insulator has only two possible values. This is easiest to understand by considering the surface of a crystal (Fig 1a). Surface states can exist within the bulk energy gap, and they disperse with momentum **k** in a two-dimensional Brillouin zone. According to Kramers' theorem, time-reversal symmetry requires that all states come in degenerate pairs, at **k** and –**k**. There are four special momenta, $\Gamma_{a=1-4}$, where **k** and –**k** coincide (Fig. 1b) — as well as **k** = 0, the periodicity of the Brillouin zone creates three additional points.

At $\mathbf{k} = \Gamma_a$, the surface states are doubly degenerate. Between any pair $\Gamma_{a,b}$, the degeneracy is lifted by spin–orbit interactions. As shown in Fig. 1c,d, there are two distinct ways in which the states can connect. In the trivial case (Fig. 1c), it is possible to eliminate the surface states by pushing all of the bound states out of the gap. Between Γ_a and Γ_b, the bands will intersect the Fermi energy an even number of times. In contrast, in Fig. 1d the edge states cannot be eliminated. The bands will intersect the Fermi energy an odd number of times — a number that cannot be zero. Which of these alternatives occurs is determined by the topological class of the bulk band structure. In a strong topological insulator, the Fermi surface for the surface states encloses an odd number of degeneracy points. This is impossible in an ordinary two-dimensional electron system. The surface of a topological insulator defines a new two-dimensional 'topological metal' phase.

In their photoemission work, Hsieh *et al.*[1] registered an odd number of surface bands crossing the Fermi energy between two degeneracy points (as in Fig. 1d), which establishes that $Bi_{1-x}Sb_x$ is a strong topological insulator. This observation opens the door to a variety of experiments for probing the electronic and magnetic properties of this new electronic state. A particularly tantalizing prospect is that the proximity effect between an ordinary superconductor and the surface states of a topological insulator leads to a state that supports non-abelian excitations[11], which could be used for fault-tolerant quantum computation[12]. Observation of such excitations would be a first step towards realizing this goal.

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HISTORY OF QUANTUM MECHANICS

The path to agreement

Werner Heisenberg's trip to Heligoland in June 1925 is a legend. Plagued by hay fever, the 23-year old escaped to the pollen-free island in the North Sea, to return with deep insight that would change the way we think about quantum mechanics. Electrons in atoms, he came to realize, do not move in sharp orbits with definite radii and periods of rotation. As a consequence, their motion should not be described by a coordinate that depends on time, but by an array of transition amplitudes. Heisenberg, Max Born and Pascual Jordan — Paul Dirac made independent contributions — expanded the approach into what would become known as the matrix-mechanics formulation of quantum mechanics.

Only a year later, Erwin Schrödinger (pictured) presented a different formalism: wave mechanics, which uses a vastly different mathematical language — differential equations rather than the algebraic approach of matrix mechanics. Already Schrödinger was considering the relation between his own theory and the quantum mechanics of Heisenberg, Born and Jordan. In 1926, in a paper originally published in *Annalen der Physik*, he presented arguments leading to the conclusion that the two so different approaches are indeed equivalent.

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But to what degree Schrödinger proved the equivalence between the two frameworks has been the subject of some recent debate — is it actually a 'myth' that Schrödinger established the equivalence of matrix mechanics and wave mechanics, that is, that they describe the same physics? Contributing to the discussion, Slobodan Perovic argues that providing a fully fledged general proof was never the goal of Schrödinger's paper (*Studies in History and Philosophy of Modern Physics* doi: 10.1016/j.shpsb.2008.01.004; 2008).

Rather, in Perovic's view, the case has to be discussed in a specific context — the context of Niels Bohr's model of the atom. Both matrix mechanics and wave mechanics were constructed against the background of Bohr's model, and Schrödinger's main goal was, according to Perovic, to establish the coherence of the two approaches with that model. This served, not least, to underline the significance of wave mechanics. Matrix mechanics, after all, had been more successful in explaining the spectral lines of the hydrogen atom — a fact that explains, in part, why Schrödinger focused on showing explicitly (and successfully) how matrices can be constructed from eigenfunctions, whereas he only sketches rather than proves the reciprocal equivalence.

The full proof of the mathematical equivalence of matrix mechanics and wave mechanics followed only a couple of years later, notably after the Copenhagen interpretation was framed. This influential interpretation of quantum mechanics is rooted in the equivalence of the two approaches — but an equivalence, Perovic argues, in the context of Bohr's model, rather than the full proof of the isomorphism of the mathematical frameworks underlying the approaches.

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