

as to be now the highest level. This destabilization of  $a_{2u}$  is contrary to all other evidence on a wide range of metal porphyrins and phthalocyanins<sup>2,3</sup> which have shown that the  $a_{2u}$  level is always below the  $3d$  levels and that it is the ligand  $b_{1u}$  level which is closest in energy to the  $d$ -orbitals. I believe it is dangerous to draw conclusions by comparison with the  $g$ -values of oxidized metal-free TPP because these are so similar to those for reduced metal-free TPP<sup>5</sup>.

The second oxidation product, III, arises by loss of an electron from either I or II, and in this case only one configuration is realistic, namely,  $e_g^4$ ,  $b_{2g}^2$ , with the species formulated as  $[\text{Ni}^{\text{IV}}\text{TPP}]^{2+}$ .

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## Relativistic Bohr Formula

SYNGE<sup>1</sup> has discussed a relativistic extension of the Bohr formula,  $h\nu' = E' - E$ , for the frequency of the emitted light quantum between two levels of a quantum system:

$$h\nu'_{(m' \rightarrow m)} = (m'^2 - m^2)/2m'c, \quad (1)$$

where  $m'$  and  $m$  are the total masses of the two states. Equation (1) is a simple consequence of the covariant energy-momentum conservation at the vertex  $m' \rightarrow m + \gamma$ , evaluated in the rest frame of the state with mass  $m'$ . In the rest frame of  $m$ , we have instead of (1),  $h\nu = (m'^2 - m^2)/2mc$ . It is also clear that equation (1) can be written formally as  $h\nu' = E' - E$ , in the rest frame of  $m'^2$ , because  $E' = m'$  in the rest frame of  $m'$ , and because the correct equation is  $h\nu' = m' - E$ , all quantities in the rest frame of  $m'$ . If the emitted quantum is not the light quantum but any system (or system of particles) with the total invariant mass  $\mu$  we obtain, again by the energy-momentum conservation at the vertex  $m' \rightarrow m + \mu$ , instead of (1),

$$h\nu' = (m'^2 - m^2 + \mu^2)/2m'c, \quad (2)$$

where  $h\nu'$  is the total relativistic energy of the emitted quantum.

Equations (1) and (2) are purely kinematic and formal. In this letter we are interested in the explicit evaluation of these equations, which is not trivial. The explicit evaluation of (1) is not possible in terms of the usual Schrödinger or even the Dirac equation for the atom, because these equations give us the energy of one of the particles in the field of an infinitely heavy nucleus, whereas in (1) or (2) we need the total invariant masses  $m'$  and  $m$ . There is, however, a new version of the dynamics of the hydrogen atom including recoil effects to all orders. In this version the atom is treated as a whole as a single relativistic system with internal degrees of freedom and obtains the mass formula<sup>3</sup>

$$m_n^2 = m_1^2 + m_2^2 + 2m_1m_2n/(n^2 + (Z\alpha)^2)^{1/2}, \quad (3)$$

where  $m_1$  and  $m_2$  are the masses of the constituents. In the Dirac-type atom with spin  $1/2$ —spin 0 constituents,  $n$  is

replaced by

$$n' + \left[ \left( j + \frac{1}{2} \right)^2 - (Z\alpha)^2 \right]^{1/2}, \quad n' = 0, 1, 2, 3, \dots$$

Now we can evaluate equation (1) explicitly

$$h\nu'_{(n' \rightarrow n)} = m_1m_2 \left[ \frac{n'}{\sqrt{(Z\alpha)^2 + n'^2}} - \frac{n}{\sqrt{(Z\alpha)^2 + n^2}} \right] / C[m_1^2 + m_2^2 + 2m_1m_2n'/(Z^2\alpha^2 + n'^2)^{1/2}]^{1/2} \quad (4)$$

The recoil effects are small for the hydrogen atom, but are of considerable importance in other systems (nuclei, hadrons, exotic atoms, and so on). In particular, (3) and (4) are valid, in contrast to Dirac or Klein-Gordon equations, for the case  $Z\alpha \geq 1$  (this is the case for the atom formed out of the spinless particles having both electric and magnetic charges, called dyonium<sup>4</sup>, where  $Z\alpha = 137/4$ ), in which case (4) becomes

$$h\nu' \cong \frac{m_1m_2}{(m_1^2 + m_2^2)} \frac{(n' - n)}{Z\alpha}; \quad n', n \ll \alpha; \quad m_1^2 + m_2^2 \gg \frac{2m_1m_2}{\alpha}$$

in contrast to the usual small  $Z\alpha$  limit:

$$h\nu' \cong \frac{m_1m_2}{(m_1 + m_2)^2} (Z\alpha)^2 \left( \frac{1}{n^2} - \frac{1}{n'^2} \right); \quad n', n \gg \alpha.$$

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