Can ABC be Used for Model Selection?

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- Also proposed ABC approach to model selection when M_1 nested in M_2: infer under M_2 with prior such that M_1 gets half of the weight

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- Question: which summary statistic s should be used?

Sufficient statistics

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- But as they point out, this does not hold for all types of models!

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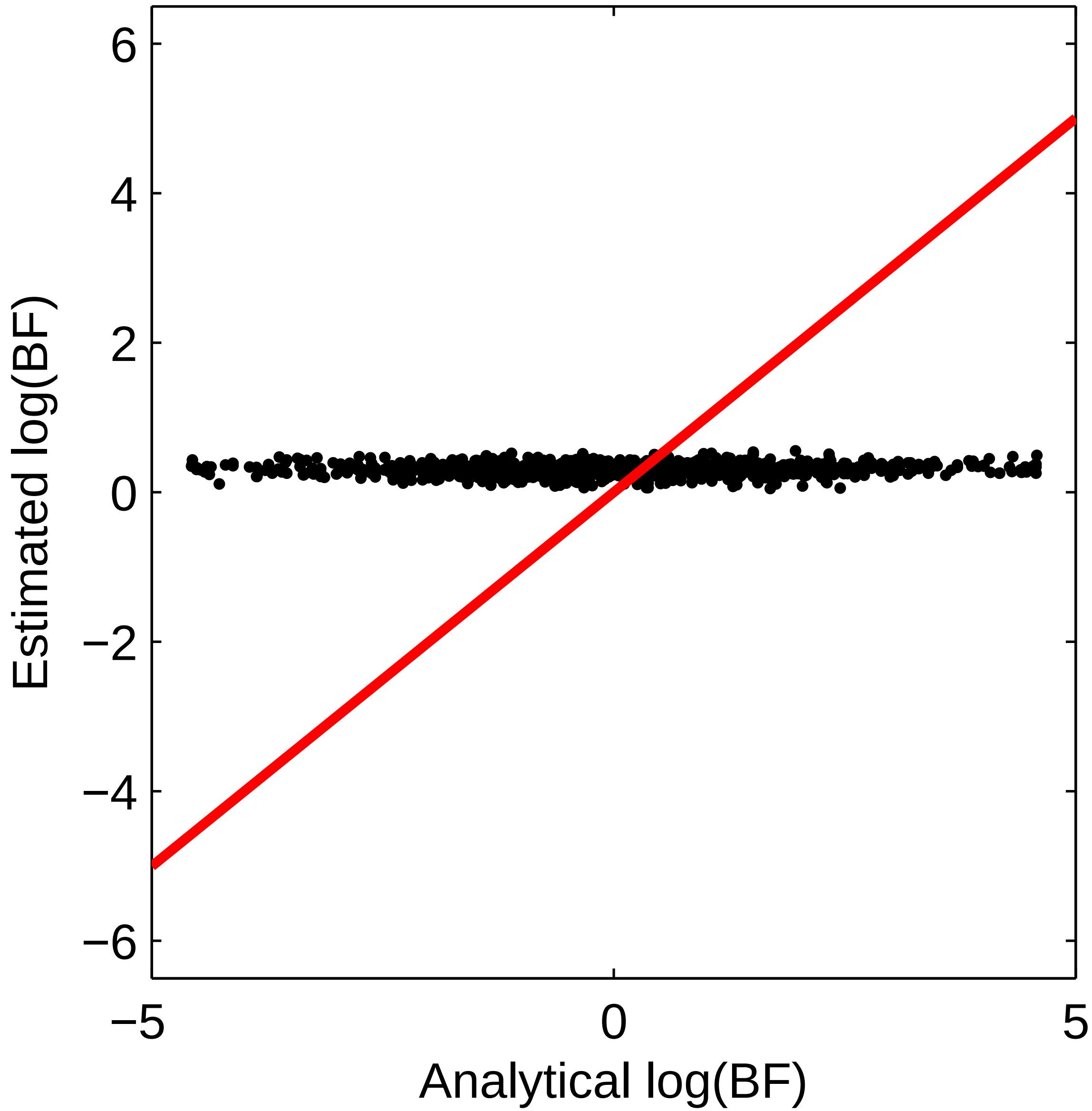
- We have $s_1 = \sum_{i=1}^{n} x_i$ sufficient for both M_1 and M_2
- But s_1 is not sufficient *for comparing* M_1 and M_2

Example using s_1



Theorem

If M_1 and M_2 are both nested in model M, then any summary statistic sufficient for model M is sufficient *for comparing* M_1 and M_2

Proof

$$p(x|M_1) = \int_\theta p(x|\theta, M_1)p(\theta|M_1)d\theta$$

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Similarly $p(x|M_2) = p(x|s(x), M)p(s(x)|M_2)$ and therefore:

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\frac{p(x|M_1)}{p(x|M_2)} = \frac{p(x|s(x), M)p(s(x)|M_1)}{p(x|s(x), M)p(s(x)|M_2)} = \frac{p(s(x)|M_1)}{p(s(x)|M_2)}
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- M reduces to M_2 if we take $\theta_1 = 0, \alpha_1 = 0, \alpha_2 = 1$
- Both M_1 and M_2 are nested within M
- M is an exponential family model for which $[s_1(x), s_2(x), t_1(x), t_2(x)]$ is sufficient.

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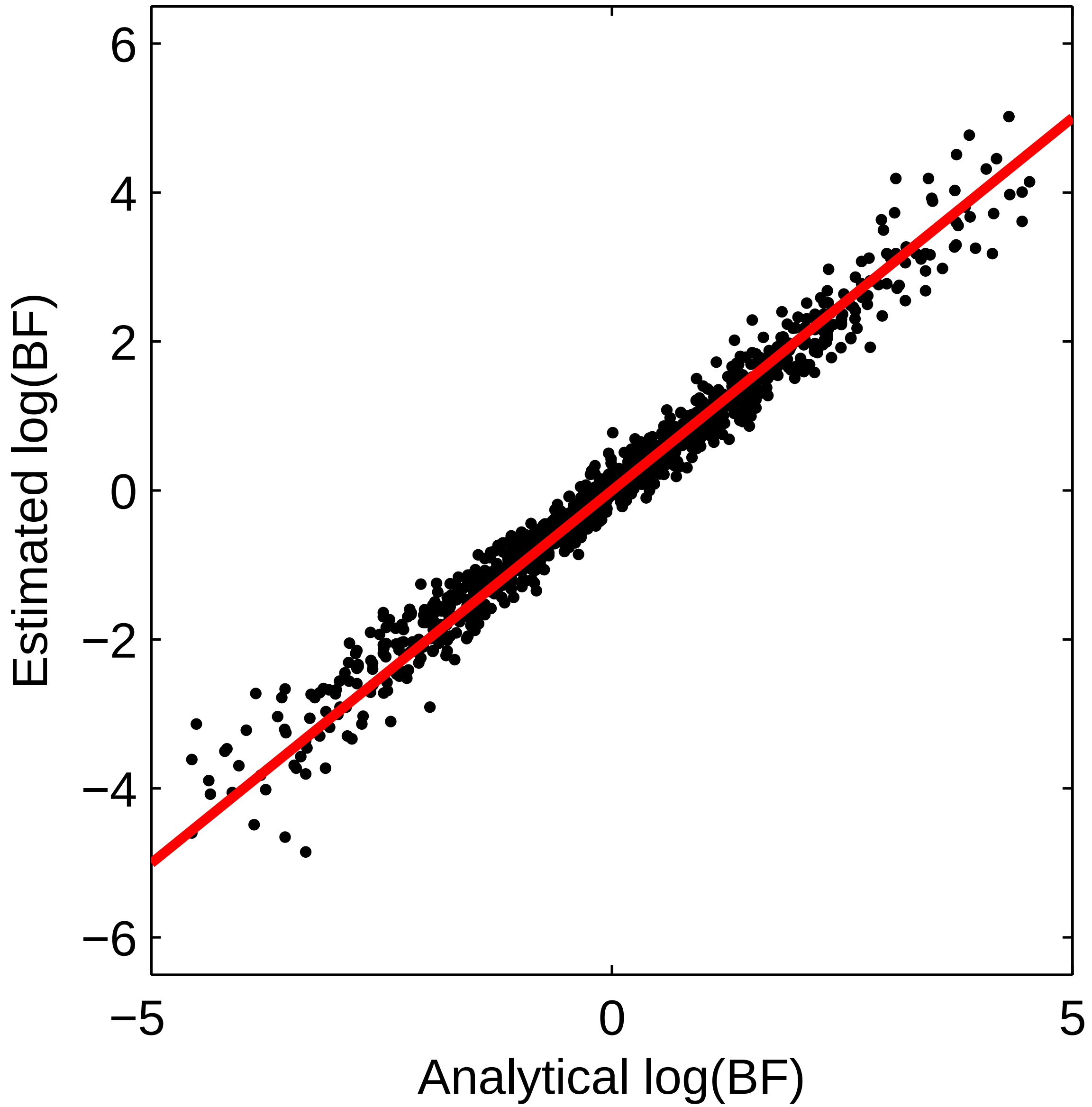
Example using (s_1, s_2)



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- We have described a method for constructing a summary statistic sufficient for comparing models
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- Recent advances in this setting (cf previous talks!) may be useful guides to choosing statistics for the model selection problem