

it. Instead, the interface locally deforms into sharp folds, whose size decreases rapidly with the strength of the stirring⁶. These singularities of the surface eventually become unstable, providing the channels through which air can enter⁷. Unless all scales down to that of a micrometre are properly modelled, there is no hope of describing air entrainment even qualitatively.

Two ideas have proved their worth in the quest for a deeper mathematical understanding of these multiscaled phenomena. First, the observation that there is a rather generic tendency of hydrodynamics to generate small-scale motion naturally couples small and large scales. Mathematically, this tendency is best captured in the limit that the formation of small scales continues with no end, so the underlying equations form a singularity^{8,9}. The description of singularities is aided by the fact that they are self-similar, meaning that their structure remains the same under a change of length scale.

Second, it still remains to be established how the largest and the

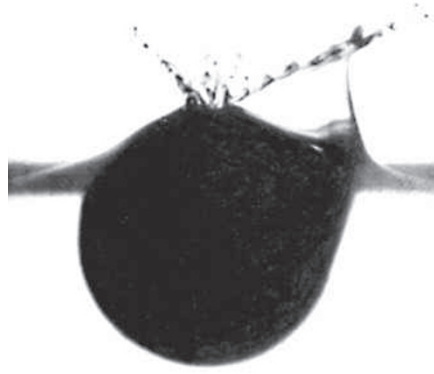


Figure 2 Wonky splash. A sphere of polished serpentine (2.57 cm in diameter), dusted on one side, falling into water from a height of 14 cm. Despite the perfect symmetry of the impact, the splash is plainly asymmetric. Image reproduced from ref. 2.

smallest features (the impacting sphere and the molecular interactions in the splashing problem) are to be joined together. Problems of this nature are captured by the mathematical technique

of ‘matched asymptotic expansions’^{10,11}, which describes the rules by which two functions characterizing two different length scales can be made compatible. In general this will be impossible, unless parameters of both parts of the solution are ‘just right’. In other words, though separated by many orders of magnitude in scale, the microscopic and the macroscopic part of the solution are intimately related. The reality of more and more of today’s problems requires us to combine the very large and the very small. Fortunately, some powerful mathematical tools open avenues for significant progress.

References

1. Duez, C. *et al. Nature Phys.* **3**, 180–183 (2007).
2. Worthington, A. M. *A Study of Splashes* (Longmans, London, 1908).
3. Pippard, B. *Notes Rec. R. Soc. London* **56**, 63–81 (2002).
4. Quéré, D. *Rep. Prog. Phys.* **68**, 2495–2532 (2005).
5. Davidson, P. *Turbulence* (Oxford Univ. Press, Oxford, 2004).
6. Jeong, J.-T. & Moffatt, H. K. *J. Fluid Mech.* **241**, 1–22 (1992).
7. Lorenceau, E., Quéré, D. & Eggers, J. *Phys. Rev. Lett.* **93**, 254501 (2004).
8. Kadanoff, L. P. *Phys. Today* **50**, 11–13 (1997).
9. Berry, M. V. *Phys. Today* **55**, 10–11 (2002).
10. Berry, M. V. *Philos. Sci.* **9**, 597–607 (1994).
11. Bender, C. & Orszag, S. *Advanced Mathematical Methods for Scientists and Engineers* (McGraw-Hill, New York, 1978).

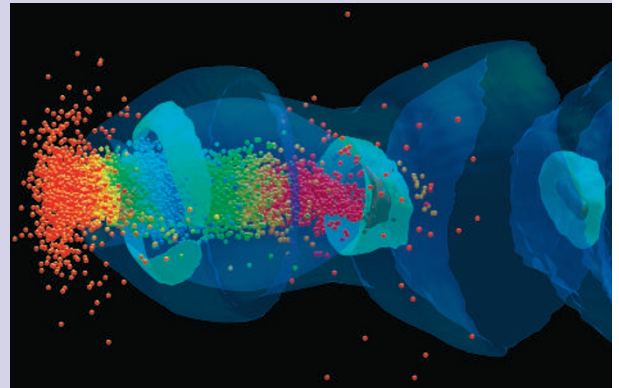
WAKEFIELD ACCELERATORS

Hybrid particle drive

The electric field within any given section of a conventional particle accelerator is limited by the breakdown field of the materials from which it is made. Consequently, getting to the relativistic energies where interesting particle physics takes place requires multiple accelerator stages spanning many kilometres. The extreme fields generated when a high-power laser is focused into a plasma have been suggested as a way of accelerating particles in a much smaller space, and potentially with less expense. On their own, accelerators that operate on this principle — known as plasma wakefield accelerators — can generate electron beams with energies exceeding 1 GeV, in a plasma just a few centimetres long. This is, of course, far below the energy demanded by particle physicists. But by using wakefield accelerator techniques in tandem with the conventional linear accelerator at the Stanford

Linear Accelerator Centre, Ian Blumenfeld and colleagues show they can use them to double the energy of electrons from a 42 GeV electron beam over a distance of less than a metre (*Nature* **445**, 741–744; 2007).

Plasma wakefield accelerators typically operate by focusing a high-power laser pulse on a tight spot in some medium, which can be a gas, liquid or solid. This creates a fully ionized plasma and drives freed electrons through the plasma at close to the speed of light. It is the wake left behind by this burst of ultra-relativistic electrons that generates the large fields used for particle acceleration. But a high-power laser is not the only means to drive electrons through a plasma to produce such a wake, a fact that Blumenfeld *et al.* clearly demonstrate by focusing their conventionally accelerated electron beam into an 85-cm-long column of lithium vapour. When the beam interacts with



Particle-in-cell simulation of the wake generated in a lithium plasma by a 42 GeV electron beam.

the vapour it transfers most of its energy to the resulting plasma, and the wake it produces accelerates a proportion of electrons in the tail of the beam to twice their original energy (see ‘particle in cell’ simulation pictured). Simulations of this interaction confirm an implied accelerating field of 53 GV m⁻¹ — a thousand times greater than can be achieved in a typical linear accelerator stage.

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