QUANTIZATION MAKES RELATIVITY COMPATIBLE WITH SUPERLUMINAL PHENOMENA

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A news[1, 2] shocked physicist society, neutrinos may move fast than light. Most physicists affirm that it is impossible according to the well established relativity. Some people try to find new theory other than relativity to explain the phenomena. In the following we would show, it may be explained within relativity, if the quantum theory is also taken into account.

The precise determination of the *neutrino velocity* v, defined as the ratio of the precisely measured distance from CERN to OPERA to the time of flight of neutrinos traveling through the Earths crust, was carried out by the experiment group OPERA[1], and found $(v-c)/c = (2.37 \pm 0.32(\text{stat.})^{+0.34}_{-0.24}(\text{sys.})) \times 10^{-5}$. Since neutrinos participate only in weak interaction, they are always practically free. If they are classical, the above definition of their velocity v is precise, and the measured value shows their motion is superluminal. But this is impossible, since the mass-velocity relation for a classical particle given by relativity shows the mass of the particle with a velocity $v > c$ will be imaginary, and therefore is un-physical[3, 4]. Suppose a particle initially has a velocity $v < c$ and then accelerated by certain mechanism, when the velocity of the particle approaches light speed c, the mass of the particle will approach infinity, and it in turn prevents the velocity reaching and crossing the limit value c. We see the contradiction appears when one assumes the particle being classical. The way out of this problem is to remember that neutrinos are not classical particles, they are quanta of the neutrino field. According to quantum theory[5], they do not move along trajectories, their space distribution at a given time is statistically described by a wave function, and its time evolution is governed by the wave equation. The position and the velocity of a neutrino could not be defined simultaneously at any given time. Moreover, one is even not able to trace a single neutrino, to identify a neutrino at time t and position x being the one originally at time t_0 and position x_0. The measured quantity v is not the neutrino velocity. One has to reconsider what is the true meaning of the observed superluminal phenomena.

The wave function describing the neutrino motion also describes a wave, it is the neutrino wave. In this experiment it is a wave packet produced at CERN and moves to Gran Sasso Laboratory(LNGS). Since the dimension of the wave packet is negligibly small comparing with the distance between CERN and LNGS, one may consider it as a point, its position and velocity are well defined at each time. The velocity of the wave packet as a whole is its group velocity $\mathbf{v}_g = \mathbf{k}c^2/\omega = \mathbf{p}c^2/E$, in which $\mathbf{k}$ and ω are wave vector and circular frequency of the wave and $\mathbf{p}$ and E are momentum and energy of the neutrino described by the wave respectively. There are relativistic relations $\mathbf{p} = \hbar\mathbf{k}$,$E = \hbar\omega$,$\omega = \sqrt{k^2+\kappa^2}\ c$, and $E = \sqrt{p^2+m^2c^2}\ c$, with m and $\kappa \equiv mc/\hbar$ to be the mass and the reciprocal of Compton wavelength for the neutrino respectively. The dimension of the wave packet here is assumed to be quite large compare to its wavelength so that its wave vector and therefore the circular frequency are quite well defined. Momentum $\mathbf{p}$ and energy E are good quantum numbers of the neutrino. From above relations we see that the group velocity of the neutrino wave packet is exactly the classical velocity of neutrinos described by it, and is less than the light speed c for massive neutrinos. It seems nothing is superluminal.

One thing is crucial. Waves associated to massive particles are dispersive. Wave packets not only displace but also expand. It means different points in the wave packet move with different velocities. Especially, the wave front may move with a velocity larger than the group velocity. Could it be superluminal?

The neutrino wave Ψ is governed by Dirac equation

$$(\gamma_\mu \partial_\mu + \kappa)\Psi = 0\,, \tag{1}$$

in which γ_μ with $\mu = 1, 2, 3, 4$, are covariant Dirac matrices satisfying

$$\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}\,. \tag{2}$$

However, the wave function satisfying Dirac equation (1) also satisfies Klein-Gordon equation

$$\partial_\mu\partial_\mu\Psi - \kappa^2\Psi = 0\,. \tag{3}$$

On the other hand, for free particles, from a wave function Ψ_0 with four components satisfying Klein-Gordon equation (3) one may construct a solution of Dirac equation. Take $\Psi_1 \equiv (\gamma_\mu\partial_\mu - \kappa)\Psi_0$. If $\Psi_1 \neq 0$, $\Psi \equiv \Psi_1$ is a solution of the Dirac equation (1), because of the relation (2). If $\Psi_1 = 0$, one may construct $\Psi \equiv \gamma_5\Psi_0$ with $\gamma_5 \equiv \gamma_1\gamma_2\gamma_3\gamma_4$, which is a solution of (1), because of $\gamma_5\gamma_\mu = -\gamma_\mu\gamma_5$ for $\mu = 1, 2, 3, 4$. It means, if one is interested in the space distribution of neutrinos only, he may ignore the complexity introduced by the spinor property of neutrino wave function, and consider a one component wave function Φ instead.

The motion of wave packet is one dimensional along the straight line from CERN to LNGS. Our problem is therefore one dimensional along this direction x. To consider the wave packet expansion, let us use a coordinate system moving with the packet, in which the center of the wave packet does not move. This is the wave packet coordinate system S_0. Suppose initially the wave function is

$$\Phi(x,0) = \begin{cases} \cos(\frac{\pi x}{a}) & \text{if } -0.5a \le x \le 0.5a \\ 0 & \text{otherwise}, \end{cases} \quad (4)$$

which is centered at the origin of this coordinate system, and is non-zero only in the interval $-0.5a < x < 0.5a$. The statistical interpretation of wave function is somewhat complicated in relativistic quantum theory. But the qualitative aspect is clear. Anyway, the probability of finding a particle in a region with zero wave function is zero. Contrarily, in the region with nonzero wave function, the probability of finding a particle is nonzero. Therefore, equation (4) shows that at $t = 0$ one cannot find any neutrino outside the region $0.5a \le x \le 0.5a$.

A solution of Klein-Gordon equation (3) satisfying the initial condition (4) is

$$\Phi(x,t) = \int_{-\infty}^{\infty} \frac{a\cos(ka/2)}{\pi^2 - k^2a^2} \cos(kx - \sqrt{k^2+\kappa^2}\, ct)\mathrm{d}k\,, \quad (5)$$

at time t. It is not difficult to work out the integral in this solution to a required precision. Take $a = 10\lambda_c$, it means the initial wave packet length is 10 times Compton wave length $\lambda_c \equiv \hbar/mc$. The calculated wave function at time $t = 10a/c$ is shown in Fig.1. We see the wave front

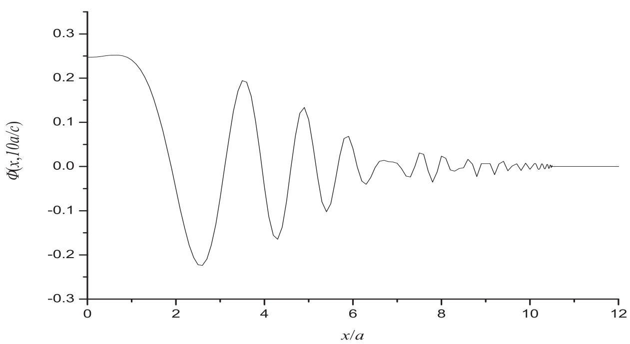

FIG. 1: Wave function in the case $a = 10\lambda_c$ at time $t = 10a/c$.

initially at $x = 0.5a$ has moved to $x = 10.5a$ in the time interval $(0, t)$, and therefore moves with light speed c. If the wave front keeps sharp, so that the wave function at time t keeps being zero in the region $x \ge 10.5a$, and therefore the probability of finding a neutrino in this region is zero, everything would seem normal. Any particle found in the region $x < 10.5a$ at time t may be thought to come from somewhere in the initial region $0.5a \le x \le 0.5a$ with a velocity not exceed the light velocity c. But this is not the case. During propagation the wave front diffuses. The wave function near $x = 10.5a$ is shown in Fig.2 with large scale. It is clearly non-zero in the region $x > 10.5a$ at time $t = 10a/c$, though it decays rapidly

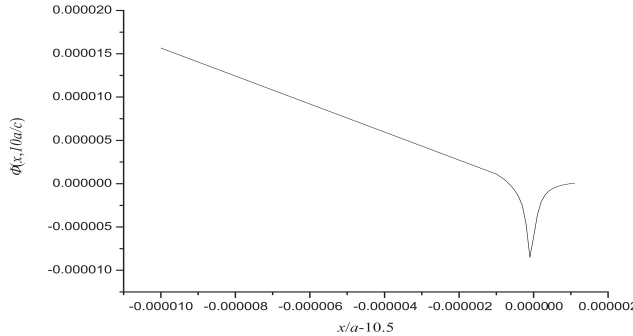

FIG. 2: Wave function in the vicinity of $x = 10.5a$ at time $t = 10a/c$.

with the increase of distance $x - 10.5a$. One therefore may find neutrino in this region. When this happens, the superluminal problem appears. Classically, this means a particle initially in the region $0.5a \le x \le 0.5a$ moved to a point with $x > 10.5a$ in a time duration $\Delta t = 10a/c$, the velocity exceeds light speed c. But this is not the true picture. According to above relativistic quantum theoretical analysis the neutrino does not move along a trajectory, its velocity is not defined. The superluminal problem for the neutrino itself is not justified. What is superluminal is the neutrino wave propagation. It is solved from relativistic wave equation, therefore is a result of relativity, rather than its opposite. Superluminal phenomena is therefore explained by relativity itself, if it is quantized.

The superluminal effect is something like the barrier penetration effect. Barrier is a space in which the potential energy of a particle is higher than its total energy. Since the kinetic energy is non-negative, a particle cannot penetrate into the barrier because of the energy conservation in classical mechanics. But according to quantum mechanics, the potential energy, the kinetic energy and the total energy of a particle cannot be determined simultaneously, the above classical restriction does not work. Since the wave function solved from Schrödinger equation is non-zero somewhere in the barrier, but decays rapidly with the increase of distance from the barrier surface, the probability of finding particle in the barrier is not zero. The barrier penetration is therefore possible. Of course, correspondence principle requires that quantum mechanics should approach classical mechanics when the planck constant becomes unimportant. There is a penetration depth, one can find the particle in barrier only within this depth from the barrier surface. Superluminal effect may be viewed as a kind of penetration effect. The quantum essence makes a particle be able to appear in a zone which is forbidden by classical mechanics. In supperluminal effect this zone is $v > c$ in velocity space. There is also a microscopic depth of penetration, one can find particle in the zone only within this depth from the surface $v = c$.

Let us return to the laboratory coordinate system S, in which CERN and LNGS are at rest, but the wave packet moves from CERN to LNGS with its group velocity. For two events happened at a pair of space-time points (x_1, t_1) and (x_2, t_2) one may define a velocity

$$v \equiv (x_2 - x_1)/(t_2 - t_1)\,, \qquad (6)$$

irrelevant to the picture of motion being classical or not. For a given pair of events, the defined velocity is different in different inertial coordinate systems. They are related by relativistic theorem for velocity addition. In our case, the velocity v_0 defined in the system S_0 and the velocity v defined in the system S are related by

$$v = \frac{v_0 + v_g}{1 + \frac{v_0 v_g}{c^2}}\,. \qquad (7)$$

Since for $v_g < c$, $\mathrm{d}v/\mathrm{d}v_0 > 0$, and for $v_0 = c$, $v = c$, we see for $v_0 > c$ one has $v > c$. It means a phenomenon appears to be superluminal in system S_0 must be superluminal too in the system S. The above analysis also shows, when the superluminal phenomenon happens nothing needs to be radiated. It explains the superluminal phenomena seen in the OPERA experiment, at least qualitatively. A quantitative comparison between experimental observation and theoretical calculation needs more details of experimental conditions.

Anyway, the relativity allows something moves fast than light. Although it could not be a classical particle but may be waves. For fermions, including neutrinos, it must be quantum waves and no classical limits. However, there are bosons, their quantum waves may approach classical limits, by boson condensations. Above analysis shows these classical waves may propagate fast than light. Therefore the superluminal communication is possible. But, it is a custom to believe any superluminal process violates causality. Consider a superluminal connection between two space like points (x_1, t_1) and (x_2, t_2), satisfying $(x_1 - x_2)^2 > c^2(t_1 - t_2)^2$. Under Lorentz transformations the sign of their time difference $t_1 - t_2$ may be different in different coordinate systems. It means, one sees $t_1 > t_2$ in a coordinate system, but $t_1 < t_2$ in another. If one defines the earlier event to be the cause, and the later event to be its result, he has to assert the event 1(2) to be the cause(result) when he observes in one coordinate system, but assert the same event to be the result(cause) when he observes in another coordinate system. The definite causal relation seems absent. However this point has to be checked carefully. The above interpretation on causality is observational. The cause should appear earlier than the result. But there are always some subjective elements in observations, for an example the choice of the coordinate system. On the other hand, the causality should be totally objective. The cause makes the result in an object.To observe this relation, one had better choose a coordinate system, in which the object is at rest. The laboratory system S, in which laboratories in CERN and LNGS are at rest, is suitable to observe the creation, displacement and detection of neutrinos. The wave packet system, in which the center of packet is at rest, is suitable to observe the expansion of neutrino wave packet. In these systems the causality is clear. Neutrinos created in CERN move to LNGS and absorbed there by detectors. No contradiction appears. One may of course choose another coordinate system S' moving from CERN to LNGS with a velocity $0 < v' < c$ satisfying $v'v > c^2$, in which v is the velocity observed in this experiment. In this system S', the observer would see neutrinos emerge from detectors in LNGS, move to CERN and disappear there. The causality seems violated. However, in our view, this is only a mistake made by the observer. He took a wrong coordinate system. It makes him see a distorted scenery. The objective causality is therefore hidden.

Beside superluminal effect, the structure of diffused wave front may contain some other useful information, including the mass of its quantum, the shape and dimension of the initial wave, and so on. A further research of their possible applications is interesting. For example, it may offer a new way for measuring the neutrino mass.

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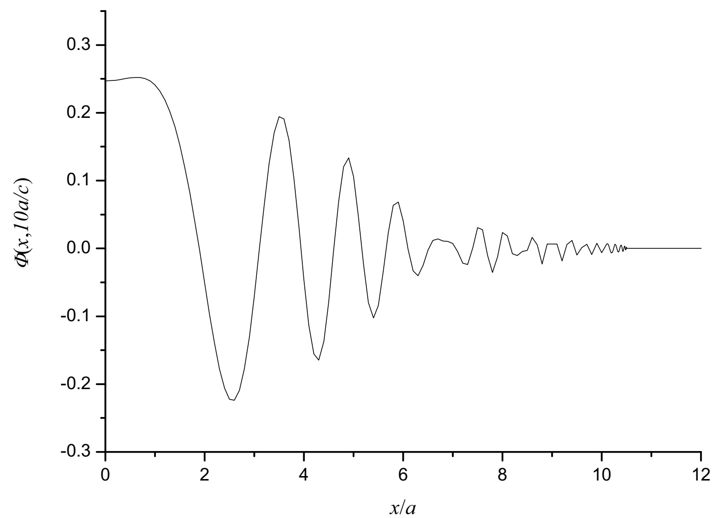
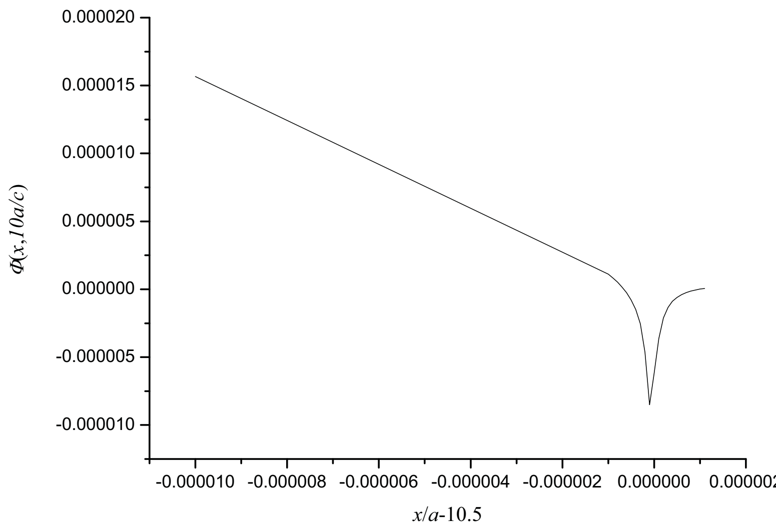


$\Phi(x,10a/c)$
0.3
0.2
0.1
0.0
-0.1
-0.2
-0.3
0
2
4
6
8
10
12
x/a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-0.000010
Φ(x,10a/c)
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0.000000
0.000002
x/a-10.5