions from (L_{II}, L_{III}) levels to the common K level whereas the pairs (L_{II}M_I, L_{III}M_I), (L_{II}M_{IV}, L_{III}M_{IV}), (L_{II}N_I, L_{III}N_I), (L_{II}O_{IV}, L_{III}O_{IV}) and so on correspond to transitions from a common level of origin to (L_{II}, L_{III}) as the final levels. So a pair of X-ray terms (L_{II}, L_{III}) is common either as initial levels or as final levels of transition in the emission process of these doublets. Moreover, the doublets are situated on the frequency scale in such a way that the differences between the ν/R values of their constituents are constant and equal to the difference between two X-ray terms, L_{II} and L_{III}. If such a rule could be found for X-ray spin doublets of multiply ionized atoms one could find the energy difference between those common but unknown X-ray terms which actually take part in the emission of these doublets. Here I report such a rule of constancy of $\Delta\nu/R$ for constant Z for X-ray spin doublets of multiply ionized atoms.

In my study I analysed the experimental data for X-ray spectra of multiply ionized atoms (satellite spectra) taken from ref. 7; I find that spin doublets satisfying the required rule do exist (Table 1). Columns 2, 3 and 4 of Table 1 clearly show that $(\Delta\nu/R)$ values for the doublets in question remain nearly constant for a particular element Z . Thus there is a pair of common unknown X-ray terms, say (L_s , $L_{s'}$), which is involved either as initial levels or as final levels of transition in the emission of all the three doublets considered here. The $(\Delta\nu/R)$ value for a particular Z (see Table 1) is, therefore,