Fish play Minority Game as humans do

Ruey-Tarng Liu, Fei Fang Chung, and Sy-Sang Liaw

Department of Physics, National Chung Hsing University

250 Guo-Kuang Road, 402 Taichung, Taiwan

Previous computer simulations of the Minority Game (MG)[1] have shown that the average agent number in the winning group (i.e., the minority group) had a maximal value such that the global gain was also maximal when an optimal amount of information was available to all agents[2]. This property was further examined[3–8] and its connection to financial markets has also been discussed[9]. Here we report the results of an unprecedented real MG played by university staff members who clicked one of two identical buttons (A and B) on a computer screen while clocking in or out of work. We recorded the number of people who clicked button A for 1288 games, beginning on April 21, 2008 and ending on October 31, 2010, and calculated the variance among the people who clicked A as a function of time. We find that variance per person decreases to a minimum and rises to a value close to 1/4 which is the expected value when agents click buttons randomly. Our results are consistent with previous simulation results[2] for the theoretical MG and suggest that our agents had employed more information for their strategies as their experience playing the game grew. We also carried out another experiment in which we forced 101 fish to enter one of the two symmetric chambers (A and B). We repeated the fish experiment 500 times and found that the variance of the number of fish that entered chamber A also decreased to a minimum and then increased to a saturated value, suggesting that fish have memory[10,11] and can employ more strategies when facing the same situation again and again[12].

The Minority Game (MG) is a simple evolutionary game designed for studying, among other things, how the actions of selfish players can be coordinated by an invisible hand to cooperate for global benefit. In the game, N players have to independently choose one of two sides (**A** and **B)** and those on the minority side win. In a theoretical analysis, one must assume that agents have a fixed number of strategies and a fixed level of memory for the winning history. Under these assumptions, many interesting properties have been discovered[13]. One interesting property that has attracted much attention is that the global gain, defined as the portion of agents who have won the game, is maximal when the agents employ an optimal amount of information[2]. The global gain is related to the variance of the number of players choosing a particular side (e.g., side **A)** in a simple way such that variance is negatively correlated to the extent of global gain. Therefore the value of variance per player $\chi(\mathbf{A})$ decreases to a minimum as the amount of information available to all agents increases, and then increases to a value close to 1/4, indicating that when there is too much information all player choices are effectively random (Figure 1).

Obviously, human players of a real MG do not have a fixed memory and most likely will not use binary tables for strategies as in a theoretical MG model, and understanding of which essential features of the theoretical MG model would remain in a real game could have practical applications. Two reports have been written on real MGs played by humans[14,15] but the numbers of players in these two experiments (15 and 5, respectively) are so small that their conclusions are not statistically significant.

Beginning in April 2008, with the help of the Office of Personnel and the Computer and Information Network Center of National Chung Hsing University (NCHU), we set up a time clock system for university staff to play the MG while clocking in and out of work. All university staff ($N = 200 \sim 600$) were required to click one of two buttons (**A** or **B)** shown on their office work stations to clock in before 08:30 and clock out after 16:45. Each button click was recorded automatically

with instances denoted as $n(\mathbf{A})$ and $n(\mathbf{B})$, to determine the winning button. The general concept of the MG was explained to all staff, and they were instructed by memo how to use the system. Staff members were incentivized to play the MG seriously by giving away gift certificates redeemable at local convenience stores to the top three winners every month. For the first 20 games, we purposely set button $\mathbf{A}$ on the left-hand side of the screen and button $\mathbf{B}$ on the right-hand side. Button $\mathbf{B}$ had won all these games, suggesting that more people tended to click the button on the left-hand side without paying attention to the game. However, the number of winners oscillated and approached $N/2$, indicating that more and more people were playing the game seriously. Starting from April 21, 2008, we switched the positions of buttons $\mathbf{A}$ and $\mathbf{B}$ randomly at every game. By October 31, 2010 a total of 1288 games had been recorded. In the first phase, consisting of the first 458 games, the winning buttons of the previous four games were provided on the screen for reference. In the second phase, consisting of the final 880 games, the winning buttons of previous four games were still provided but these results had been modified without the knowledge of the players in favor of button $\mathbf{B}$ by adding a value $0.09N$ to $n(\mathbf{A})$[16].

We calculated the variance per player $\chi(\mathbf{A})$ and studied its variation as a function of time. Note the number of agents N_i at time t_i is not a constant in our games (Figure 2) since the number of university staff members working on any given day was subject to fluctuation. Thus each $n(\mathbf{A})$ has to be multiplied by a factor $\sqrt{<N>}/N_i$ before calculating its variance, where $<N>$ is the average value of N_i in the time period considered. (See **Methods** for details.) To find the time evolution of $\chi(\mathbf{A})$, we divided the total 1288 games into 18 time periods (with each period marked by a horizontal thick bar in Figure 2) so that each period contains no less than 50 games and N_i does not vary too dramatically within most periods. In phase 1, $\chi(\mathbf{A})$ drops to a minimum and rises back to near 1/4. The evolution of the variance, as compared with the theoretical variance as a function of information in the standard MG (Figure 1), shows the agents employed more information in playing the

game as they gained experience with it. According to the simulation results shown in Figure 1, $\chi(\mathbf{A})$ will saturate to 1/4 when even more information is available to the agents. For our games, since $\chi(\mathbf{A})$ has passed the minimum and reached 1/4, we did not expect it to vary significantly if more games were played the same way. We therefore reactivated the game in phase 2, starting from game #459, by adding 9% of N_i as virtual players who always chose button **A**. The manipulated results were provided as a reference to the agents, who were not aware of the addition of the virtual players. Apparently, the strategies learned from phase 1 did not work in phase 2 and $\chi(\mathbf{A})$ began to increase (Figure 2). It took another 350 games before the agents adapted to the effect of the virtual players, at which time $\chi(\mathbf{A})$ decreased again, this time to a lower minimum than seen in phase 1. A smaller variance means, on average, more agents in each game and so a larger global gain. The increase in global gain by adding persistent virtual players was consistent with what had been previously predicted in the theoretical MG model[16]. After reaching a minimum, $\chi(\mathbf{A})$ increased to near 1/4 and we concluded our human experiment.

We also carried out a separate fish experiment, the experimental setup of which is shown in Figure 3. The Mosquito fish (Gambusia affinis) is a common fish about 3 cm long and is ubiquitous in Taiwan's ponds and rivers. Using a slow-moving mesh, we forced $N = 101$ Mosquito fish in a tank into two symmetric chambers **A** and **B**, lit from overhead by a lamp to reveal their entrances. The whole system was covered by a large piece of black cloth to isolate the fish from environmental cues. Once all the fish had entered a chamber, the number of fish in each chamber was quickly counted and recorded. We then reduced the water volume in the losing chamber so that the 50-plus fish inside were forced into a crowded cluster for a few seconds. Presumably, fish do not like being forced to be in direct contact with one another and would learn[10–12] to enter the other chamber when forced to choose next time. We repeated this fish MG 500 times (a few fish died during the repeated experiments and were replaced by new fish). Variance per fish $\chi(\mathbf{A})$ as a function of time is given in the inset of Figure 2. Qualitatively, the evolution of $\chi(\mathbf{A})$ is similar to that in the human

experiment, and reaches a minimum even faster than in the human case. However, $\chi(\mathbf{A})$ is surprisingly large in the fish experiment, ranging from 0.55 to 1.25. Upon closer investigation, we found that the Mosquito fish have a tendency to form groups when escaping[17]. Assuming that a group averages 3 fish when entering the chambers, the variance per group is then given by one third of the value presented in Figure 2, thus its value begins with 0.4, drops to a minimum of 0.19, and then saturates to 1/4, a pattern very similar to the human case.

Methods

In the human game, we divided the 1288 games into 18 samples such that the number of games L in each sample is no less than 50 while minimizing the variance of N_i in each sample. To calculate the variance of the number of agents choosing button $\mathbf{A}$, we modified the value $n(\mathbf{A})$. While pretending that all games in a given sample were played by $<N>$ agents, we multiplying it by the factor $<N>/N_i$, where $<N>$ is the mean of N_i in the sample $<N>=\frac{1}{L}\sum_{i=1}^{L} N_i$. Since the standard deviation of a sample is proportional to the square root of the sample size, we compared variance per agent between different samples by multiplying $n(\mathbf{A})$ by another factor $1/\sqrt{<N>}$. Thus the actual value used in calculating variance per agent $\chi(\mathbf{A})$ of the number of agents choosing button $\mathbf{A}$ **was** $n(\mathbf{A})\sqrt{<N>}/N_i$.

We have checked for possible errors due to the inconstancy of N_i using random samples of length L. Take $<N>=400$ and $N_i=<N>+r_i$, where r_i is a random integer between -100 and 100. We used $n(\mathbf{A})\sqrt{<N>}/N_i$ to calculate the variance per agent $\chi(\mathbf{A})$ and found that the random fluctuation of r_i has little effect on $\chi(\mathbf{A})$. The average value of $\chi(\mathbf{A})$ is close to 1/4 as expected. The standard deviation[18] of $\chi(\mathbf{A})$ is inversely proportional to $\sqrt{L}$. For $L=50$, the standard deviation is close to 0.05.

References

1. Challet, D. & Zhang, Y. C. Emergence of cooperation and organization in an evolutionary game. *Physica A* **246**, 407-418 (1997).

2. Savit, R., Manuca, R. & Riolo, R. Adaptive competition, market efficiency, and phase transition. *Phys. Rev. Lett.* **82**, 2203-2206 (1999).

3. Hart, M., Jefferies, P., Hui, P. M. & Johnson, N. F. Crowd-anticrowd theory of multi-agent market games. *Eur. Phys. J.* **B20**, 547-550 (2001).

4. Cavagna, A. Irrelevance of memory in the minority game. *Phys. Rev. E* **59**, R3783-R3768 (1999).

5. Challet, D. & Marsili, M. Phase transition and symmetry breaking in the minority game. *Phys. Rev. E* **60**, R6271-R6274 (1999).

6. Challet, D., Marsili, M. & Zecchina, R. Statistical mechanics of systems with heterogeneous agents: minority games. *Phys. Rev. Lett.* **84**, 1824-1827 (2000).

7. Liaw, S.-S. & Liu, C. The quasi-periodic time sequence of the population in minority game. *Physica A* **351**, 571-579 (2005).

8. Liaw, S.-S., Hung, C.-H. & Liu, C. Three phases of the minority game. *Physica* A **374**, 359-368 (2007).

9. Farmer, J. D. Physicists attempt to scale the ivory towers of finance. *Comput. Sci. Eng.*, 26-39 (1999).

10. Brown, C. Familiarity with the test environment improves escape responses in the crimson spotted rainbowfish, *Melanotaenia duboulayi. Anim. Cogn.* **4**, 109-113 (2001).

11. Warburton, K. Learning of foraging skills by fish. *Fish and Fisheries* **4**, 203-215 (2003).

12. Kendal, J. R., Rendell, L., Pike, T. W. & Laland, K. N. Nine-spined sticklebacks deploy a hill-climbing social learning strategy. *Behavioral Ecology* **20**, 238-244 (2009).

13. Challet, D., Marsili, M. & Zhang, Y.-C. *Minority Games*, Oxford (2005).

14. Platkowski, T. & Ramsza, M. Playing minority game. *Physica A* **323**, 726-734 (2003).

15. Bottazzi , G. & Devetag, G. A laboratory experiment on the minority game. *Physica A* **324**, 124-132 (2003).

16. Liaw, S.-S. Maximise global gain in the Minority game. *Res. Lett. Phys.*, ID410950 (2009).

17. Bisazza, A. & Marin, G. Sexual selection and sexual size dimorphism in the eastern mosquitofish *Gambusia holbrooki* (Pisces: Poeciliidae). *Ethology, Ecology and Evolution* **7**, 169–183 (1995).

18. Cooper, G. R. & McGillem, C. D. *Probabilistic methods of signal and system analysis*, Oxford (1998).

Correspondence and requests for materials should be addressed to S.-S. Liaw (liaw@phys.nchu.edu.tw)

Supplementary Information
1. Human MG records (file: HumanMG.txt)
2. Fish MG records (file: FishMG.txt)

Figure 1

Theoretical variance per agent $\chi(\mathbf{A})$ as a function of information $\beta = \log(2^M / N)$ in the

standard MG, where *M* stands for the memory length in the theoretical model. The dashed line

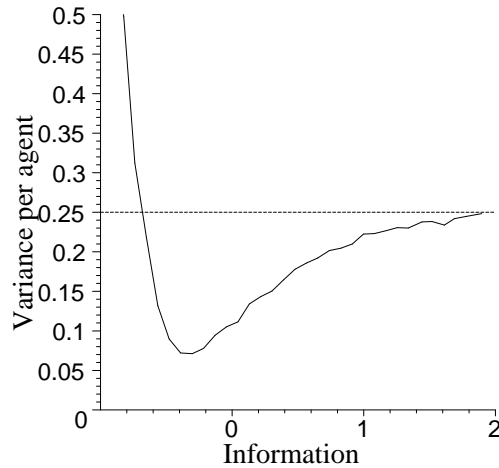indicates the variance value 1/4 for the case of random choices.

Figure 2

Evolution of the variance per agent $\chi(\mathbf{A})$ for human MG and fish MG (inset). In the fish experiment,

$N = 101$. In the human MG, the number N_i of players (dot) was not constant. In both phases

(diamond for phase 1, circle for phase 2), the variance per agent was qualitatively similar: decreasing to

a minimum and then increasing to near the value 1/4. Variances per agent were calculated for periods

(indicated by solid bars) each containing more than 50 games. The dashed line indicates the variance

value 1/4 for the case of random choices. Mosquito fish aggregate into groups, each containing on

average 3 fish so that its $\chi(\mathbf{A})$ is three times greater than the results of the human MG.
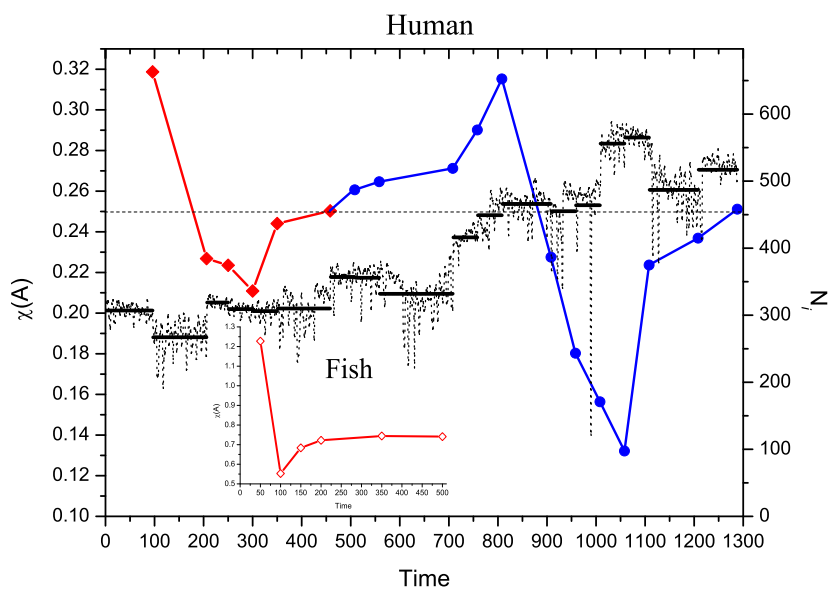
Figure 3

Fish MG experimental setup, in which 101 Mosquito fish in tank **C** were forced by a moving mesh to enter either chamber **A** or **B** which were lighted by a lamp above. Tank **C** was covered by a piece of black cloth to isolate the fish from environmental cues.