Multi-site precipitation downscaling via an expanded conditional density network

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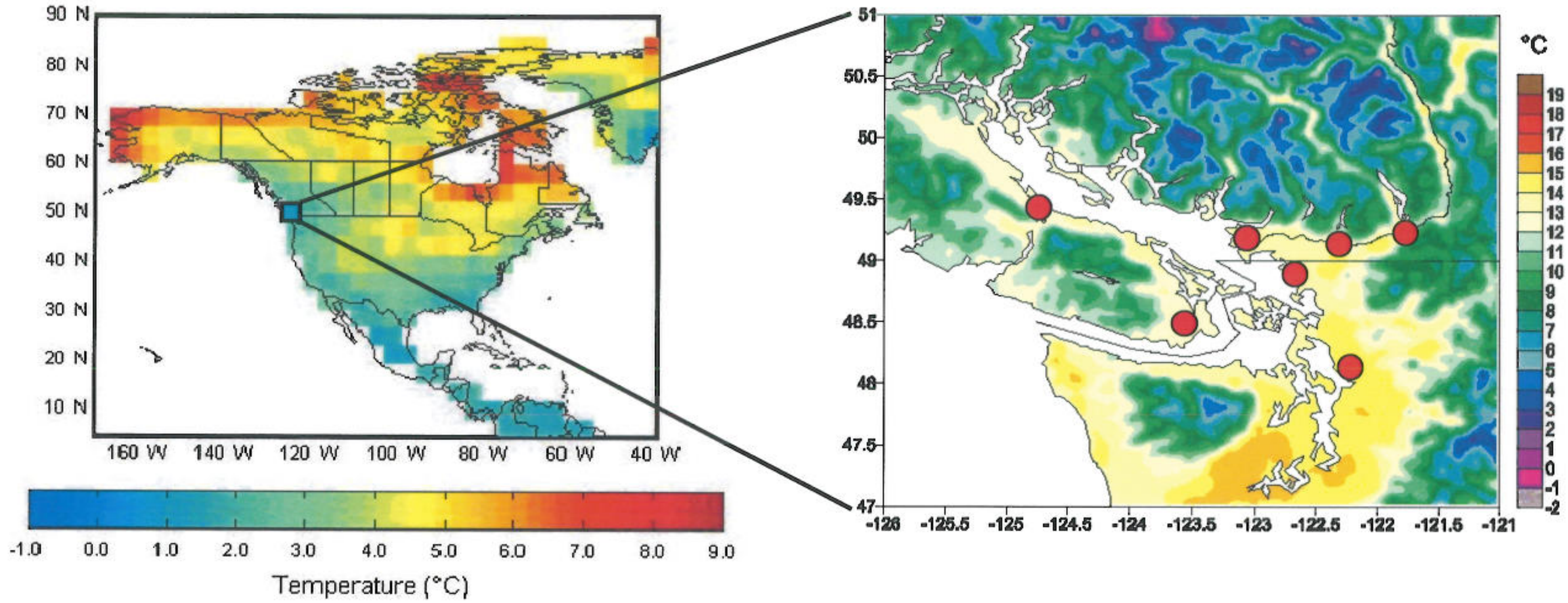
Outline

1. Downscaling
2. Neural network vs. Density Network
3. Precipitation
4. Multi-site precipitation
5. Expanded downscaling
6. Expanded Poisson-gamma density network
7. Application to stations in Kootenay region

Climate Scenarios & the Need for Downscaling



Global Climate Model (GCM)

~ 100-1000 km

Impact Assessments

~ 1 km or at station(s)

Mismatch in Scales

Solution - Downscaling

Global Climate Model

Downscaling ⇒ **Develop intermediate downscaling model to bridge the gap in scales**

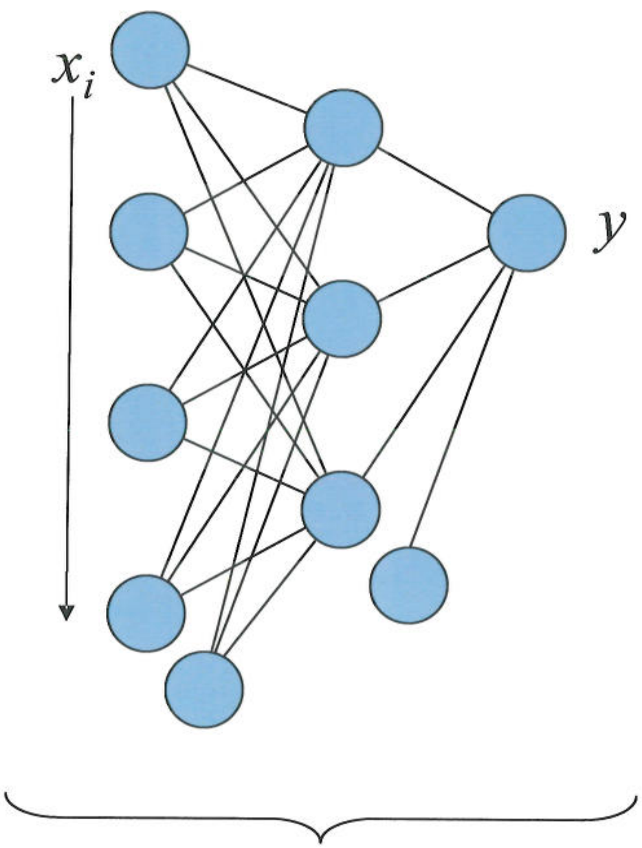
1. **Dynamical model**
2. **Statistical model**
   - linear regression
   - neural network

   etc.

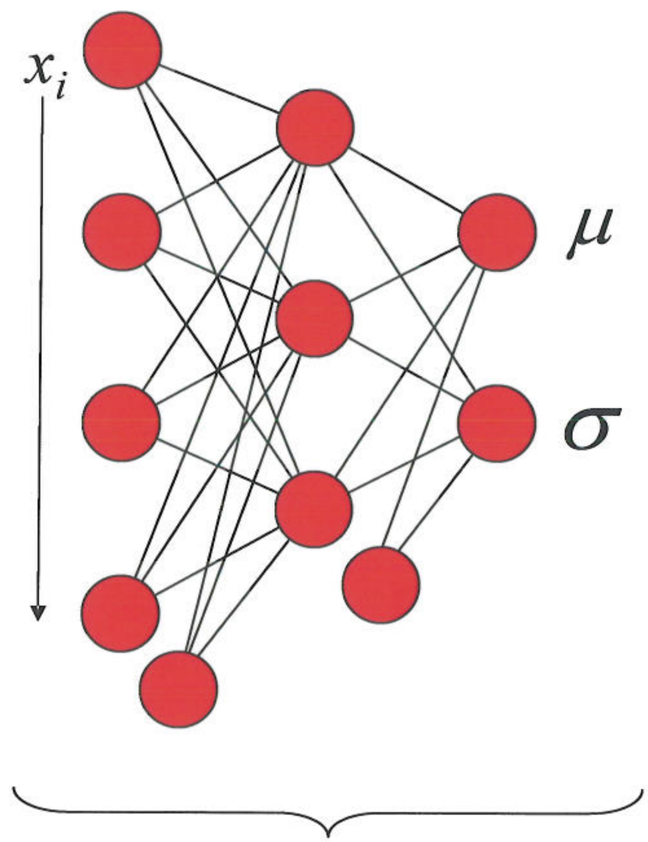
Impact Assessment

Neural Network versus Density Network

e.g.,



conditional mean, constant variance
(homoscedastic)

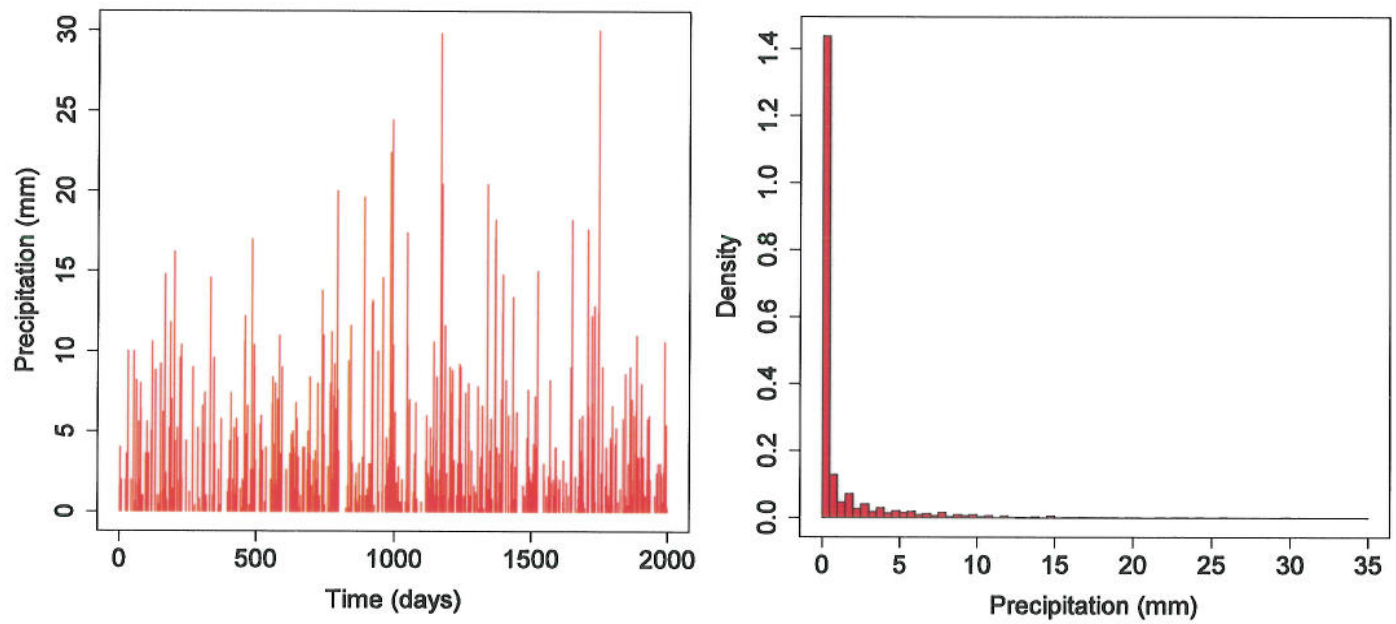


conditional mean & variance
(heteroscedastic)

→ **normally distributed variables like temperature**

Properties of Precipitation – Implications for Modelling

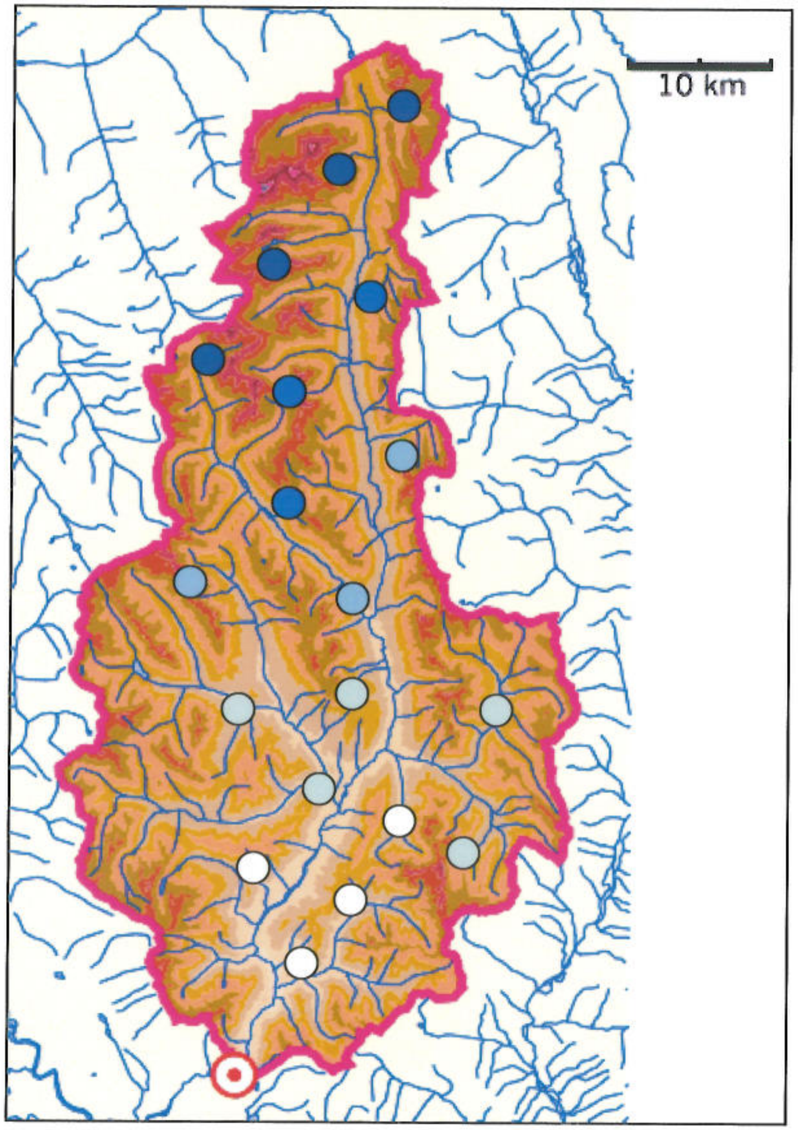
- discontinuous in space and time
- non-Gaussian distribution



→ separate models for **(1)** precipitation occurrence,
(2) transformed amounts

→ application to multi-site precipitation?

Problem - Spatial Relationships Between Stations



Even with a multi-site model:

- inconsistency between downscaled and observed spatial relationships between stations

Implications for watershed modelling:

- events may be too wet/dry, patterns/gradients wrong

Solution: Change the criterion used to calibrate the model

Downscaling → **Expanded Downscaling** (Bűrger, 1996)

$$C = \underbrace{\sum_{t=1}^{N}\sum_{k=1}^{K}\left(y_k(t) - y_k^{obs}(t)\right)^2}_{\text{Error term}} + \alpha \underbrace{\sum_{i=1}^{K}\sum_{j=i+1}^{K}}_{\text{Covariance constraint}}\left(\mathrm{cov}(y_i, y_j) - \mathrm{cov}(y_i^{obs}, y_j^{obs})\right)^2$$

Error term

Covariance constraint

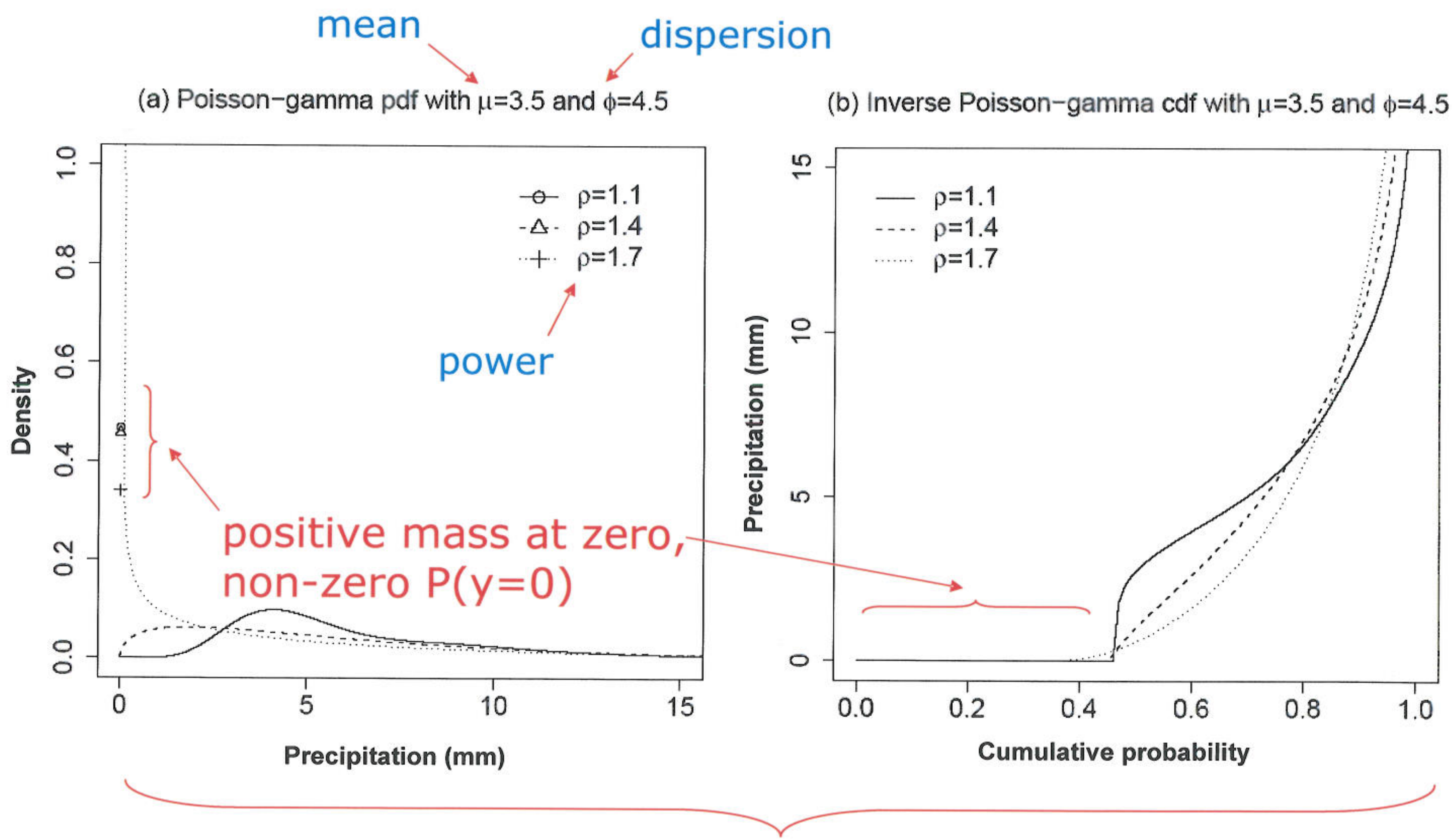
Preserves linear relationships between stations

Putting It All Together

1. Probability distribution (discrete & continuous)
2. Density network model
3. Expanded downscaling calibration criterion

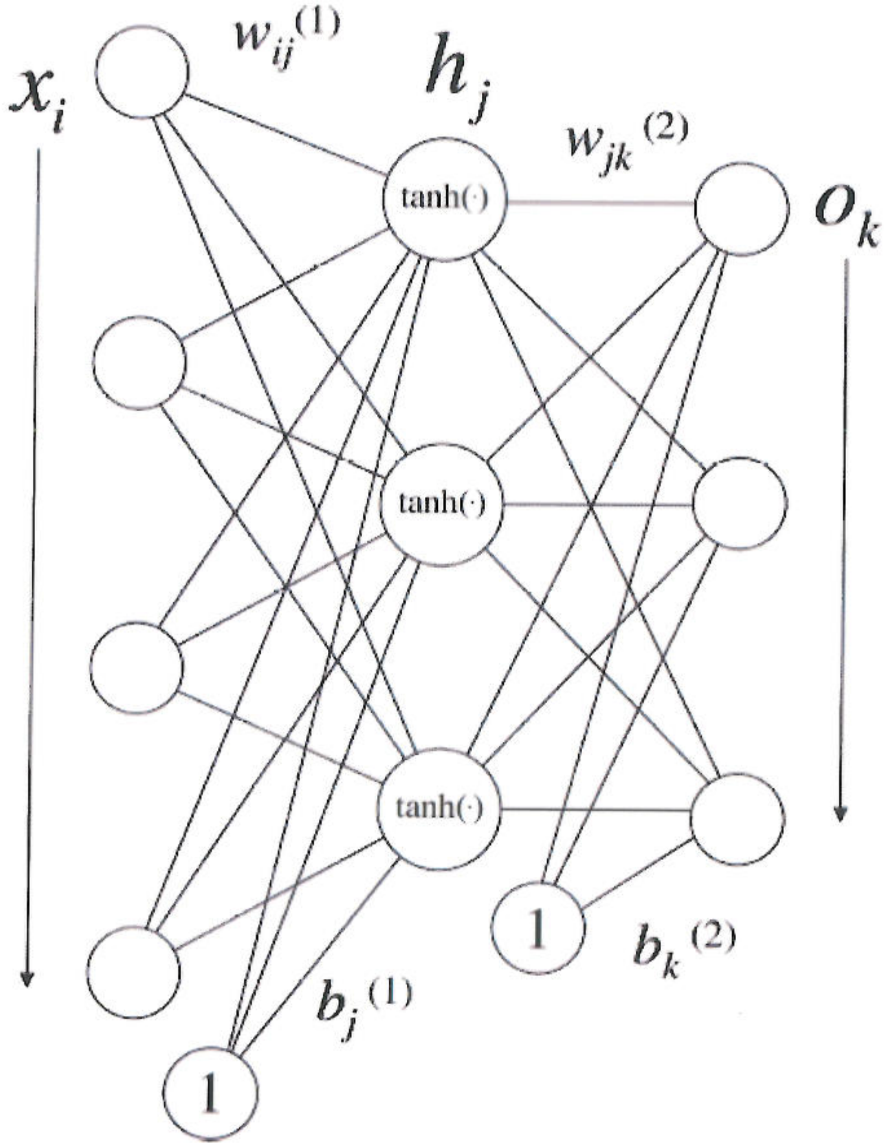
Probabilistic Multi-site Precipitation Downscaling

1) Poisson-gamma Probability Distribution



Model precipitation **occurrence** & **amount** at the same time

2) Poisson-gamma Density Network



→ $\mu = \exp(o_1)$ (mean)

→ $\rho = 1 + \dfrac{1}{1+\exp(o_2)}$ (power)

→ $\phi = \exp(o_3)$ (dispersion)

Fit via maximum likelihood

Using a Density Network to Predict Precipitation

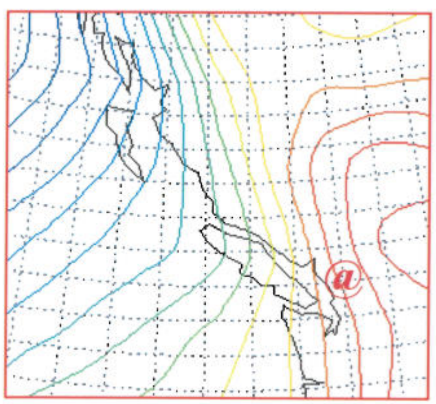
Density Network | Sample from Distribution

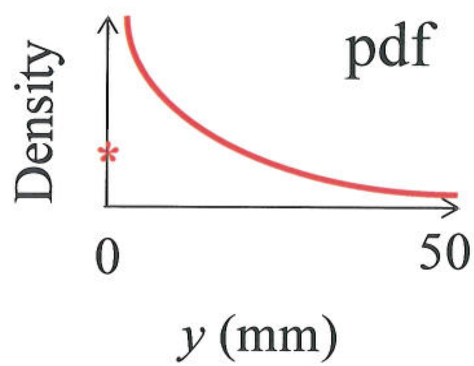
$$\mathbf{x}(t) \xrightarrow{\text{Density Network}} [\mu(t), \rho(t), \phi(t)] \xrightarrow{\text{Sample from Distribution}} y(t)$$

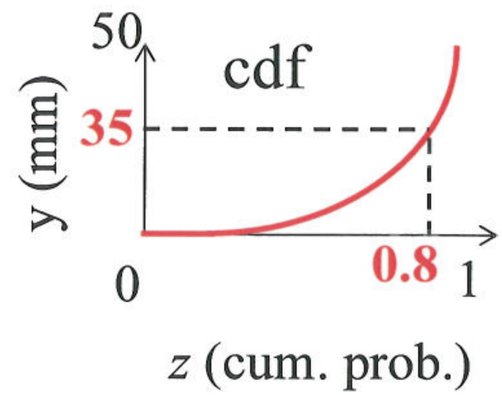
Model Inputs

Poisson-Gamma Parameters

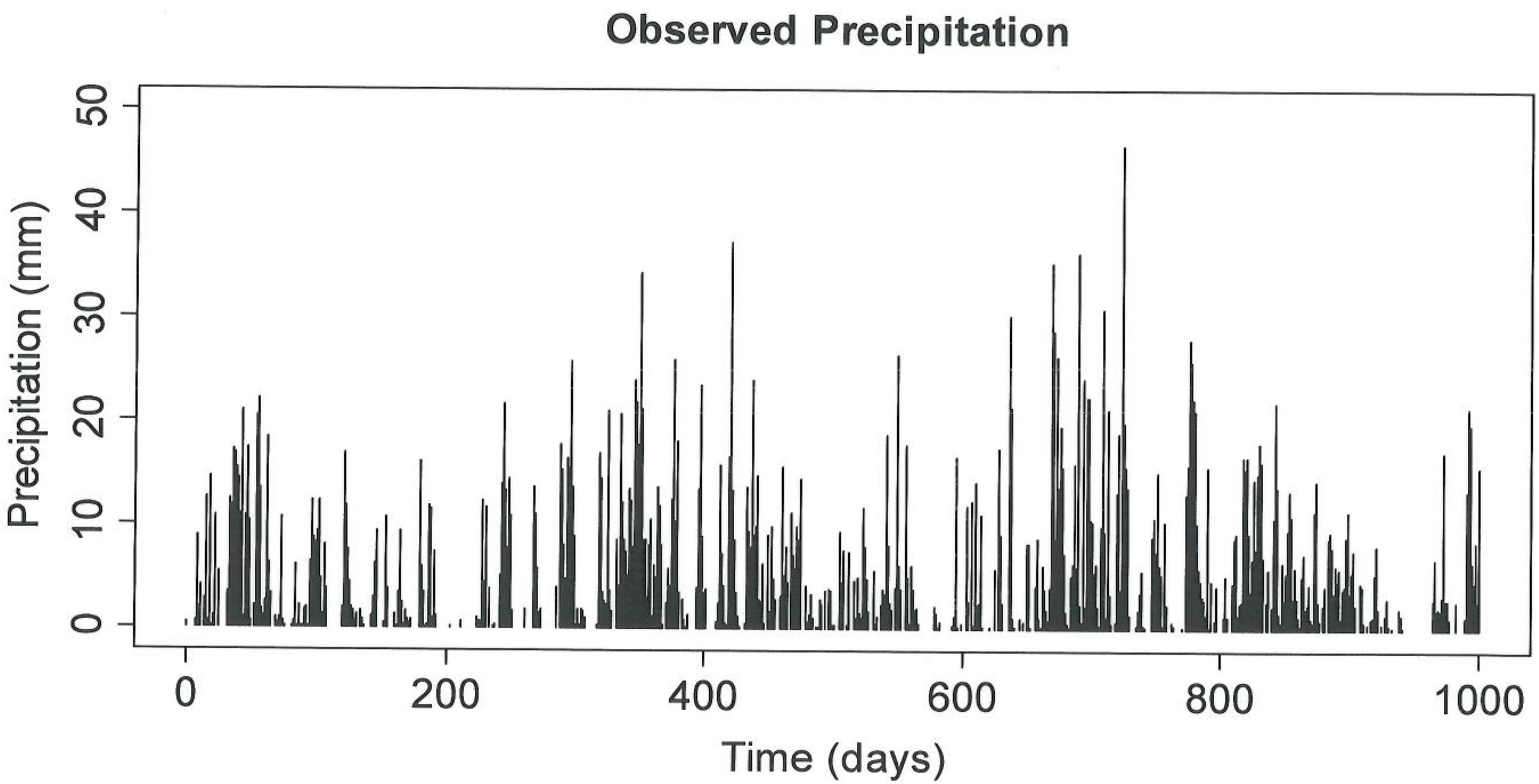
Precipitation (mm)





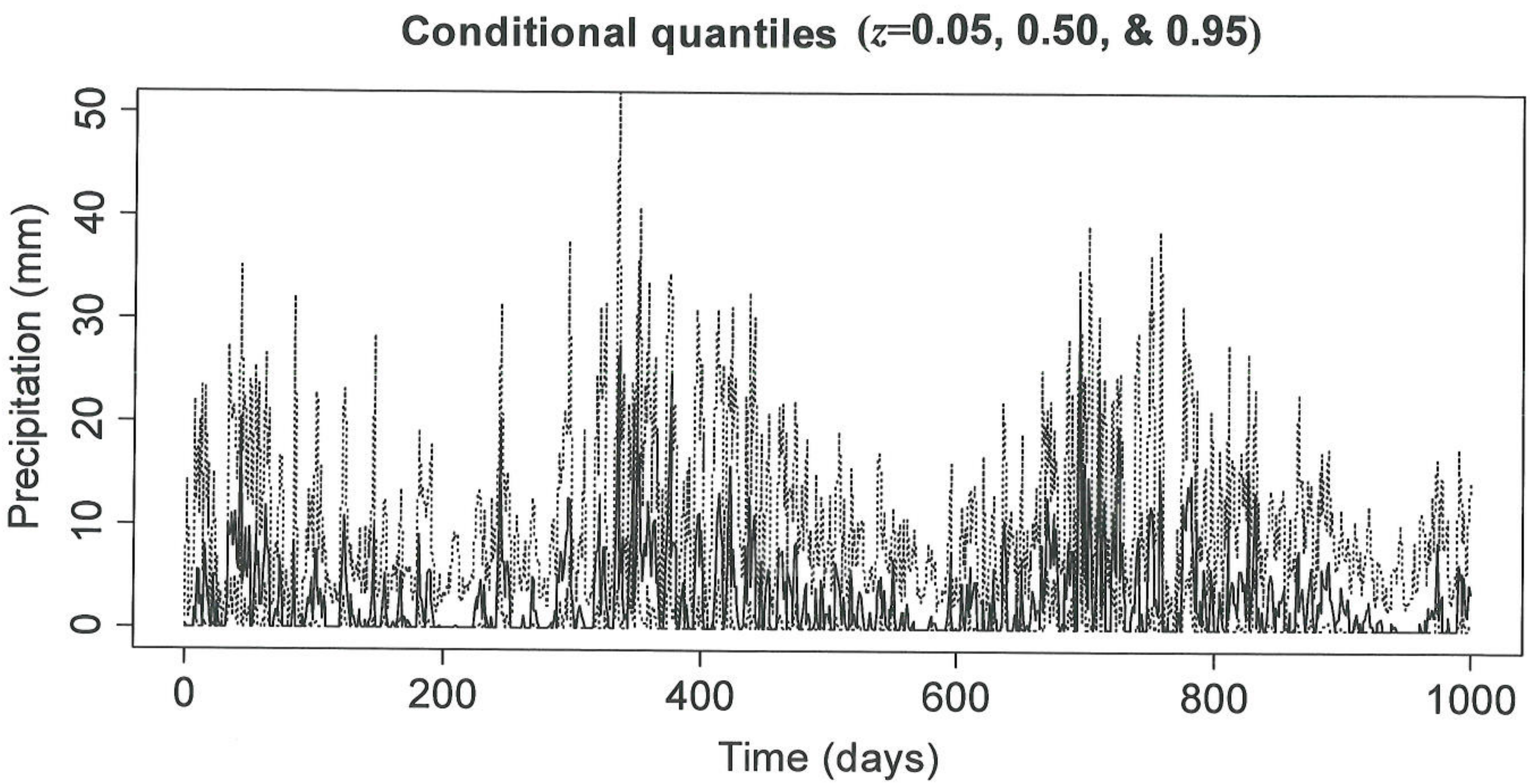


e.g.,



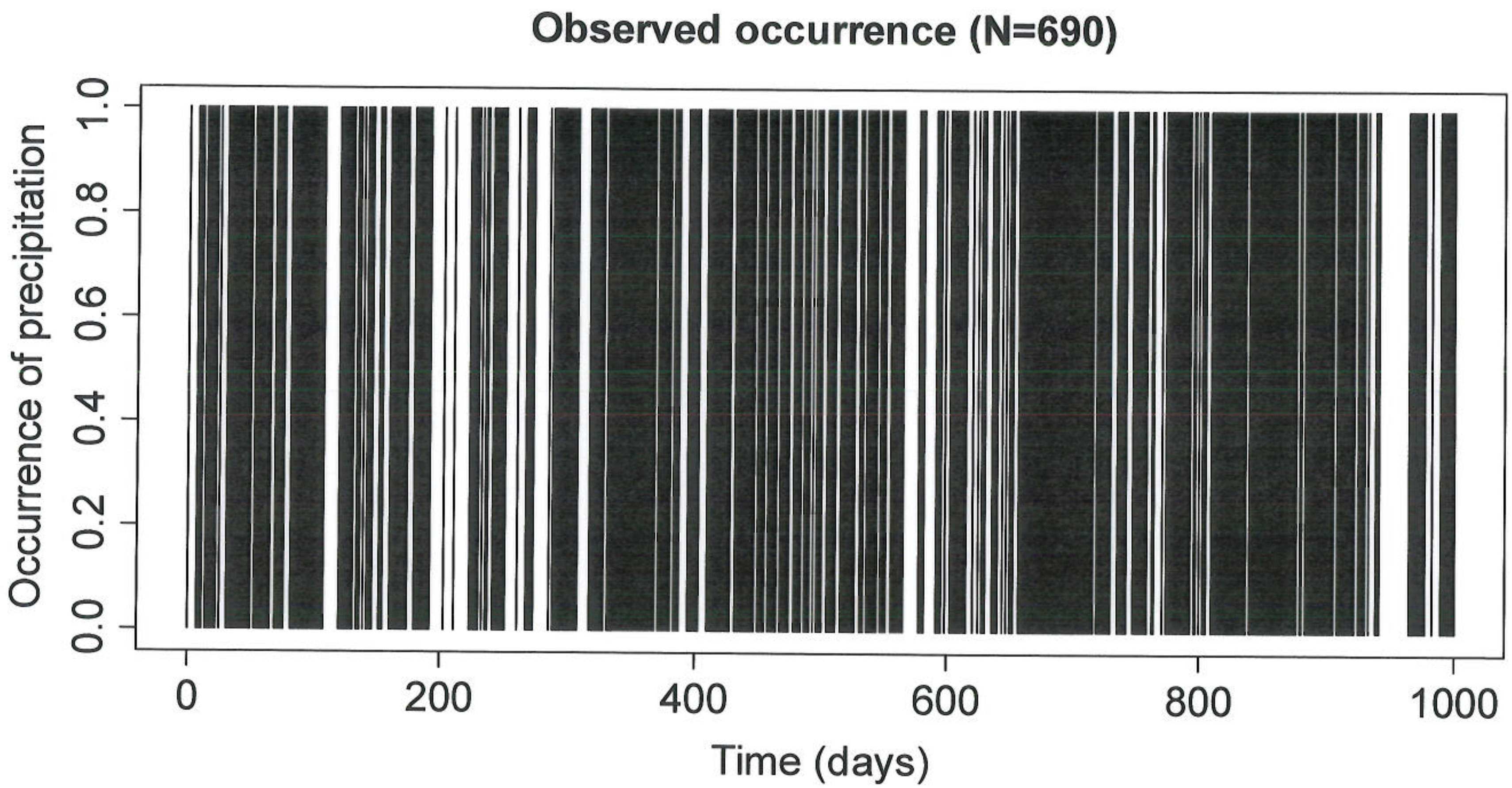
Observed Precipitation
Precipitation (mm)
0
10
20
30
40
50
0
200
400
600
800
1000
Time (days)

e.g.,

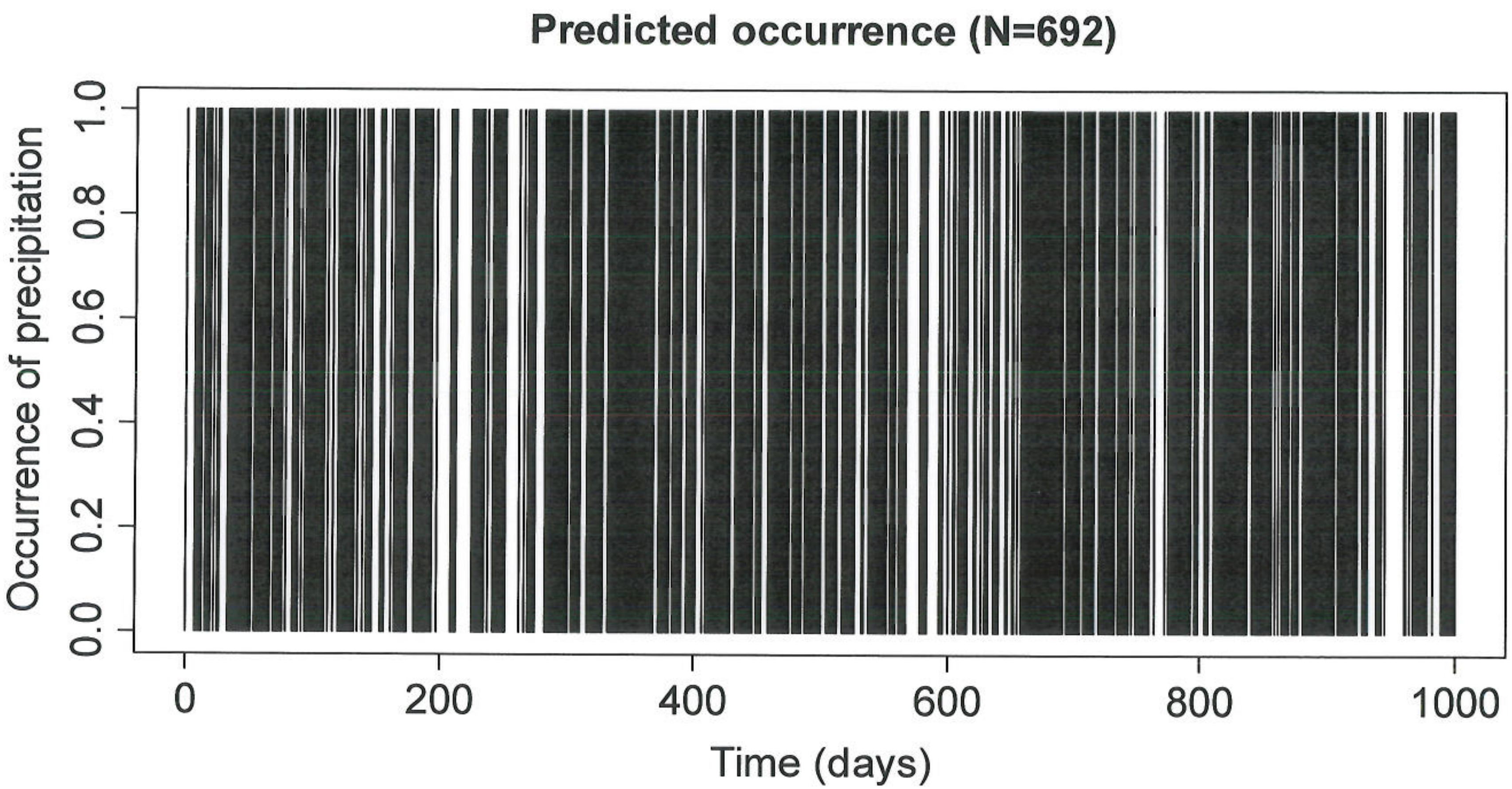


Conditional quantiles (z=0.05, 0.50, & 0.95)
Precipitation (mm)
0
10
20
30
40
50
0
200
400
600
800
1000
Time (days)

e.g. (cont.)


Observed occurrence (N=690)
Occurrence of precipitation
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000
Time (days)

e.g. (cont.)



Predicted occurrence (N=692)
Occurrence of precipitation
0.0
0.2
0.4
0.6
0.8
1.0
0
200
400
600
800
1000
Time (days)

3) Expanded Poisson-gamma Density Network

Expanded downscaling for multi-site precipitation in a probabilistic framework:

$$C_1 = \underbrace{-\sum_{t=1}^{N}\sum_{k=1}^{K} \ln P\left(y_k^{obs}(t) \mid \mathbf{x}(t); \mu(t), \phi(t), \rho(t)\right)}_{\text{Negative Log-Likelihood}} + \alpha \underbrace{\sum_{i=1}^{K}\sum_{j=i+1}^{K}\left(\mathrm{cov}(y_i, y_j) - \mathrm{cov}(y_i^{obs}, y_j^{obs})\right)^2}_{\text{Covariance constraint}}$$

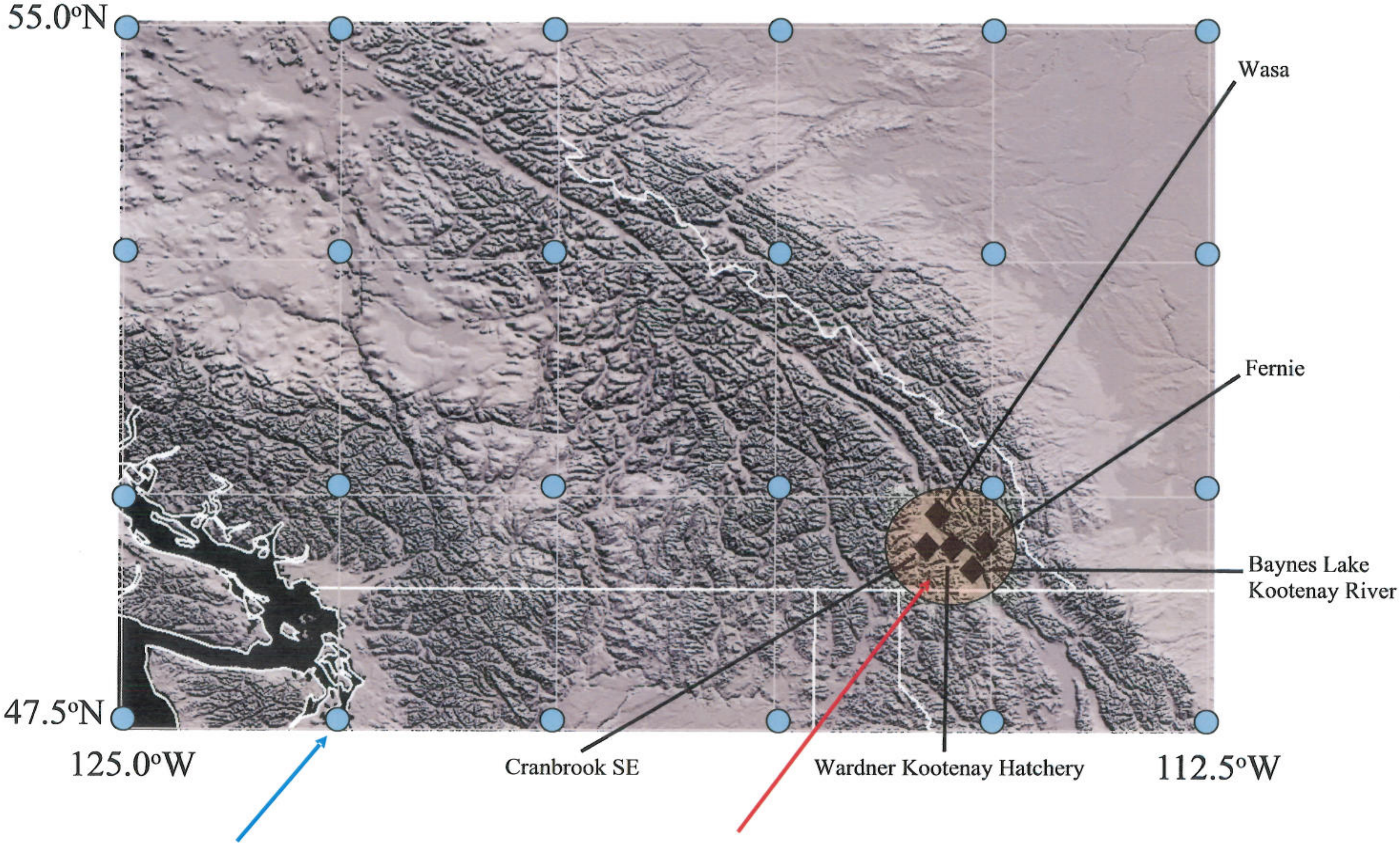
Negative Log-Likelihood *Covariance constraint*

$$C_2 = -\sum_{t=1}^{N}\sum_{k=1}^{K} \ln P\left(y_k^{obs}(t) \mid \mathbf{x}(t); \mu(t), \phi(t), \rho(t)\right) + \alpha_1 \sum_{i=1}^{K}\sum_{j=i+1}^{K}\left(\mathrm{cor}(\mu_i, \mu_j) - \mathrm{cor}(y_i^{obs}, y_j^{obs})\right)^2$$

$$+\alpha_2 \sum_{k=1}^{K}\left(\langle \phi_k \mu_k^{\rho_k} \rangle + \mathrm{var}(\mu_k) - \mathrm{var}(y_k^{obs})\right)^2 + \alpha_3 \sum_{k=1}^{K}\left(\mu_k - \langle y_k^{obs} \rangle\right)^2$$

Modified constraints required for computational reasons

Test – Daily Precipitation in Kootenay Region



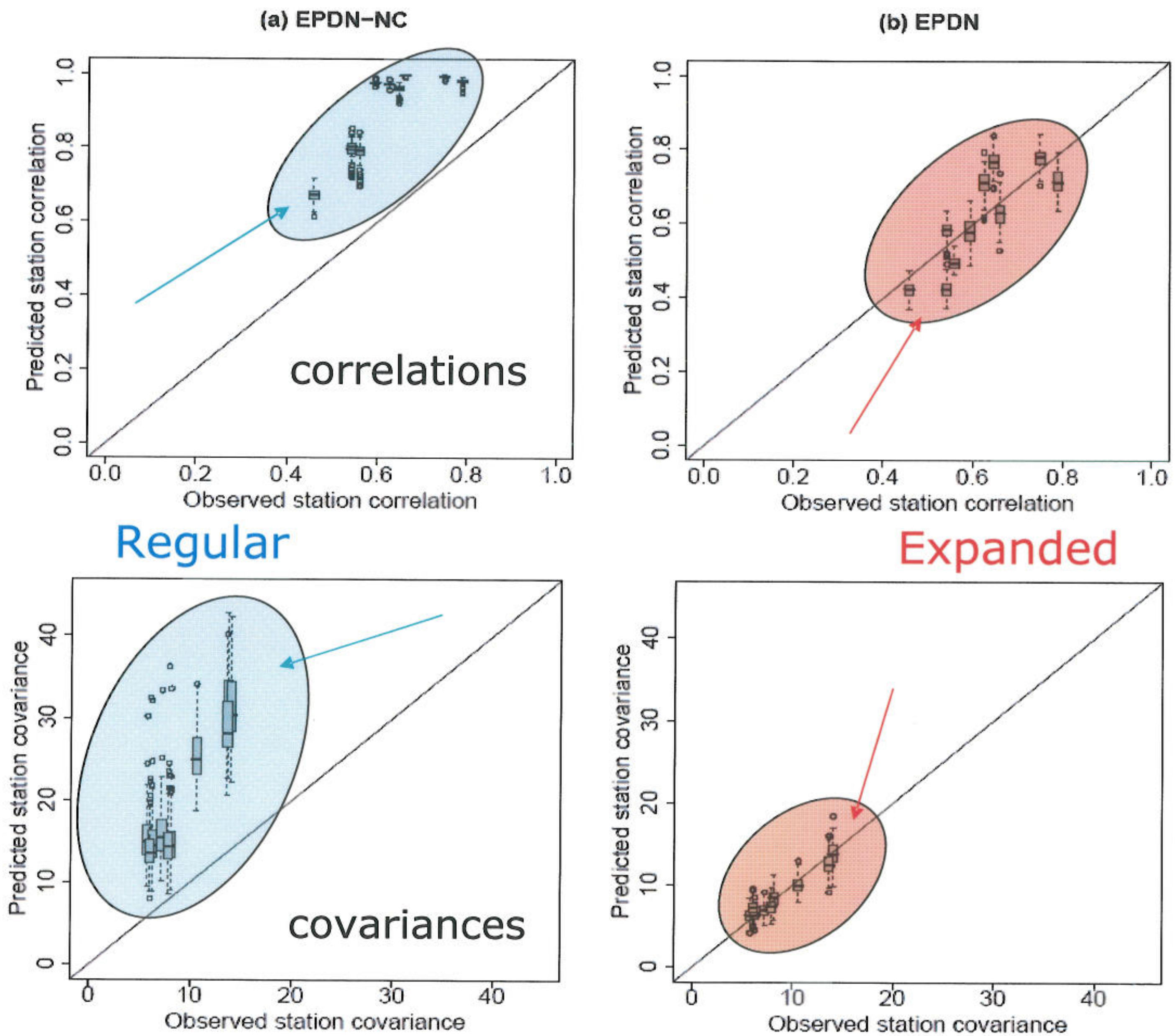
Model Calibration & Testing

1) Inputs → synoptic-scale surface circulation, mid-troposphere circulation, & low-level moisture fields; outputs → daily precipitation at five stations

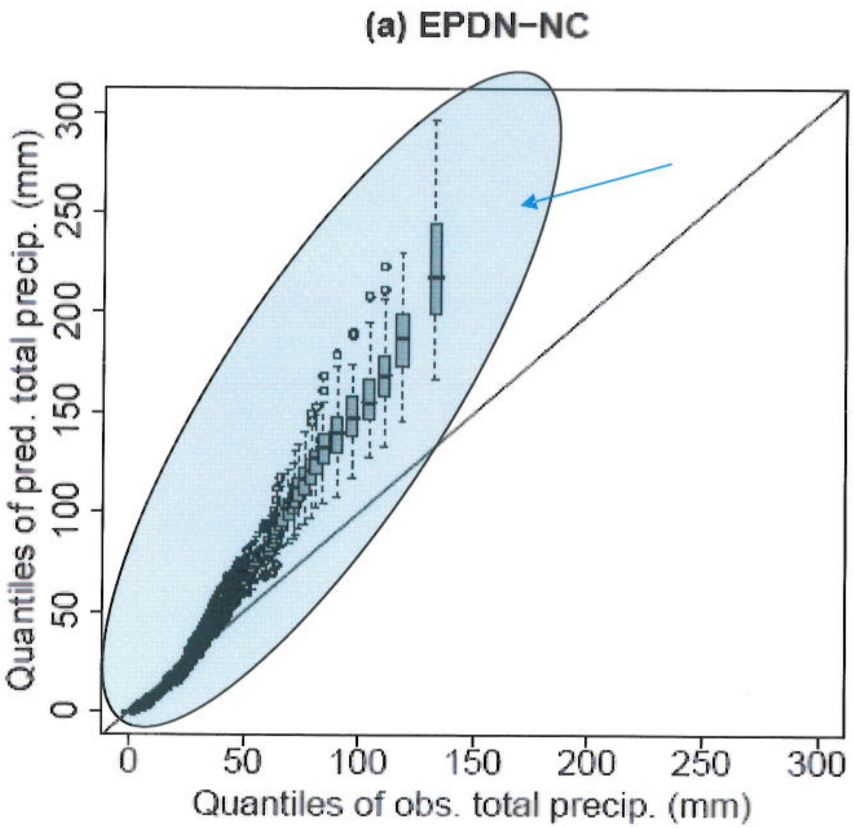
2) Expanded Poisson-gamma density network calibrated on 1985-1994 data & tested on 1995-2001 data

3) Comparison between "expanded" (results in **red**) and "regular" (results in **blue**) multi-site downscaling algorithms

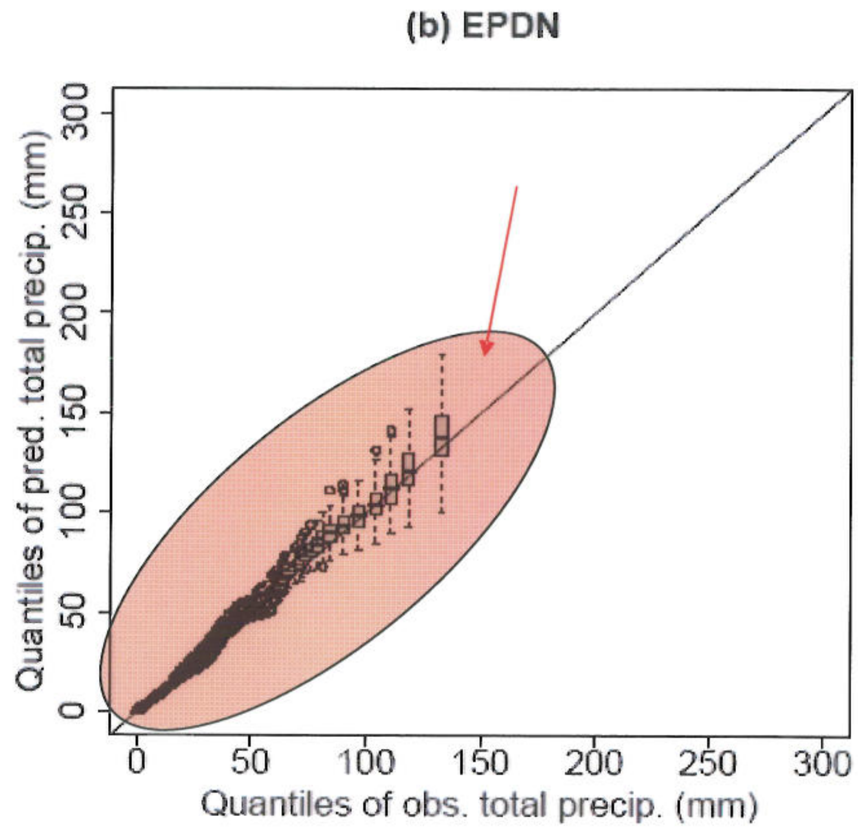
Test Results – Station Correlations & Covariances



Test Results – Quantile-Quantile Plot of Regional Precipitation



Regular



Expanded

Conclusions

1) Poisson-gamma density network can predict the conditional distribution of multi-site precipitation at a daily time step

2) Prediction parameters of the Poisson-gamma distribution allows precipitation occurrence and precipitation amounts to be modelled at the same time

3) Principles of expanded downscaling can be applied in a probabilistic modelling framework to allow realistic representation of spatial relationships between stations