

Statistical Notes calculation

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INTRODUCTION

The accurate determination of sample size is a crucial aspect of study design. If the study sample is too small, a real effect may exist, and be observed, yet lack statistical significance thus resulting in a false negative conclusion. On the other hand, a study that is larger than it needs to be will absorb funding that could have been better used elsewhere, and possibly delay the release of important results. Sample size calculations need to be presented in funding applications where they are rightly subjected to close scrutiny, and they should also be clearly reported when publishing results.

Estimating sample size is complex and for all but the simplest studies a statistician should be consulted. However, the questions that a statistician will ask are largely predictable by someone with an elementary understanding of the principles involved in calculating sample size. In this paper, we describe the method and the information required for a simple calculation moving on to consider some aspects of more complex study designs that may influence sample size.

COMPONENTS OF SAMPLE SIZE CALCULATION

Difference to be detected

The process of estimating the optimal number of subjects for a particular study requires researchers to think forward to the data analysis stage. Significant tests, comparing two or more groups, seek to determine whether an observed difference is a chance finding or whether it represents a true difference between the groups. When planning the study, it is necessary to decide on the size of difference that the study will be able to detect. This should be the smallest change that is considered to be clinically important. Ideally, we would want to detect an improvement, no matter how small, but because sample size increases sharply when seeking small

changes, pragmatic considerations become important.

Most studies gather data on several outcomes; from these the single most important one should be identified, commonly referred to as the 'primary outcome'. Sample size should then be based on the anticipated difference in primary outcome between the groups under study. In the case of several outcomes having equal importance, so no primary outcome can be identified, sample size should be calculated for each outcome of interest and the largest resultant value used.

In the context of a clinical trial in which the primary outcome measure is a categorical variable (exacerbations of asthma within a predetermined period, for example), baseline disease prevalence, (i.e. frequency of exacerbation's in the control group) needs to be known, together with the change in disease prevalence that the study seeks to detect. If the primary outcome is a continuous variable (eg FEV1 or PEF variability), the expected difference in the mean value is required, together with the standard deviation of the data in the control group.

Sometimes the required values will be obtainable from the literature or from a pilot study; if this is not the case and reliable estimates are unavailable 'best guess' clinical judgement must suffice.

Significance and power

Significance tests can result in Type I or Type II errors. A Type I error, or false positive, occurs when new treatment is declared to be better than the control but is in fact no different. Conversely, to conclude that the new treatment is no different when in fact it is constitutes a Type II error or false negative result. The risk of such errors is directly related to sample size when planning the study, it is therefore necessary to decide what risk of error is considered acceptable.

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Date submitted: 05/01/0
Date accepted: 28/02/0

Prim. Care Respir. J
2001 10(1):15-1

Table 1. Formulae to calculate number in each arm of a two group trial

Continuous primary outcome	$n \geq \left\{ \frac{T_1 F_{2df}}{diff} \right\}^2$	diff = difference in mean value σ = standard deviation in control group
Categorical primary outcome	$n > \left\{ \frac{T_1 \sqrt{2p + \bar{p}} + 2\sqrt{p_1(1-p_1) + p_2(1-p_2)}}{p_1 - p_2} \right\}^2$	p = prevalence overall p ₁ = prevalence, control group p ₂ = prevalence, intervention group
T ₁ and T ₂ Probability (Type I error) T ₁	1% 2.576	5% 1.960
		10% 1.645
Power T ₂	90% 1.282	80% 0.842
		70% 0.52

The probability of a Type I error is often referred to as the **significance level**, typically set at 0.05 (5%). The **power** of a study is one minus the probability of a Type II error, the most commonly accepted chance of a Type II error is 0.2 (20%), giving a power of 0.8 (80%). In some situations, other values of significance and power will be more appropriate. For example, if a new treatment has unpleasant side effects it may be appropriate to reduce the false positive risk to one-percent (0.01), or for a treatment that possibly represents a major therapeutic breakthrough, power may be increased to 90% or above.

Example

A randomised-controlled trial is to be carried out to compare the efficacy of a new bronchodilator with salbutamol in patients with established asthma. The primary outcome is morning peak expiratory flow (PEF), and a difference of greater than 20 litres per minute would be considered clinically important. The trial is required to have 80% power at the five-percent significance level. From the literature, it is estimated that adult asthmatics have an average morning PEF of 400 l/min, with a standard deviation of 100 l/min.

From equation 1, with $\mu_1 = 1.96$ and $\mu_2 = 0.842$, the study would require at least 250 patients in each arm.

Losses to follow-up

For a number of reasons participants pull out of longitudinal research studies and are lost to follow up. Usually this results in exclusion from the data analysis, effectively reducing the sample size. Since losses to follow up are almost inevitable it is wise to compensate at the design stage by calculating sample size normally and then multiplying up by an appropriate factor. The factor used depends on the expected losses to follow up, best estimated from previous studies of a similar nature. In the absence of any prior knowledge a dropout rate of 20% may be assumed, if the sample size is then increased by 25% it will be returned to the original value by a 20% dropout rate.

Example

A randomised-controlled trial designed to compare the efficacy of a new bronchodilator with salbutamol requires at least 250 patients in each arm to have 80% power of detecting a five-percent difference in PEF at the five-percent significance level. In the absence of data from pilot studies, it is assumed that up to 20% of

subjects may be lost to follow-up. In order to account for dropouts, it is necessary to increase the sample size by 25%, thus requiring at least 313 patients to be recruited to each arm of the trial. Losing 20% of the 313 patients then returns the total to 250.

UNEQUAL GROUP

The most statistically efficient study design will always have equally sized groups, although sometimes other considerations still make an unbalanced study the best option. If a case-control design is used to study a rare disease the number of available cases may be limited, or if a new treatment is particularly expensive, it may be cost effective to compare a small number of treated patients with a larger untreated number, for example. To compensate for the lower efficiency of an unbalanced design the overall sample size needs to increase - reducing the number of patients in one arm of a trial thus requires an increase of greater magnitude in the other arm. As the design moves further from a 1:1 ratio, to 2:1, 3:1 or 4:1 greater compensation is required. The statistical inefficiency of unbalanced trials should therefore be weighed against the practical considerations mitigating against a balanced study design.

CONCLUSION

The final step of sample size calculation is a simple matter of entering numbers into a formula, but as with any formula it is very much a case of 'rubbish in, rubbish out'. This paper has described the principal pieces of information that researchers need to make available to statisticians: the effect size which needs to be reliably detected; a measure of dispersion for continuous outcomes; the acceptable risk of error (significance and power) and predicted losses to follow-up. Care taken at this stage will help ensure that the results obtained can be readily and reliably interpreted.

ACKNOWLEDGMENT

We thank Claire Cook for her constructive comment on an earlier draft of this manuscript.

FURTHER READING

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