Pressure buildup during CO_2 injection in brine aquifers using the Forchheimer equation

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Abstract

If geo-sequestration of CO_2 is to be employed as a key emissions reduction method in the global effort to mitigate climate change, simple yet robust screening of the risks of disposal in brine aquifers will be needed. There has been significant development of simple analytical and semi-analytical techniques to support screening analysis and performance assessment for potential carbon sequestration sites. These techniques have generally been used to estimate the size of CO_2 plumes for the purpose of leakage rate estimation. A common assumption has been that both the fluids and the geological formation are incompressible. Consequently, calculation of pressure distribution requires the specification of an arbitrary radius of influence.

In this talk, a new similarity solution is derived using the method of matched asymptotic expansions. By allowing for slight compressibility in the fluids and formation, the solution improves on previous work by not requiring the specification of an arbitrary radius of influence. A large-time approximation of the solution is then extended to account for non-Darcy inertial effects using the Forchheimer equation. Both solutions are verified by comparison with finite difference solutions. The results show that inertial losses will often be comparable, and sometimes greater than, the viscous Darcy-like losses associated with the brine displacement, although this is strongly dependent on formation porosity and permeability.

Introduction

- Currently there is a great interest in geo-sequestration of CO_2.
- This involves capturing CO_2 at the point of generation, compressing it to a supercritical fluid, and then sequestering it at depth within a suitable permeable geological formation.
- There has been significant development of simple analytical and semi-analytical techniques to support screening analysis and performance assessment for potential carbon sequestration sites.
- These have generally been used to estimate the size of CO_2 plumes for the purpose of leakage rate estimation.
- A common assumption is that both the fluids and formation are incompressible.
- Consequently, calculation of pressure distribution requires the specification of an arbitrary radius of influence.
- In this presentation we improve on previous work by allowing for slight compressibility in the fluids and formation and accounting for inertial effects by applying the Forchheimer equation.

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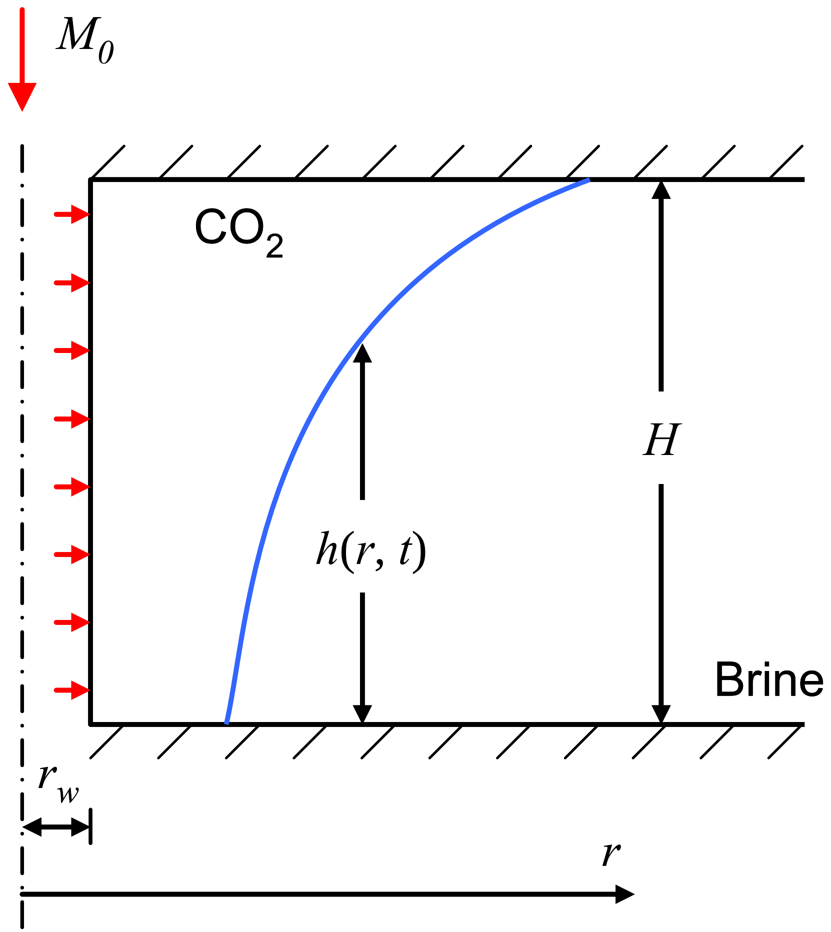
Key assumptions

- Following Nordbotten et al. (2005) we consider a fluid pressure, p [$ML^{-1}T^{-2}$] that includes an assumption of negligible capillary pressure, and which applies over the entire thickness of a confined porous formation of vertical extent H [L].
- The CO_2 and brine are assumed to be separated by a sharp interface, located at an elevation h [L] above the base of the formation.
- The CO_2 zone is fully saturated with CO_2 whilst the brine zone is fully saturated with brine.



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Boundary value problem

Continuity equation for the CO_2

$$\frac{\partial}{\partial t}\left[\phi\rho_o(H-h)\right]=-\frac{1}{r}\frac{\partial}{\partial r}\left[r\rho_o(H-h)q_o\right]$$

Continuity equation for the brine

$$\frac{\partial}{\partial t}(\phi\rho_w h)=-\frac{1}{r}\frac{\partial}{\partial r}(r\rho_w h q_w)$$

where :

q_o = volumetric flux of CO_2 [LT^{-1}]

q_w = volumetric brine of CO_2 [LT^{-1}]

ρ_o = density of CO_2 [ML^{-3}]

ρ_w = density of brine [ML^{-3}]

ϕ = porosity [-]

Boundary and initial conditions :

$$\begin{array}{lll}
p=0, & r\geq 0, & t=0 \\
p=0, & r\rightarrow\infty, & t>0 \\
rq_o=M_0/(2\pi H\rho_o), & r=r_w, & t>0 \\
h=H, & r\geq 0, & t=0 \\
h=H, & r\rightarrow\infty, & t>0 \\
rq_w=0, & r=r_w, & t>0
\end{array}$$

The Forchheimer equation

Inertial effects are incorporated through the Forchheimer equation.

For low fluxes these reduce to Darcy's law.

Flux equation for CO_2

$$\frac{\mu_o}{k} q_o + b\rho_o q_o |q_o| = -\frac{\partial p}{\partial r}$$

0

Flux equation for brine

$$\frac{\mu_w}{k} q_w + b\rho_w q_w |q_w| = -\frac{\partial p}{\partial r}$$

0

where :

$b =$ Forchheimer parameter [L^{-1}]

$k =$ permeability [L^2]

$p =$ pressure [$ML^{-1}T^{-2}$]

$\mu_o =$ viscosity of CO_2 [$ML^{-1}T^{-1}$]

$\mu_w =$ viscosity of brine [$ML^{-1}T^{-1}$]

$\rho_o =$ density of CO_2 [ML^{-3}]

$\rho_w =$ density of brine [ML^{-3}]

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Key parameters

Dimensional analysis reveals that there are three important dimensionless groups:

$$\alpha = \frac{M_0 \mu_o (c_r + c_w)}{2\pi H \rho_o k} \qquad \beta = \frac{M_0 k b}{2\pi H r_w \mu_o} \qquad \gamma = \frac{\mu_o}{\mu_w}$$

compressibility parameter

inertial parameter

viscosity ratio

$b =$ Forchheimer parameter [L^{-1}]
$c_r =$ compressibility of formation [$M^{-1}LT^2$]
$c_w =$ compressibility of brine [$M^{-1}LT^2$]
$H =$ formation thickness [L]
$k =$ permeability [L^2]
$M_0 =$ mass injection rate [MT^{-1}]
$\mu_o =$ viscosity of CO_2 [$ML^{-1}T^{-1}$]
$\mu_w =$ viscosity of brine [$ML^{-1}T^{-1}$]
$\rho_o =$ density of CO_2 [ML^{-3}]
$\rho_w =$ density of brine [ML^{-3}]

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Solution strategy for $\beta = 0$

- Assuming no inertia (i.e. $\beta = 0$) and an infinitesimal well (i.e. $r_w \to 0$) then allows application of the Boltzmann transform ($x = r^2 / t$).
- The problem then reduces to two coupled ordinary differential equations.
- Expanding the dependent variables about α and assuming $\alpha << 1$ then leads to two simplified and solvable problems. One for the near-field and for far-field.
- These are then joined using the method of matched asymptotic expansion.

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The small α approximation

The CO_2-brine interface (after Noordbotten et al., 2006)

$$h_D = \begin{cases} 0, & x \le 2\gamma \\ \dfrac{(2\gamma/x)^{1/2} - 1}{\gamma - 1}, & 2\gamma < x < 2/\gamma \\ 1, & x \ge 2/\gamma \end{cases}$$

- $x \le 2\gamma$: CO_2 only
- $2\gamma < x < 2/\gamma$: 2-phase region
- $x \ge 2/\gamma$: brine region

The pressure buildup (new result)

$$p_D = \begin{cases} -\dfrac{1}{2}\ln\left(\dfrac{x}{2\gamma}\right) - 1 + \dfrac{1}{\gamma} - \dfrac{1}{2\gamma}\left[\ln\left(\dfrac{\alpha}{2\gamma^2}\right) + 0.5772\right], & x \le 2\gamma \\ -\left(\dfrac{x}{2\gamma}\right)^{1/2} + \dfrac{1}{\gamma} - \dfrac{1}{2\gamma}\left[\ln\left(\dfrac{\alpha}{2\gamma^2}\right) + 0.5772\right], & 2\gamma < x < \dfrac{2}{\gamma} \\ -\dfrac{1}{2\gamma}\left[\ln\left(\dfrac{\alpha x}{2\gamma}\right) + 0.5772\right], & x \ge \dfrac{2}{\gamma} \end{cases}$$

$$h_D = h/H$$

$$r_D = r/r_w$$

$$t_D = \frac{M_0 t}{2\pi\phi H r_w^2 \rho_o}$$

$$x = r_D^2 / t_D$$

$$p_D = \frac{2\pi H \rho_o k p}{M_0 \mu_o}$$

$$\alpha = \frac{M_0 \mu_o (c_r + c_w)}{2\pi H \rho_o k}$$

$$\beta = \frac{M_0 k b}{2\pi H r_w \mu_o}$$

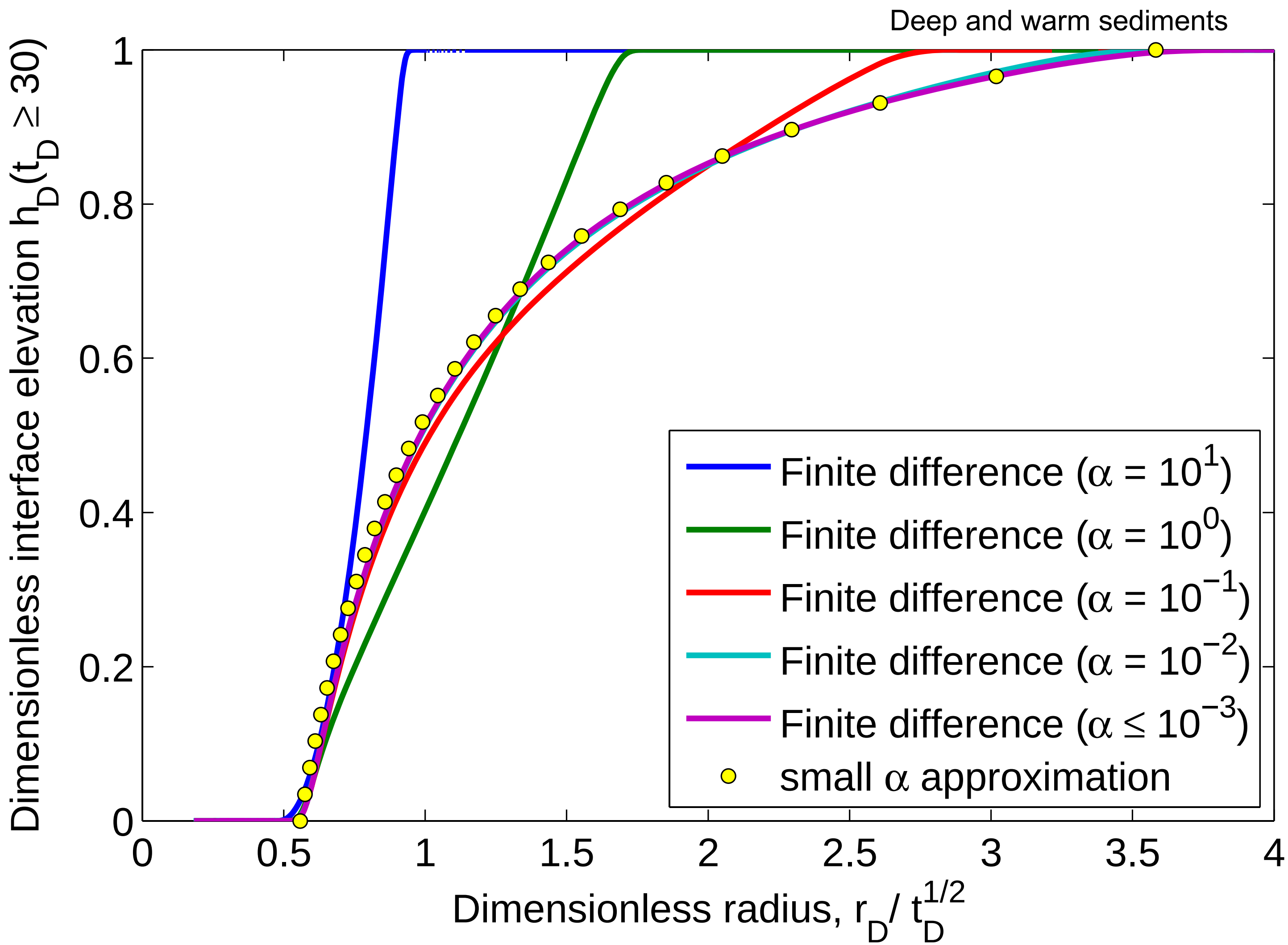
$$\gamma = \frac{\mu_o}{\mu_w}$$

Viscosity and density

Fluid properties of CO_2 and brine phases, representing the range of subsurface conditions (temperature and pressure) found in continental sedimentary basins (after Gasda et al., 2008).

Parameter	Deep and warm	Shallow and cold
μ_o (Pa.s)	0.395×10^{-4}	0.577×10^{-4}
μ_w (Pa.s)	2.535×10^{-4}	11.875×10^{-4}
ρ_o (kg/m^3)	479	741
ρ_w (kg/m^3)	1045	1121

The CO_2-brine interface



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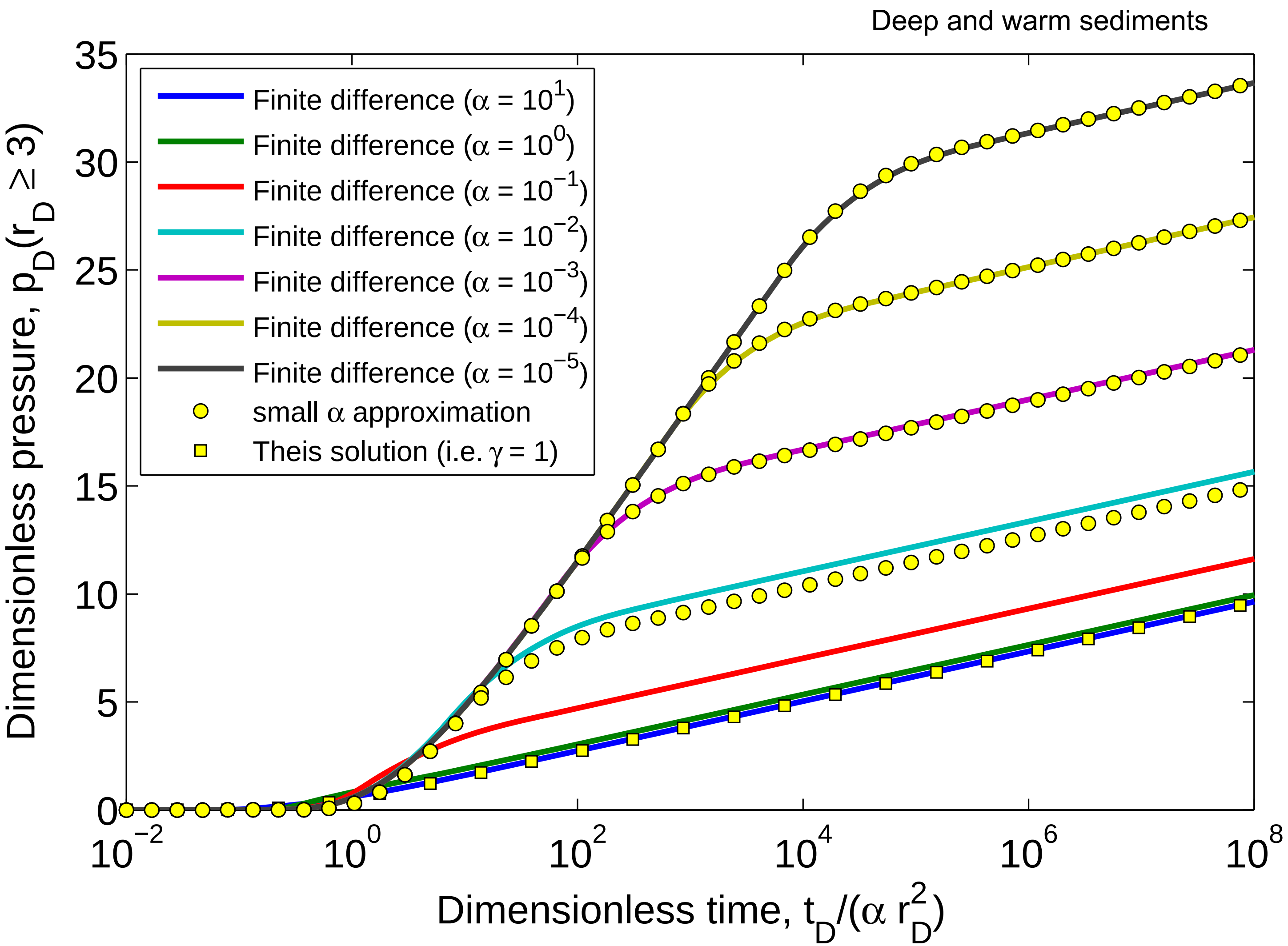
Pressure buildup



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Solution strategy for $\beta > 0$

- In the far-field, flow velocities are greatly reduced and therefore the system behaves similar to when $\beta = 0$.
- In fact the far-field is characterised by the previous equation.
- There is an inner region where the system is effectively at steady state.
- Matching the asymptotic expansions leads to the large-time approximation

$$p_D \approx -\frac{1}{2}\ln\left(\frac{x}{2\gamma}\right) - 1 + \frac{1}{\gamma} - \frac{1}{2\gamma}\left[\ln\left(\frac{\alpha}{2\gamma^2}\right) + 0.5772\right] + \frac{\beta}{r_D}$$

$$h_D = h / H$$

$$r_D = r / r_w$$

$$t_D = \frac{M_0 t}{2\pi\phi H r_w^2 \rho_o}$$

$$x = r_D^2 / t_D$$

$$p_D = \frac{2\pi H \rho_o k p}{M_0 \mu_o}$$

$$\alpha = \frac{M_0 \mu_o (c_r + c_w)}{2\pi H \rho_o k}$$

$$\beta = \frac{M_0 k b}{2\pi H r_w \mu_o}$$

$$\gamma = \frac{\mu_o}{\mu_w}$$

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Pressure buildup

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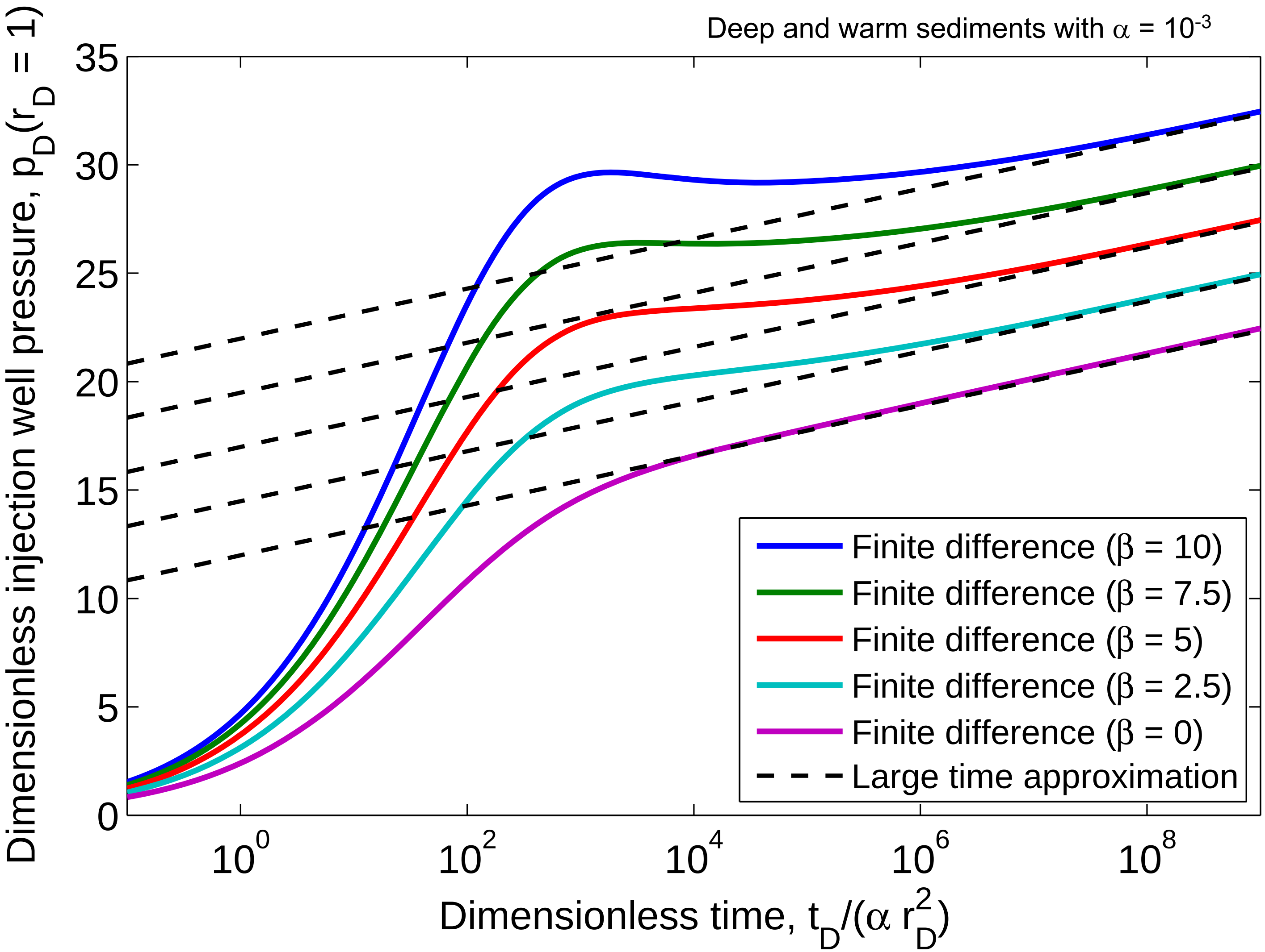

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Pressure losses

The large time approximation for pressure buildup can be rearranged to get

$$p_D \approx -\frac{1}{2}\left[\ln\left(\frac{\alpha x}{4}\right)+0.5772\right]-\frac{1}{2\gamma}\left[\ln\left(\frac{1}{\gamma^3}\left(\frac{\alpha\gamma}{2}\right)^{1-\gamma}\right)+1.4228(\gamma-1)\right]+\frac{\beta}{r_D}$$

Viscous pressure loss in brine
(i.e. Cooper and Jacob, 1946)

L_1 - Viscous pressure loss in CO_2

L_2 - Inertial pressure loss in CO_2
(similar to Wu, 2002)

Possible scenarios

Zhou et al. (2008) consider a typical scenario:

- Mass injection rate, M_0 = 120 kg/s
- Rock compressibility, c_r = 4.50E-10 Pa^{-1}
- Brine compressibility, c_w = 3.50E-10 Pa^{-1}
- Aquifer thickness, H = 125 m
- Well radius, r_w = 0.1 m

For deep and warm sediments

k (m^2)	ϕ	α	L_1 (viscous loss)	L_2 (inertial loss, i.e. β)
1E-12	0.10	1.01E-05	24.0	61.2
1E-14	0.10	1.01E-03	11.6	6.1
1E-12	0.20	1.01E-05	24.0	1.4
1E-14	0.20	1.01E-03	11.6	0.1

Strong dependence on porosity is due to the Geertsma (1974) correlation, $b = 0.005\phi^{-5.5}k^{-0.5}$

Summary and conclusions

- New analytical solutions have been presented to estimate pressure buildup during CO_2 injection in brine aquifers.
- These improve on previous work by accounting for compressibility and inertial effects.
- It was found that for large times the pressure contribution due to viscous and inertial losses in the CO_2 plume become constant.
- Furthermore, it was found that inertial losses are likely to be comparable and sometimes greater than those associated with the two-phase displacement.
- The new solutions are easy to code up in spreadsheet software and should greatly aid fast and cost-effective screening to quickly identify sites suitable for the injection procedure.
- Look out for:

  - S. A. Mathias, A. P. Butler, H. Zhan (2008) Approximate solutions for Forchheimer flow to a well. *ASCE Journal of Hydraulic Engineering* 134(9): 1318-1325. doi:10.1061/(ASCE)0733-9429(2008)134:9(1318)
  - S. A. Mathias, P. E. Hardisty, M. R. Trudell, R. W. Zimmerman (2008) Approximate solutions for pressure buildup during CO_2 injection in brine aquifers. *Transport in Porous Media*. doi:10.1007/s11242-008-9316-7