a straight line the chain would be similar in angular extent to that reported by Arp[7] and Burbidge and Burbidge[8]. PKS 0531+19 is identified with a 17.7 mag E type galaxy[9]. It is unfortunate that the present position measurements do not yield even tentative optical identifications for the new sources as the area is heavily obscured. More accurate positions are needed before the possible association of these sources can be discussed further and detailed measurements of their radio structures would also be of interest.

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Received September 28, 1972.

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On Lowering a Rope into a Black Hole

Penrose[1] and Bekenstein[2] have considered the idea of slowly lowering a particle at the end of a rope into a black hole—or rather its ergosphere—and thereby letting the gravitational field do work on the apparatus at the upper end of the rope. In effect we have an excellent form of waste disposal which could in principle convert waste heat and material refuse into useful energy.

If the particle is at rest with respect to the black hole (or more strictly relative to its timelike Killing vector $\mathbf{K}^a$), then its energy per unit rest mass is $V = \sqrt{-\mathbf{K}_a\mathbf{K}^a}$. Thus if one lowers the particle into the ergosphere where $V=0$ one should be able to extract a total energy of 100% of the rest mass of the particle.

Here I calculate the stresses in the rope for a general stationary spacetime. A simple formula is derived relating the tension at any point of the rope to the gravitational potential V. It is used to show: first, that no matter how strong the rope is it must certainly break before the particle reaches the ergosphere; second, if the energy (including rest mass energy) per unit proper length of the rope σ is independent of the tension T and the material obeys the weak energy condition[3] then no more than $(1-e^{-1}) = 63.212\%$ of the rest mass of the particle can be extracted.

For a realistic rope or hawser made of piano wire, say, the fraction is $10^{-10}\%$.

The rope is made up of particles whose 4-velocity is $\mathbf{U}_a$. Each fibre of the rope describes a ribbon in spacetime spanned by $\mathbf{U}_a$ and $\mathbf{t}_a$ where $\mathbf{t}_a$ is a unit spacelike vector orthogonal to $\mathbf{U}_a$. The material of the rope can only support stresses S in the $\mathbf{t}_a$ direction. The energy momentum tensor $\mathbf{T}_{ab}$ is thus given by

$$\mathbf{T}_{ab} = \mu\mathbf{U}_a\mathbf{U}_b + S\mathbf{t}_a\mathbf{t}_b \qquad (1)$$

where μ is the energy density. To be realistic we must have that $\mu - S \geq 0$ (the weak energy condition[3] and the dominant energy condition[4] both agree in this case). This will ensure sound speeds less than light. $\mathbf{T}_{ab}$ obeys the equation

$$\mathbf{T}_a{}^b{}_{;b} = 0 \qquad (2)$$

Multiplying (2) by $\mathbf{U}_a$ and $\mathbf{t}_a$ respectively, one obtains

$$-\frac{d\mu}{ds} - \mu\mathbf{U}^b{}_{;b} + S\dot{\mathbf{t}}^a\mathbf{U}_a = 0 \qquad (3)$$

$$\mu\mathbf{U}^a\mathbf{t}_a + \frac{dS}{dt} + S\mathbf{t}^b{}_{;b} = 0 \qquad (4)$$

$$\frac{d\mu}{ds} = \mu_{,a}\mathbf{U}^a; \ \frac{dS}{dt} = S_{,a}\mathbf{t}^a; \ \mathbf{U}^a = \dot{\mathbf{U}}^a{}_{;b}\mathbf{U}^b; \ \dot{\mathbf{t}}^a = \mathbf{t}^a{}_{;b}\mathbf{t}^b$$

If A is the cross sectional area of the rope, then

$$\mathbf{t}^b{}_{;b} = \frac{1}{A}\frac{dA}{dt} \qquad (5)$$

whence

$$\mu A\dot{\mathbf{U}}^a\mathbf{t}_a + \frac{d}{dt}(SA) = 0 \qquad (6)$$

Clearly $\sigma = \mu A$ and $T = SA$

whence
$$\sigma\dot{\mathbf{U}}_a\mathbf{t}^a + \frac{dT}{dt} = 0 \qquad (7)$$

This equation shows how the tension varies along the rope. So far the discussion has been quite general. I now consider a spacetime containing a Killing vector $\mathbf{K}^a$. If $\mathbf{U}^a = \frac{\mathbf{K}^a}{V}$, that is, the rope is at rest with respect to the black hole, then

$$\dot{\mathbf{U}}_a = V_a; \ V\mathbf{U}^a{}_{;a} = 0; \ \dot{\mathbf{t}}^a\mathbf{U}^a = 0$$

so $\frac{d\mu}{ds} = 0$. That is, μ is independent of time and T is given by

$$\frac{dT}{\sigma} = \frac{dV}{V} \qquad (8)$$

It is clear that as $V \to 0$ the RHS diverges and so T must diverge whence I deduce that the rope must always break before the ergosphere has been reached.

If σ is independent of T

$$\frac{T}{\sigma} = \log\frac{1}{V} + \text{constant}$$

At $V=1$ (that is, at infinity) $T = T_\infty$; at the end of the rope $T = W$ the weight of the particle (which becomes infinite at the ergosphere anyhow), so $T_\infty = W + \sigma\log\frac{1}{V}$. If we are to have $\frac{T_\infty}{\sigma} \leq 1$, we must have $\log\frac{1}{V} \leq 1$ whence $V > e^{-1}$ and the maximum possible energy extracted is $(1-e^{-1})\,mc^2$. For a non-rotating black hole this means that we cannot get closer than 1.14 Schwarzschild radii.

For a practical rope $\frac{T}{\sigma}$ is less than the value corresponding to breakage which is $\approx 1.2\times10^{-12}$ so the available energy in practice is less than $mc^2\times1.2\times10^{-12}$ and we could get no closer than 5×10^{11} Schwarzschild radii on our rope.

I thank Dr S. W. Hawking for discussions.

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Received September 13, 1972.

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