emitted close to the surface. Photonuclear reactions will fragment heavy nuclei of high energies if the surface temperature is greater than 5×10^6 K. This will be important for the acceleration of heavy nuclei close to surface in the early stages of the pulsar. Again photon–photon collisions resulting in pair production of electrons will degrade γ-rays of high energy. If the pulsed γ-rays of energy $\gtrsim 6\times10^{11}$ eV observed by Grindlay[13] from NP 0532 originate at the surface of the neutron star, this implies a surface temperature of $<4\times10^6$ K, in order that γ–γ collisions are not effective. This limit is independent of the magnetic field, which if it is high enough can also induce pair production. Further, if the temperature is $>5\times10^6$ K, even γ-rays of ~100 MeV, which are unaffected by the magnetic field will be converted into electron pairs in photon–photon collisions. Details of these aspects will be discussed elsewhere.

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Derivation of the Differential Equation for the Simpler Cosmological Models

SEVERAL simplified expositions of cosmology have appeared in the past few years in response to the increased interest in the subject[1–7]. In these expositions the dimensionless scale factor $R(t)$ at a universal cosmic time t plays an important part. It governs the universal expansion of all linear dimensions of the model, and is introduced through the equation

$$\mathbf{r}_i(t) = R(t)\,\mathbf{r}_i(t_1) \qquad (1)$$

where i labels a typical galaxy (or "particle" in the model), $\mathbf{r}_i$ is the distance of this galaxy from a galaxy $i=0$ which acts as origin, and t_1 is a standard time at which the galaxies are labelled. The differential equation for R is then

$$\dot{R}^2 - BR^{-1} - DR^2 - E = 0 \qquad (2)$$

where $\dot{R} = \mathrm{d}R/\mathrm{d}t$, and B, D and E are constants. The argument, in its Newtonian framework, is based on a homogeneous and isotropic model universe in which the galaxies are assumed to be smeared out to give a mass density $\rho(t)$ which is constant in space at any one time. Equation (2) is then obtained as an energy equation for a spherical distribution. There then arise some awkward problems concerning the significance of the origin, the significance of the surface of the sphere considered and the effect of the possibly infinite amount of matter outside the sphere. But authors usually manage to make it plausible that none of these effects change equation (2).

It does not seem to be generally known that the approach just outlined can be formulated even more simply. I have used this method for a number of years in lectures at University College, Cardiff. The essential point is that one dispenses with the assumption of a continuous distribution of matter and the assumption of precise homogeneity and isotropy. One merely writes down the energy equation for the galaxies considered, which are defined as lying within a polyhedron whose corners are certain named galaxies:

$$\mathcal{E} = \tfrac{1}{2}\sum_i m_i\,[\dot{r}_i(t)]^2 - G\sum_{i<j}\frac{m_i m_j}{r_{ij}(t)} - \frac{\lambda}{6}\sum_j m_j\,[r_j(t)]^2 \qquad (3)$$

where λ arises from the *ad hoc* cosmological term and may be put equal to zero. Equation (1) and equation (3) then give equation (2). The coefficients in equation (2) may then be interpreted as

$$B = G\sum_{i<j}\frac{m_i m_j}{r_{ij}(t_1)}\Big/A, \qquad A = \tfrac{1}{2}\sum_i m_i\,[r_i(t_1)]^2$$

$$D = \frac{\lambda}{6}\sum_i m_i\,[r_i(t_1)]^2/A$$

$$E = \mathcal{E}/A$$

If one now introduces the smeared-out density $\rho(t)$ and assumptions of homogeneity and isotropy, the summations over the galaxies in the polyhedron can be replaced by integrations over an approximating sphere of radius $q(t_1)$(say). One then finds

$$A = (2\pi/5)\rho(t_1)[q(t_1)]^5, \quad AB = (16\pi^2/15)G[\rho(t_1)]^2[q(t_1)]^5$$

$$AD = (2\pi\lambda/15)\rho(t_1)[q(t_1)]^5$$

and this leads to the usual results. The simple proportionality of $\mathcal{E}$ and a power of $q(t_1)$ is found only if the energy due to the cosmological term in equation (3) goes as $[r_j(t)]^2$.

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Hydrogen Bonding in Gypsum

IN our recent letter on the symmetry of the SO_4 ion in gypsum[1] we gave two W–O_1 distances of 2.816 ± 0.002 and 2.896 ± 0.002 Å, without a discussion of their significance. Since the publication of our letter, Dr Falk of the National Research Council of Canada has brought to our attention infrared studies of water in gypsum[2]. These show that two hydrogen bonds differing by 0.02 Å are expected from the difference between the two OH stretching frequencies of HDO molecules of 90 cm^{-1}. Our difference of 0.080 ± 0.003 Å indicates that the hydrogen bondings of the two OH groups differ considerably in length and strength, and much more than predicted from the spectroscopic data. In the original neutron diffraction work on gypsum, Atoji and Rundle[3] reported no significant difference in the hydrogen bond lengths in gypsum, but we have reprocessed their data and found a difference of 0.065 ± 0.018 Å.

The short hydrogen bond is always associated with the atom H_1 (Fig. 1) and is found between both O_w and O_1, and O_w