FRACTIONAL EXCHANGE STATISTICS

A home for anyon?

Exotic quasiparticles known as 'anyons' have intriguing fundamental and practical properties. A proposal for a solid-state structure in which anyons might 'live' provides fresh ideas for getting a practical handle on them.

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From a physicist's point of view, having two spatial dimensions is special: a pair of particles trading places behave very differently in two dimensions than they do in three. In three dimensions, any two sets of paths taken by two identical particles in the process of exchanging their positions can be continuously morphed into one another. But in two dimensions, particles can wind around each other in two distinct ways, clockwise or anticlockwise. A profound consequence of this observation for quantum mechanics is that in two dimensions, exchanging identical particles twice is not equivalent to leaving them alone. The particles' wavefunction after swapping places twice may differ from the original one; particles with such unusual exchange statistics are known as anyons. By contrast, in three dimensions, exchanging particles twice cannot change their wavefunction, leaving us with only two possibilities: bosons, whose wavefunction remains the same even after a single exchange, and fermions, whose exchange only changes the sign of their wavefunction. The theoretical possibility of particles taking 'any phase' on being interchanged is currently attracting interest, not least due to the realization that anyonic statistics of quasiparticles (especially of the 'non-abelian' kind, in which exchanges result in unitary rotations of state vectors in a degenerate Hilbert space) could be used to manipulate quantum information in a way that might lead to the building of a fault-tolerant 'topological' quantum computer[1] (for a review, see ref. 2). A number of practical systems that could support anyons have been discussed, but Conan Weeks and colleagues[3] (page 796 of this issue) propose a candidate — based on a superconducting film grown on top of a semiconductor that hosts a two-dimensional electron gas (2DEG) — whose properties set it apart from the other possibilities, and that provides fresh perspectives on the possibility of generating and manipulating anyons.

Frank Wilczek, who coined the term 'anyons', proposed[4] a physical picture to explain how their unusual exchange statistics may come about: imagine a situation in which an electric charge q and a magnetic flux tube Φ — such as the magnetic flux created by a solenoid — are bound together to form a 'dyon' (a hypothetical particle with both electric and magnetic charges). If one such particle is moved around another, the phase acquired by the wavefunction is $2q\Phi$ (twice the Aharonov–Bohm phase; in natural units). Wilczek argued that if either the charge of this dyon is a fraction of the electron charge e, or its magnetic flux is a fraction of the fundamental flux quantum $2\pi/e$ (or both), then the resulting phase may differ from a multiple of 2π, signalling anyonic statistics.

The most important feature captured by Wilczek's toy model is that anyonic statistics are inextricably connected with the phenomenon of fractionalization (that is, excitations in the system carry fractions of fundamental quantum numbers of its constituent particles). Fractional quantum Hall states — where the transverse electrical conductivity is quantized in non-integer multiples of e^2/h — are the prime example of such behaviour, and other condensed-matter systems (for instance, frustrated magnets and rotating Bose–Einstein condensates) have also been considered theoretically. The proposal by Weeks *et al.*[3] recalls Wilczek's toy model[4] in that it provides a physical implementation for creating a dyon binding fractionalized electric charge to fractionalized magnetic flux. What makes their system different from those considered previously is that fractionalization is achieved not as a result of strong correlations between the constituent particles, but rather through a combination of two weakly interacting systems with different properties — a vortex lattice in the superconducting layer, and electrons in the doped semiconducting layer.

The key to this construction[3] is the fact that a superconducting flux quantum is half of the fundamental flux quantum $2\pi/e$ (because the charge of a Cooper pair is twice that of an electron). Threading the flux generated by the superconducting vortex through a semiconducting layer in the integer quantum Hall state locally expels half of the electron charge by pushing it towards the boundary. Several assumptions are implicit here. For example, the precision of charge quantization is not enforced by any intrinsic energy scales associated with strong correlations — unlike in fractional quantum Hall systems — but rather by the strength of coupling between the two subsystems. Excitations carrying half a fundamental flux are not eigenstates in the integer quantum Hall regime; a superconducting layer must 'force' them in. Also, the notion of any exchange statistics has a meaning only for quantum particles. Therefore, the resulting dyonic objects should be truly quantum before they can be called anyons. Although electrons in the 2DEG considered by Weeks *et al.*[3] are indeed quantum objects with long dephasing lengths and times, the superconducting vortices are not necessarily so. These issues must be overcome experimentally before the proposal can be put to test.

Whether or not such a test proves successful, their idea may have an interesting offshoot: it could pave the way for manipulating quasiparticles in the fractional quantum Hall state with filling fraction 5/2, which is the leading candidate for a system with non-abelian anyons[2,5] — anyons that would be of particular value with a view to fault-tolerant topological quantum computing. Unlike the integer quantum Hall state, the 5/2 fractional quantum Hall state should support excitations with flux 1/2 and charge 1/4. Using superconducting vortices to bind such excitations could provide a 'handle' with which to manipulate anyons in this system. An ability to control anyons is a prerequisite for building, eventually, a topological quantum computer, so any new ideas as well as experimental developments in this direction are welcome.

References

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